4 Standard Model Physics

In the past twenty years we have witnessed two important successes of the Standard Model: the matching between the predicted top mass and its actual value measured at the Tevatron [38–40] (Fig. 7) and the indication of a light Higgs in the mass region where it has been observed at LHC [41–43] (Fig. 8). With the Higgs discovery electroweak precision measurements are becoming a crucial test of our "standard model" and have the potential of developing into important tools for indirect information on new physics. Another success is the amazing capability of describing, often within small uncertainties, the wealth of collider data.

In the present chapter we regard Standard Model physics as a major tool for discovering new physics. The chapter starts with a discussion on Higgs physics, which is central for the experimental programme at high energy accelerators in the next decades. Then a brief report on the status and prospects of vector bosons physics follows, including a description of key points related to our understanding of strong interactions. Furthermore, because of their relevance for the present discussion, top-quark physics prospects are presented. Perspectives in the field of theoretical calculations and Monte Carlo generators, which are essential to analyse the data and interpret the measurements, conclude the chapter.

![Fig. 7: The bands show the indirect determination of the top mass from electroweak fits as a function of time.](image)

The light blue band and green band are without and with the information coming from the Higgs boson mass, respectively. The points represents the combinations of direct measurements of the top mass in various years. Courtesy of Roman Kogler.

4.1 Higgs Properties and Couplings

The discovery of a scalar boson at the LHC [42, 43] has started up a new phase in the experimental exploration of the electroweak symmetry-breaking (EWSB) mechanism of the Standard Model (SM).
The bands show the indirect determination of the Higgs boson mass from electroweak fits as a function of time. The light blue bands and green band are without and with the information coming from direct Higgs searches, respectively. The grey band shows the 90% CL interval (without direct searches). The points with error bars represent the combinations of direct measurements of the Higgs boson mass in various years. Courtesy of Roman Kogler.

The observed resonance is, within present experimental errors, well compatible with a minimal structure of the Higgs sector. Nevertheless, the determination of the different properties of the new particle with increasing precision is expected to be a powerful tool to explore what could be beyond the SM description of fundamental interactions. In particular, on the one hand, deviations in the scalar boson couplings to the electroweak vector bosons $V = W, Z$ would require further degrees of freedom to keep the $VV$ scattering unitary. On the other hand, anomalies in the Yukawa couplings to matter fields could possibly point to a non-standard mechanism for the generation of quark and charged-lepton masses.

The key role of the Higgs sector in elementary particle physics makes it an important goal to understand the properties of the LHC resonance as accurately as possible [44,45] and to clarify the real nature of the newly discovered Higgs-boson particle, in particular relevant questions are

- is the new boson really the SM Higgs boson?
- is it an elementary or a composite particle?
- is it the only one, or are there other Higgs fields?
- is it natural?
- is it really responsible for the masses of all elementary particles?
- is it at the origin of the matter-antimatter asymmetry?
- is it responsible for the inflationary expansion of the Universe?
The size of the deviation in the Higgs couplings to SM particles depends in general on the energy scale where the New Physics becomes relevant. For instance, in the MSSM (with tan β = 5), one expects a deviation in the Yukawa couplings of the heaviest down-type fermions of the order \( g_{hbb}^{SM} / g_{hbb} \sim 1 + 1.7\% (1 \text{ TeV} / m_A)^2 \) (where \( m_A \) is a heavy pseudoscalar mass), while, in minimal composite Higgs models, the Higgs coupling to vector bosons is affected according to \( g_{hVV} / g_{hVV}^{SM} \sim 1 - 3\% (1 \text{ TeV} / f)^2 \), (where \( f \) is the composite scale) [46]. The solution of the SM hierarchy problem, which in general implies the NP scale to be of the order of 1 TeV, entails then coupling deviations of at most a few percents. Hence, a percent-level accuracy on Higgs couplings is needed in order to pinpoint relevant NP effects in the Higgs precision measurements.

A list of relevant measurements discussed in this section is

- Precision in measuring couplings
- Rare and “invisible” decay modes
- Search for CP mixing in the Higgs sector
- New trends to measure natural width.

### 4.1.1 Precision coupling measurements

As already stated, the recently discovered “125 GeV” Higgs particle by ATLAS and CMS at LHC, is compatible within current experimental and theoretical uncertainties with the properties of the Higgs boson predicted by the Standard Model. It is of paramount importance to clarify the nature of this new object and its role in EWSB. Deviations of physics properties from those predicted in the Standard Model would unambiguously indicate new physics beyond this theory. Hence, the precise measurement of this particle’s properties represent a major goal of future experiments.

The results presented here are relative to what was presented at the ECFA LC2013 Workshop (May 2013), the ECFA HL-LHC Workshops (October 2013 and October 2014) and the kick-off Workshop on FCC (February 2014).

Following the approach and benchmarks recommended in [44], measurements of couplings are currently implemented using a leading-order tree-level motivated framework. Other frameworks, based on SM Effective Field Theory (EFT), are currently in development and are discussed in [47]. The present framework is based on the assumption that the width of the Higgs boson is narrow, justifying the use of the narrow-width approximation. Hence the predicted rate for a given channel can be decomposed in the following way:

\[
\sigma \cdot BR (i \rightarrow H \rightarrow f) = \frac{\sigma_i \cdot \Gamma_f}{\Gamma_H} \tag{1}
\]

where \( \sigma_i \) is the production cross section through the initial state \( i \), \( BR \) and \( \Gamma_f \) are the branching ratio and partial decay width into the final state \( f \), respectively, and \( \Gamma_H \) the total width of the Higgs boson.

The coupling scale factors \( \kappa_i \) associated with the SM particle \( j \) are defined in such a way that the cross sections \( \sigma_j \) and the partial decay widths \( \Gamma_j \) scale with \( \kappa_j^2 \) compared to the SM prediction (i.e., \( \kappa_i \equiv g_{hii} / g_{hii}^{SM} \), where \( g_{hii} \) represents the coupling). Table 7 shows the precision on Higgs boson coupling \( \kappa_i \) for several elementary particles that can be achieved by ATLAS and CMS independently, assuming 300 fb\(^{-1}\) of data at LHC, and 3000 fb\(^{-1}\) of data at HL-LHC. The necessary reduction of theory uncertainties which is required to have an impact of less than 10% on the total uncertainty, at LHC and HL-LHC, it is shown in Table 8. The factors \( \lambda_{ij} \) express the ratio of coupling scale factors, so that for example \( \lambda_{\tau Z} = \kappa_{\tau} / \kappa_Z \); the \( \kappa_{gZ} \) parameter is defined as \( \kappa_g \cdot \kappa_Z / \kappa_H \), where \( \kappa_H^2 \) is the scaling factor with respect the total Higgs boson width.
Table 7: Precision on the measurements of $\kappa_\gamma$, $\kappa_W$, $\kappa_Z$, $\kappa_\mu$, $\kappa_t$, $\kappa_{\gamma\gamma}$, and $\kappa_\mu$. These values are obtained at $\sqrt{s} = 14$ TeV using an integrated dataset of 300 fb$^{-1}$ at LHC, and 3000 fb$^{-1}$ at HL-LHC. Numbers in brackets are % uncertainties on the measurements estimated under [no theory uncertainty, current theory uncertainty] for ATLAS and [optimistic, pessimistic] systematic experimental and theoretical uncertainties for CMS.

<table>
<thead>
<tr>
<th>$L$ (fb$^{-1}$)</th>
<th>Exp.</th>
<th>$\kappa_\gamma$</th>
<th>$\kappa_W$</th>
<th>$\kappa_Z$</th>
<th>$\kappa_\mu$</th>
<th>$\kappa_t$</th>
<th>$\kappa_{\gamma\gamma}$</th>
<th>$\kappa_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 ATLAS</td>
<td>[9, 9]</td>
<td>[9, 9]</td>
<td>[8, 8]</td>
<td>[11, 14]</td>
<td>[22, 23]</td>
<td>[20, 22]</td>
<td>[13, 14]</td>
<td>[24, 24]</td>
</tr>
<tr>
<td>3000 ATLAS</td>
<td>[4, 5]</td>
<td>[4, 5]</td>
<td>[4, 4]</td>
<td>[5, 9]</td>
<td>[10, 12]</td>
<td>[8, 11]</td>
<td>[9, 10]</td>
<td>[14, 14]</td>
</tr>
<tr>
<td>CMS</td>
<td>[2, 5]</td>
<td>[2, 5]</td>
<td>[2, 4]</td>
<td>[3, 5]</td>
<td>[4, 7]</td>
<td>[7, 10]</td>
<td>[2, 5]</td>
<td>[10, 12]</td>
</tr>
</tbody>
</table>

Table 8: Estimation of the deduced size of theory uncertainties, in percent (%), for different Higgs coupling measurements, requiring that each source of theory systematic uncertainty affects the measurement by less than 30% of the total experimental uncertainty and hence increase the total uncertainty by less than 10%. A dash "-" indicates that the theory uncertainty from existing calculations [44,48,49] is already sufficiently small to fulfill the condition above for some measurements. The same applies to theory uncertainties not mentioned in the table for any measurement. The impact of the jet-bin and $p_T$ related uncertainties in $gg \to H$ depend on analysis selections and hence no single number can be quoted. Therefore the range of uncertainty values used in the different analysis is shown. (MHOU stands for missing higher-order uncertainties. Note that these uncertainties have been recently further reduced thanks to N3LO calculations [50].)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Status</th>
<th>Deduced size of uncertainty to increase total uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>by $\leq 10%$ for 300 fb$^{-1}$</td>
</tr>
<tr>
<td>Theory uncertainty (%)</td>
<td>2014</td>
<td>$\kappa_{\gamma Z}$</td>
</tr>
<tr>
<td>$gg \to H$</td>
<td>[44,48,49]</td>
<td>8</td>
</tr>
<tr>
<td>PDF</td>
<td>incl. QCD scale (MHOU)</td>
<td>7</td>
</tr>
<tr>
<td>$p_T$ shape and 0j $\to$ 1j mig.</td>
<td>10–20</td>
<td>-</td>
</tr>
<tr>
<td>1j $\to$ 2j mig.</td>
<td>13–28</td>
<td>-</td>
</tr>
<tr>
<td>1j $\to$ VBF 2j mig.</td>
<td>18–58</td>
<td>-</td>
</tr>
<tr>
<td>VBF 2j $\to$ VBF 3j mig.</td>
<td>12–38</td>
<td>-</td>
</tr>
<tr>
<td>VBF PDF</td>
<td>3.3</td>
<td>-</td>
</tr>
<tr>
<td>$t\bar{t}H$</td>
<td>PDF incl. QCD scale (MHOU)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-</td>
</tr>
</tbody>
</table>

In the fit [51,52], it is assumed that no particles other than those in SM are contributing to NLO electroweak loops, nor decay modes contribute to the full width of this particle other than those predicted by SM. As it can be seen, depending on the type of coupling, an accuracy of a few % can be achieved for bosons, and of about 5% for fermions, by each of the two experiments. Ratio of couplings allow a test free from assumptions on the total width; the findings are reported in Fig. 9 for ATLAS [51]; similar results are expected by CMS.

The precision on couplings can considerably increase at ILC, as pictorially shown in Fig. 10. Colliders such as ILC or FCC-ee are indeed Higgs-boson factories in a very clean environment. The dominant Higgs production cross sections are shown in Fig. 11 (left plot). They correspond to the associated production $e^+ e^- \to Z H$ (dominant at 250 GeV), and to the vector-boson-fusion (VBF) scattering $e^+ e^- \to H \nu \nu$, $H e^+ e^-$ (that gets the upper hand at $\sqrt{s} > 450$ GeV). At $\sqrt{s} \simeq 250$ GeV, a statistics of $8 \times 10^4$ Higgs bosons will be collected in a first stage ($3.7 \times 10^5$ after full program with 1.15 ab$^{-1}$). At
ATLAS Simulation Preliminary
\( \sqrt{s} = 14 \) TeV: \( \mathcal{L}_{\text{det}} = 300 \) fb\(^{-1} \); \( \mathcal{L}_{\text{det}} = 3000 \) fb\(^{-1} \)

\[
\begin{array}{c}
\kappa_{Z} \\
\lambda_{WZ} \\
\lambda_{W} \\
\lambda_{Z} \\
\lambda_{ZZ} \\
\lambda_{\mu Z} \\
\lambda_{\phi Z} \\
\lambda_{\gamma Z} \\
\lambda_{\tau Z} \\
\lambda_{\Lambda Z}
\end{array}
\]

\[\Delta \lambda_{XY} = \Delta \left( \frac{\kappa}{\kappa_{Y}} \right)\]

**Fig. 9:** Relative uncertainty on the expected precision for the determination of coupling scale factor ratios \( \lambda_{XY} \) in a generic fit without assumptions on the total Higgs boson width [51], assuming a SM Higgs Boson with a mass of 125 GeV and LHC at 14 TeV, 3000 fb\(^{-1} \). The \( \kappa_{gZ} \) parameter represents the ratio \( \frac{\kappa_{gZ}}{\kappa_{H}} \), where \( \kappa_{H} \) is the scaling factor with respect the total Higgs boson width. The hashed areas indicate the increase of the estimated error due to current theory systematics uncertainties.

\( \sqrt{s} \simeq 500 \) GeV (where the lower total cross section is more than compensated by the larger integrated luminosity), an additional statistics of \( 1.2 \times 10^{5} \) will be collected in a second stage \( (4 \times 10^{5} \) after full program with 1.6 ab\(^{-1} \), with a dominant contribution from VBF.

The associated production \( e^{+}e^{-} \rightarrow ZH \) is the key process that allows \( e^{+}e^{-} \) colliders to make a model-independent measurement of the Higgs couplings (unaffected by assumptions on the production mechanism), which is not attainable at hadron colliders. Thanks to the excellent signal-to-background (S/B) ratio, by applying the four-momentum conservation to the two-body \( ZH \) final state, one can indeed reconstruct the Higgs-system production properties just through the observation of its recoil \( Z \) system, independently from the Higgs decay mode. Notably any possible Higgs invisible decay would contribute to this measurement just as any visible Higgs decay. The outcome of these features is that one can make an absolute (that is independent from the Higgs decay BR’s) determination of \( g_{HZZ} \), the Higgs coupling to the \( Z \). Indeed, the Higgs production cross section \( \sigma(ZH) \sim g_{HZZ}^{2} \) can be measured in a model-independent way through the normalization of the \( Z \) recoil-mass distribution with no assumption on the Higgs interaction with other particles. In Fig. 11 (right plot), the recoil-mass distribution for the process \( e^{+}e^{-} \rightarrow ZH \rightarrow \mu\mu X \) is shown as obtained by a full detector simulation [53]. A precise measurement of the Higgs mass \( (\Delta m_{H} \lesssim 100 \) MeV) can also be obtained from the shape of the distribution.

32
An absolute measurement of Higgs BRs for all possible decay channels (including invisible and exotic decays, as well as SM decays which are overwhelmed by background at the LHC, like $H \rightarrow jets$) can then be made by tagging different Higgs final states, thus obtaining a model-independent determination of the quantities $\sigma(ZH) \cdot BR(H \rightarrow ii)$ for different Higgs decays, and hence an absolute measurements of $BR(H \rightarrow ii)$ (here $i$ stands for any boson or fermion coupled to $H$). The latter in turn can be combined with the direct coupling measurement from the production cross section in order to obtain a direct (model independent) determination of the Higgs total width. For instance, starting from the $H \rightarrow ZZ$ branching ratio, one has $\Gamma_H = \Gamma(H \rightarrow ZZ)/BR(H \rightarrow ZZ) \propto \sigma(ZH)/BR(H \rightarrow ZZ)$.

Similarly, at larger collision energies, one can use the $e^+e^- \rightarrow H\nu\nu$ cross section to get a different
Table 9: Model-Independent (top) and Model-Dependent (bottom) precision on Higgs-boson couplings [5]. For the latter, the fitting technique most closely matches that used at the LHC, and no non-SM production or decay modes are assumed.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>$L$ (ab$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ILC(250)</td>
</tr>
<tr>
<td>$\gamma\gamma$</td>
<td>0.25</td>
</tr>
<tr>
<td>$gg$</td>
<td></td>
</tr>
<tr>
<td>$WW$</td>
<td></td>
</tr>
<tr>
<td>$ZZ$</td>
<td>1.3%</td>
</tr>
<tr>
<td>$t\bar{t}$</td>
<td>14%</td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>1.6%</td>
</tr>
<tr>
<td>$\tau^{+}\tau^{-}$</td>
<td>2.3%</td>
</tr>
<tr>
<td>$e\nu$</td>
<td>2.8%</td>
</tr>
<tr>
<td>$\mu^{+}\mu^{-}$</td>
<td>91%</td>
</tr>
<tr>
<td>$\Gamma_V(h)$</td>
<td>4.9%</td>
</tr>
<tr>
<td>$bhb$</td>
<td>8%</td>
</tr>
<tr>
<td>BR(invis.)</td>
<td>&lt;0.9%</td>
</tr>
</tbody>
</table>

The final precision presently expected for the Higgs couplings at the ILC is reported in Table 9, as summarized in [5], for the various ILC staging scenarios. These results assume ($e^{-}, e^{+}$) polarizations of (-0.8, 0.3) at 250 and 500 GeV, and (-0.8, 0.2) at 1 TeV, plus a 0.5% theory uncertainty. A comparison with the HL-LHC potential is presented in Fig. 12. The top part of Table 9 refers to a model-independent fit, with no assumption on or between $g_{HW}$ and $g_{HZZ}$, nor on the saturation of the total width by invisible decays. It requires the measurement of the recoil $HZ$ process at low energies. For BR($H \rightarrow invis.)$, the numbers quoted are the 95% confidence upper limits on the branching ratio. Note that the measurement of the $g_{Ht}$ and of the trilinear $g_{HHH}$ coupling requires $\sqrt{s} \approx 500$ GeV. The corresponding accuracies presented in Table 9 are quite preliminary, and considerable improvements are foreseen in the ongoing analyzes [54].
Fig. 12: Model-Dependent (left) and Model-Independent (right) projected precision on Higgs-boson coupling scaling factors. Here $\kappa_i \equiv g_{hii}/g_{hii}^{(SM)}$. Green bands refer to HL-LHC projections; blue bands refer to combinations of ILC and HL-LHC outputs.

Table 10: Expected precisions on the Higgs coupling scaling factors from a constrained 7-parameter fit assuming no non-SM production or decay modes. The fit assumes generation universality [46]. The range of values shown for LHC and HL-LHC corresponds to a conservative scenario (current theory uncertainties) and an optimistic scenario (theory uncertainties scaled by a factor 1/2).

<table>
<thead>
<tr>
<th>Facility</th>
<th>LHC</th>
<th>HL-LHC</th>
<th>ILC500</th>
<th>ILC500-up</th>
<th>ILC1000</th>
<th>ILC1000-up</th>
<th>CLIC</th>
<th>TLEP (4 IPs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s}$ (GeV)</td>
<td>14,000</td>
<td>14,000</td>
<td>250/500</td>
<td>250/500</td>
<td>250/500/1000</td>
<td>250/500/1000</td>
<td>350/1400/3000</td>
<td>240/350</td>
</tr>
<tr>
<td>$\int L , dt$ (fb$^{-1}$)</td>
<td>300/expt</td>
<td>3000/expt</td>
<td>250+500</td>
<td>1500+1600</td>
<td>250+500+1000</td>
<td>1500+1600+2500</td>
<td>500+1500+2000</td>
<td>10,000+2600</td>
</tr>
<tr>
<td>$\kappa_\gamma$</td>
<td>5 - 7%</td>
<td>2 - 5%</td>
<td>8.3%</td>
<td>4.4%</td>
<td>3.8%</td>
<td>2.3%</td>
<td>$-5.5 &lt; \kappa_\gamma &lt; 5.5$</td>
<td>1.45%</td>
</tr>
<tr>
<td>$\kappa_\phi$</td>
<td>6 - 8%</td>
<td>3 - 5%</td>
<td>2.6%</td>
<td>1.1%</td>
<td>1.1%</td>
<td>0.67%</td>
<td>3.6/0.79/0.56%</td>
<td>0.79%</td>
</tr>
<tr>
<td>$\kappa_W$</td>
<td>4 - 6%</td>
<td>2 - 5%</td>
<td>0.39%</td>
<td>0.21%</td>
<td>0.21%</td>
<td>0.2%</td>
<td>1.5/0.15/0.11%</td>
<td>0.10%</td>
</tr>
<tr>
<td>$\kappa_Z$</td>
<td>4 - 6%</td>
<td>2 - 4%</td>
<td>0.49%</td>
<td>0.24%</td>
<td>0.50%</td>
<td>0.3%</td>
<td>0.49/0.33/0.24%</td>
<td>0.05%</td>
</tr>
<tr>
<td>$\kappa_t$</td>
<td>6 - 8%</td>
<td>2 - 5%</td>
<td>1.5%</td>
<td>0.98%</td>
<td>1.3%</td>
<td>0.72%</td>
<td>3.5/1.4/0.13%</td>
<td>0.51%</td>
</tr>
<tr>
<td>$\kappa_d = \kappa_b$</td>
<td>10 - 13%</td>
<td>4 - 7%</td>
<td>0.93%</td>
<td>0.60%</td>
<td>0.51%</td>
<td>0.4%</td>
<td>1.7/0.32/0.19%</td>
<td>0.39%</td>
</tr>
<tr>
<td>$\kappa_u = \kappa_t$</td>
<td>14 - 15%</td>
<td>7 - 10%</td>
<td>2.5%</td>
<td>1.3%</td>
<td>1.3%</td>
<td>0.9%</td>
<td>3.1/1.0/0.7%</td>
<td>0.69%</td>
</tr>
</tbody>
</table>

ee) prospects on Higgs-coupling accuracies [46]. CLIC numbers assume polarizations of (0.8, 0) for energies above 1 TeV. TLEP (FCC-ee) numbers assume unpolarized beams.

Additional information on the TLEP (FCC-ee) potential is given in Table 11. The prospects shown in the table assume $\sqrt{s} = 350$ GeV, four experiments and 2600 fb$^{-1}$ of data each (plus a data sample of 10000 fb$^{-1}$ taken at 240 GeV), and negligible theoretical uncertainties with respect the experimental ones. It is expected to measure the Higgs couplings to bosons with less than 1% uncertainty, while couplings to fermions should be measured with an uncertainty from about 0.4% ($Hb\bar{b}$) to 13% ($Ht\bar{t}$) [55].
Table 11: Relative statistical uncertainty on the Higgs boson couplings, as expected from the physics programme at $\sqrt{s} = 240$ and 350 GeV at TLEP. (The first column indicates the expected precision at TLEP when the sole 240 GeV data are considered. The substantial improvement with the inclusion of the 350 GeV data – in the second column – mostly stems from the precise total Higgs boson width measurement, which constrains all couplings simultaneously.) The numbers between brackets indicates the uncertainties expected with two detectors instead of four. For illustration, the uncertainties expected from the ILC baseline programme at 250 and 350 GeV are also given. The first three columns give the results of a truly model-independent fit, while the last two include the two assumptions made in Ref. [56] on the $W/Z$ couplings and on the exotic decays, for completeness and easier comparison. The column labelled "TLEP-240" holds for the sole period at 240 GeV for TLEP. The last line gives the absolute uncertainty on the Higgs boson branching fraction to exotic particles (invisible or not).

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Model-independent fit</th>
<th>Constrained fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TLEP-240</td>
<td>TLEP</td>
</tr>
<tr>
<td>$g_{HZZ}$</td>
<td>0.16%</td>
<td>0.15% (0.18%)</td>
</tr>
<tr>
<td>$g_{HWW}$</td>
<td>0.85%</td>
<td>0.19% (0.23%)</td>
</tr>
<tr>
<td>$g_{Ht\bar{t}}$</td>
<td>0.88%</td>
<td>0.42% (0.52%)</td>
</tr>
<tr>
<td>$g_{Hc\bar{c}}$</td>
<td>1.0%</td>
<td>0.71% (0.87%)</td>
</tr>
<tr>
<td>$g_{Hgg}$</td>
<td>1.1%</td>
<td>0.80% (0.98%)</td>
</tr>
<tr>
<td>$g_{H\tau\tau}$</td>
<td>0.94%</td>
<td>0.54% (0.66%)</td>
</tr>
<tr>
<td>$g_{H\mu\mu}$</td>
<td>6.4%</td>
<td>6.2% (7.6%)</td>
</tr>
<tr>
<td>$g_{H\gamma\gamma}$</td>
<td>1.7%</td>
<td>1.5% (1.8%)</td>
</tr>
<tr>
<td>BR$_{\text{exo}}$</td>
<td>0.48%</td>
<td>0.45% (0.55%)</td>
</tr>
</tbody>
</table>

4.1.2 Rare and invisible decay modes

Measurements of Higgs boson decay rates suppressed in the Standard Model give crucial information on the structure of new physics. In this respect, final states such as $H \rightarrow \mu^+\mu^-$ and $H \rightarrow Z\gamma$ are very attractive (in the SM, BR $H \rightarrow \mu^+\mu^- \sim 2.2 \times 10^{-4}$ and $H \rightarrow Z\gamma \sim 1.5 \times 10^{-3}$, respectively [57]). A high luminosity collider such as HL-LHC is the ideal machine for the production and the measurement of these final states.

The study of the $H \rightarrow \mu^+\mu^-$ decay channel is of particular importance as it allows the investigation of the Higgs boson coupling to second generation fermions, and can contribute to the final mass measurement. Several BSM models predict a Higgs boson $H \rightarrow \mu^+\mu^-$ decay rate significantly higher than in the SM case [58, 59]. In addition, at hadron colliders, the $H \rightarrow \mu^+\mu^-$ channel offers the best experimental mass resolution for fermionic final states, comparable to the one of $H \rightarrow \gamma\gamma$ and $H \rightarrow \ell^+\ell^-\ell^+\ell^-$. The production of the SM Higgs boson in the decay $H \rightarrow \mu^+\mu^-$ at HL-LHC is expected to be measured with an accuracy of about 12% with 3000 fb$^{-1}$, a factor three better than expected at LHC. Even with 3000 fb$^{-1}$, the uncertainty on the $H \rightarrow \mu^+\mu^-$ branching ratio (and other rare final states) will be limited by statistics, implying that a combination of results from ATLAS and CMS will yield even more precise measurements.

In the Standard Model the $H \rightarrow Z\gamma$ decay rate is about two third of the already observed $H \rightarrow \gamma\gamma$ process; current measurements from ATLAS and CMS are based on the one photon + two lepton final state, with a cross section limit ten times higher than the SM expectation. This final state can be either enhanced or suppressed in BSM models [60]; a precision on the $H \rightarrow Z\gamma$ rate of 20% – 30% can be obtained with 3000 fb$^{-1}$ at LHC [52, 61]. Studies made for ILC predict an accuracy for the $H \rightarrow Z\gamma$ rate larger than 30%, even in case of the highest centre-of-mass energy and integrated luminosity considered in the analyses that have been made; at FCC-ee this rate is expected to be measured with an uncertainty of about 12%.
Higgs boson decays, which are forbidden or strongly suppressed in the Standard Model, provides and unambiguous signature of physics beyond the SM. Among the first ones, lepton-flavour-violating decays \[62, 63\], such as \(H \rightarrow \tau \mu\) \[64\] or \(H \rightarrow \tau e\), play an important role because of the distinct experimental signature and their link to the flavour sector (Section 6.6.7). Exclusive hadronic decays, for example decays to a vector boson and a narrow resonance (e.g. \(H \rightarrow Z J/\Psi \) or \(H \rightarrow \gamma \Upsilon\)) \[65\] have SM branching ratios in the \(10^{-5} - 10^{-6}\) range and give complementary information on the couplings, providing a unique probe of the Higgs-Goldstone-vector coupling \[66\]. A comprehensive survey of other exotic Higgs boson decays is given in \[67\].

A further key study, performed by ATLAS and CMS at the HL-LHC, is the search for decays of the Higgs boson to particles that leave the experimental apparatus without being detected, e.g. to dark matter WIMPs. The process considered in the study performed is the associated \(ZH\) production, with \(Z \rightarrow \ell^+\ell^-\) and the Higgs boson decaying invisibly. Limits on the “invisible final state” branching ratio of the Higgs boson at the level of \(6-8\%\) can be set at the 95\% confidence level; in a more conservative scenario this limit would degrade by about a factor of two. Measurements using the VBF and gluon fusion production modes can further improve the results.

The search for invisible decay modes (and other final states suppressed by Standard Model) at \(e^+e^-\) colliders can be covered by the measurement of the total width, compared with the SM prediction, whose uncertainty is predicted to be a few \% (see also Section 4.1.4).

4.1.3 CP mixing studies

Studies on the measurement of properties of the Higgs boson decay vertex \(H \rightarrow \ell^+\ell^-\) in 14 TeV proton-proton collisions have been performed \[68\]. Though this channel has low sensitivity to CP mixed states in the MSSM because the \(ZZ\) system has defined positive CP properties, in more general BSM models such constraint vanishes increasing the channel sensitivity. The sensitivities on the \(HZZ\) vertex tensor couplings have been determined for an integrated luminosity of 300 \(fb^{-1}\) at LHC and 3000 \(fb^{-1}\) at HL-LHC. The decay amplitude has been described following ref. \[44\], where the \(HZZ\) scattering amplitude has been parameterized as a function of 4 complex coupling constants: \(g_1, g_2, g_3\) and \(g_4\). The result of the studies are shown in Table 12, where the 95\% confidence level intervals on the couplings have been reported. Such results show that a substantial improvement of our knowledge of the tensor structure of the \(HZZ\) vertex can be achieved.

Table 12: Expected values excluded at 95\% C.L. for the real and imaginary part of \(g_4/g_1\) and \(g_2/g_1\) couplings, assuming the Standard Model. These values are obtained at \(\sqrt{s} = 14\) TeV using an integrated dataset of 3000 \(fb^{-1}\) at HL-LHC.

\[
\begin{array}{c|c|c|c|c}
\text{Re}(g_4)/g_1 & \text{Im}(g_4)/g_1 & \text{Re}(g_2)/g_1 & \text{Im}(g_2)/g_1 \\
< -0.34 & > 0.26 & < -0.30 & > 0.11 \\
\end{array}
\]

4.1.4 New trends in measuring the Higgs boson natural width

The natural width of the 125 GeV Higgs boson is an important physics property that could reveal new physics. In particular, decays of the Higgs boson to dark matter objects, WIMPs and other possible BSM particles would increase the natural width with respect to what predicted by SM, and indicate the production of new physics. A direct measurements of this quantity is possible only at muon colliders via line shape scan in the s-channel (Section 2.5), while at the LHC/HL-LHC and at \(e^+e^-\) colliders the experimental mass resolution for detected Higgs candidates is significantly larger than the expected
width from Standard Model. As an example, the mass resolution of the $\gamma\gamma$ system at LHC experiments is about 400 times the SM predicted natural width for $m_{\gamma\gamma} \sim 125$ GeV.

Nevertheless, an indirect determination of the Higgs boson width is possible by using the interference of the Higgs boson signal ($H \rightarrow \gamma\gamma$ or $H \rightarrow ZZ$) with the same final state ($\gamma\gamma$ or $ZZ$) in the continuum [69]. In the first case the interference is studied with the on-shell peak, because the interference modifies the $H \rightarrow \gamma\gamma$ line shape, in the second case the interference happens on the off-shell side, in particular in the region where the invariant mass of the four leptons is larger than twice the $Z$ mass. Constraints from data have been recently studied by CMS [70] and ATLAS [71] on the total width of the 125 GeV Higgs boson, using its relative on-shell and off-shell production and decay rates to a pair of $Z$ bosons [72] [73], where one $Z$ boson decays to an electron or muon pair, and the other to an electron, muon, or neutrino pair. This leads to an upper limit on the Higgs boson width of about 20 MeV at 95% C.L.. More studies are needed to investigate possible routes to reduce the experimental and theory uncertainties.

4.1.5 $HH$ pair production

After the discovery of a light resonance at 125 GeV, Higgs pair production is drawing a lot of attention in the community. Indeed, among the measurements which need to be performed in future projects, the assessment of the Higgs self-coupling in processes where the Higgs boson is produced in pairs is of paramount importance. In many BSM models, double Higgs boson is significantly different from what predicted in SM. Also, Higgs boson self-coupling is strongly connected with the vacuum stability, and it has therefore important cosmological implications [47]. Early discovery of $HH$ production in RUN-2/HL-LHC would represent another major result, hopefully followed by subsequent Higgs self-coupling measurements. The Higgs self-coupling is accessible thanks to the production diagram involving a triple Higgs vertex.

With a Higgs boson at the mass measured at LHC, examples of the most promising channels are $HH \rightarrow bb\gamma\gamma$ and $bb\tau^+\tau^-$. The challenging $HH \rightarrow bbW^+W^-$ channel, initially thought to be inaccessible due to the large $tt$ background, is now currently also been investigated. The final state with the largest branching fraction, $HH \rightarrow bbbb$, could be an interesting channel if boosted topologies (that offer better signal-to-background ratio) are considered. The channel $HH \rightarrow bbZZ \rightarrow bb\ell\ell\nu$ could be another interesting final state to investigate, if the $Z$ mass constraint proves sufficient to reduce the large background.

At hadron colliders, the dominant production mechanism is gluon-gluon fusion, and at the HL-LHC is estimated to have a cross-section of $34^{+37}_{-30}\%$ fb at NLO [74] assuming $m_{H1} = 125$ GeV. Due to the destructive interference of the diagrams involving di-Higgs production, this cross-section is modified to be $71(16)$ fb if the self-coupling is assumed to be zero (twice the SM prediction). Recent calculations of next-to-next-to-leading order (NNLO) QCD corrections suggests an increase of the SM cross-section by a factor $O(20\%)$ [75, 76], thus enhancing its value to about 41 fb.

Preliminary studies on di-Higgs boson production in the $HH \rightarrow bb\gamma\gamma$ decay have been recently released by ATLAS and CMS [51, 77]. A cut based analysis has been performed using Monte Carlo samples for signal and several background processes that are expected to contribute to the production of this final state. These samples have been processed through a simplified or full detector simulation. Figure 13 shows the distribution of the diphoton invariant mass, $m_{\gamma\gamma}$, after all other selection cuts are applied. A total of 8.4 events are expected with a 5000 fb$^{-1}$ data sample, with a background contamination of 47 events, yielding a significance on HH production of $S/\sqrt{B} \approx 1.3$. A higher significance is predicted by the analysis performed by CMS, with slightly lower background expected, that would allow a measurement of the HH yield with an accuracy of about 60%; see Fig. 14. ATLAS and CMS are

\footnote{At the $e^+e^-$ colliders, an indirect measurement is possible combining the measurements of the Higgs boson production from “Higgsstrahlung” and Vector Boson Fusion processes, as described in Section 4.1.1.}
Fig. 13: The distribution of the two-photon invariant mass \( m_{\gamma\gamma} \) from the \( \gamma\gamma bb \) channel after applying all selection cuts (except the one on that variable) [78].

Fig. 14: The average expected relative uncertainty on the di-Higgs cross-section measurement is shown as a function of the b-tagging efficiency [77].

currently working to better understand these residual differences, and to investigate potential avenues for improving the sensitivity. Studies on \( HH \rightarrow b\bar{b}+\tau^+\tau^- \) final states are in progress. More detailed results from ATLAS and CMS on Higgs pair production measurement prospects at HL-LHC will be available a few months after the publication of this document.

Higgs boson pair production at \( e^+e^- \) colliders requires a centre-of-mass energy of at least the kinematic limit (\( \sim 340 \) GeV). Therefore only colliders such as ILC in the version with \( \sqrt{s} = 1000 \) TeV and 1000 fb\(^{-1}\) can expect to measure the triple Higgs coupling with an uncertainty of about 13%.

4.2 Experimental measurements with W and Z bosons and QCD

Precision measurements in the electroweak sector of the Standard Model (SM), studying the properties of W and Z vector bosons, keep their own interest even after the discovery of the SM-like Higgs boson and in view of future experimental programs. These studies are a tool for the indirect discovery of physics beyond SM, through the search for deviations from the properties predicted by the model itself induced by quantum loop corrections. In case new physics is independently discovered, they may help in characterizing it.
There are two main classes of measurements which are interesting for the next future:

- the study of precision observables related to the vector boson properties, that enter directly as input to the electroweak global fits, together with the top quark mass $m_t$ and the Higgs boson mass $m_H$.

Under the assumption that the new 125 GeV boson discovered at the LHC is the SM Higgs boson, all the parameters are known and this fit can now over-constrain the model for the first time [79];

- the study of the couplings in the gauge boson sector, aiming to search for anomalous contributions to the triple (TGC) and quartic (QGC) gauge boson couplings. This is a sector of measurements tightly related to the understanding of the properties of the Higgs boson, through the detailed study of the vector boson scattering (VBS), whose theoretical explanation is one of the reasons for introducing the Higgs boson itself.

As it will be noticed, besides the direct study of the vector bosons’ properties and couplings, a number of additional inputs are required by the global fit. The precise knowledge of the electromagnetic and QCD coupling constants $\alpha_{em}, \alpha_S$, directly affects the fit predictions. Moreover, since hadronic colliders will keep playing a fundamental role for a while in these measurements, the improved knowledge of the initial state through a precise measurement of the proton parton distribution functions (PDFs, Section 7.3.1) is of the utmost importance.

4.2.1 Precision observables

The SM parameters mostly sensitive to the Higgs boson mass value are the $W$ boson mass $m_W$, and the couplings of the $Z$ boson described by the Weinberg angle, $\sin^2 \theta_W$ [80]. The current experiment uncertainty on $m_W = 80.385 \pm 0.015$ GeV is driven by the measurements performed at the Tevatron [81], while the uncertainty on the Weinberg effective angle is dominated by the measurements performed at LEP1/SLC $\sin^2 \theta_W^{eff} = 0.23113 \pm 0.00016$ [82]. The discrepancies in the latter results obtained at the two $e^+e^-$ colliders constitutes one of the most intriguing legacies of that experimental program.

The indirect determination of $m_W$ from the global electroweak fit provides at present an uncertainty of 8 MeV [83], already smaller than the experimental precision, and that will be likely further reduced in next future through the progresses in theoretical calculations and other SM parameters knowledge [84]. This motivates the push for a reduction of the experimental uncertainty in this measurement, that can be achieved both at hadronic and leptonic colliders. The expected final precision by the analysis of the full Tevatron data sample could be of about 9 MeV, with the statistical and PDF-induced uncertainties playing a leading role. The use of data control samples to understand the detector behaviour and calibration has proved to be at the Tevatron an essential tool in reducing the other sources of systematic uncertainties, and therefore a similar result can be expected at LHC. The studies in [85] show that the $m_W$ uncertainty coming from PDFs at LHC can be even similar to the one at Tevatron, provided the effects on the cross section normalisation are removed. Therefore an improvement of the results clearly requires a corresponding improved knowledge of the PDFs themselves. The relative importance of various PDFs depends on the observables used for the $m_W$ extraction. Given the relevance of a precise determination of the $W$ mass at LHC, a special section is dedicated to the subject in this report (Section A.1). Its conclusions, based on the current trend in the development of calculations and understanding of the impact of PDFs, suggest that the 10 MeV uncertainty level is reachable, at least for measurements based on the transverse mass. Extrapolations in [84] push the target for an uncertainty at LHC towards 5 MeV in the long term, exploiting for these studies the samples collected up to the HL-LHC data taking.

Better results are expected to come from the precise knowledge of the $e^+e^- \rightarrow W^+W^-$ cross section around the production threshold, which is very sensitive to $m_W$. This would be the main observable to study at future leptonic machines, both linear (ILC) and circular ones (ee-FCC). In both cases the dominant source of uncertainty would be the detailed knowledge of the beam energy (determining the value of $\sqrt{s}$ at which the cross section is evaluated). Studies for the ILC physics case [84, 86] suggest
that a threshold scan would allow to reach an in-situ energy calibration sufficient to achieve a \(\simeq 2.5\) MeV total uncertainty on \(m_W\). Crucial ingredients of the measurement would be the beam energy calibration in situ with \(Z\gamma\) events and the use of polarised beams to evaluate background from data. Other techniques, like the full event kinematic reconstruction à la LEP2 or the study of the hadronic mass in single \(W\) production, can complement this result but they are unlikely to provide a precision better than 3 MeV. The ultimate precision in a leptonic machine could be reached by a \(WW\) threshold scan at a circular collider: studies performed for the TLEP physics case [87] claim that the resonant depolarisation method, used at LEP1 for the precise measurement of the beam energy, could be pushed to work up to 80 GeV. This could turn into a corresponding precision on \(m_W\) of about 1 MeV.

The current theoretical precision on \(\sin^2 \theta_W^{\text{eff}}\) is at the level of \(\simeq 7 \times 10^{-5}\), and it is reasonable to imagine it could be halved in next future mostly through an improved knowledge of the main input parameters. The effort of moving the experimental precision at the \(10^{-5}\) level, and understand the current discrepancy among the values obtained by LEP1 and SLC separately is therefore important. The most precise result at an hadronic collider has been obtained by CDF [88] exploiting innovative procedures to analyse \(Z \rightarrow \mu\mu\) decays, which have allowed to bring the uncertainty at the \(10^{-3}\) level. The addition of th electron channel and of D0 similar measurement can likely bring the Tevatron combined result at the level of precision of the LEP/SLC one. Recent ATLAS results [89] show that the usage of forward electrons, in the detector acceptance \(|\eta| > 2.5\), can improve the precision, even with a smaller statistics compared to the central region, thanks to a smaller dilution of the information on the quark direction, confirming the smaller sensitivity to PDFs already noticed in the study [90]. All these facts suggest that LHC could achieve a comparable level of precision as LEP1/SLC as well, within the HL-LHC completion [84]. Alternative methods to explore the \(Zb\bar{b}\) vertex, the basis for the LEP1 measurement of the Weinberg angle, have been suggested in [91], studying the associated production of \(Z\) and \(b\)-quark at LHC.

The ultimate precision reachable on \(\sin^2 \theta_W^{\text{eff}}\) is nevertheless achievable at a lepton collider, at present the only environment in which it can be expected to break the \(10^{-4}\) precision barrier. Both for circular and linear machines, the use of longitudinally polarized beams is essential to obtain the best results. Repeating the SLC measurement of the left-right polarisation asymmetry with both beams polarised can provide a jump in precision provided all the possible polarisation combinations are used to compute it in the so called "Blondel scheme" [92]. The precision in the beam energy determination also matters, given the strong \(\sqrt{s}\) dependence of the \(\gamma Z\) interference term. The projections presented in [84] shown the \(10^{-5}\) uncertainty achievable at ILC, and even better at an ee-FCC.

A lepton collider running at the \(Z\) peak can allow to repeat the LEP1 studies of the \(Z\) line shape with possibly improved precision. It is worthwhile to notice that the \(Z\) mass determination is strongly dependent on the precise knowledge of the beam energy. The use of the resonant depolarisation technique, possible in a circular collider, might push the precision beyond the LEP1 level, as shown in the projections presented in [87].

### 4.2.2 Boson couplings and vector boson scattering measurements

The study of triple and quartic gauge couplings among bosons, a fundamental prediction of the SM, relies on the study of multi-boson final states at both hadron and lepton colliders. The smallness of the cross section for these processes, compared to the production of single bosons, implies the need for high integrated luminosities in order to reach a sufficient statistical precision to improve the currently available limits on anomalous contributions, that are signatures of physics beyond the SM. Among the mechanisms producing multi-boson final states, VBS plays a special role, as its unitarization is achieved by the introduction of the Higgs mechanism. A deviation from the SM predictions for these processes would be an important evidence for the need of an extension of the minimal SM. The sensitivity to non SM contribution of the observables related to VBS increases with the centre of mass energy, and this explains why hadronic machines are the main environment where this study must be performed.
Concerning charged triple gauge couplings, as shown in [93], LHC is already competitive with earlier LEP2 and Tevatron results on the parameters $\Delta \kappa$ and $\lambda$, used to describe anomalous couplings: the former is mostly sensitive to differential distributions, and the cleaner environment provided by lepton colliders can help in the precision of the determination, and ILC at 800 GeV is clearly winning over LHC, even assuming the full HL-LHC foreseen statistics (Fig. 15). On the contrary, the latter can profit from the $\sqrt{s}$ dependence; furthermore, the sensitivity to it resides mostly in the high $p_T$ tail of diboson system, and can benefit from large statistics. This explains why HL-LHC is expected to reach performances comparable to ILC at 800 GeV, while a h-FCC hadron collider at high energy could push further this precision.

Quartic gauge boson couplings can be extracted by the study of both di and triboson final states, like $WW$, $WZ$, $ZZ$ and $Z\gamma\gamma$ respectively. Vector boson scattering processes contributing to these final states are characterised by emission of very forward jets, large jet-jet invariant mass, and large rapidity gaps (since they produce colourless objects in the central region). With the typical experimental selections used, the cross section for electroweak vector boson pair production is of $O(10)$ fb. The study of the vector boson pair mass in the VBS processes can probe the need for physics beyond the SM Higgs mechanism at higher scale. In practice there is also the transverse polarization to take into account, and the PDFs suppress the growth, so a precise measurement of the production rate at high mass is needed. These facts combined explain why HL-LHC is needed in order to have a detailed investigation of VBS, studying in details several differential distributions sensitive to non SM contributions.

Both ATLAS [94] and CMS [95] have provided preliminary studies of sensitivity of HL-LHC to QGC, using the effective field theory approach to parameterise the possible anomalous contributions. As summarised in [84], there is a clear gain in sensitivity reach by moving from the integrated luminosity of 300 fb$^{-1}$ expected by the end of LHC to the 3000 fb$^{-1}$ expected at the end of the full HL-LHC program. Limits in the new physics scale are model dependent, but in general they reach or exceed the 2 TeV.

In general, lepton colliders are not competitive in this sector, where hadronic machines can give better results by one or two orders of magnitude. It is worthwhile to notice that tribosons are specially sensitive to $\sqrt{s}$, and could provide even better limits at energies beyond the HL-LHC ones.

In parallel to these studies, another interesting process providing access to anomalous QGC is the $\gamma\gamma \rightarrow WW$ scattering. The very weak limits on new physics established by LEP2 are already improved at LHC, and can be pushed further by order of magnitudes before any lepton collider enters into operation. The experimental technique used to study this very rare process is based on the tagging of very forward protons emerging from the scattering in the so called central exclusive production mechanism, that can be achieved with the proton spectrometers situated at about 200 m from the interaction point that both ATLAS and CMS are building at present. Results are presented in [96], showing a very interesting physics potential.

4.2.3 Parton distribution functions and the strong coupling constant

From the previous discussion it is clear that the precision in the knowledge of the proton PDFs is a limiting factor for the program of precision measurements in the electroweak sector in the next 15/20 years, when it will be dominated by LHC and HL-LHC. In particular the gluon PDF is still poorly constrained by data, while becoming of crucial importance (for instance in the description of the Higgs boson production).

LHC measurements have started to be used to complement those of HERA in constraining the proton PDFs: inclusive jet and dijet production, $W$ charge asymmetry, $W + c$ production. But a quite larger program can be envisaged. The gluon PDF can be constrained by studying many different final states:

- isolated photons, or photon+jets, constraining medium x gluons through QCD Compton scattering;
- vector bosons + jets (high $p_T$): small to medium x gluons (qg dominating at $p_T > 100$ GeV);
– low mass Drell-Yan: small x gluons (if resummed calculations available; LHCb might be particularly sensitive in its acceptance);
– tt̄ production (known at NNLO): large x gluons (important for high mass BSM).

Quark PDFs understanding can benefit from measurements of the ratio of W and Z production at high \( p_T \), useful to study the quark-antiquark separation, or of the Drell-Yan process at high mass, sensitive to large x quark PDFs.

It is important to notice that the proposed electron-proton collider option suggested for LHC, the so called LHeC program, could be the ultimate tool to gain the deepest understanding on the PDFs, according to the projections made in the proposal [97].

Together PDFs, the precise knowledge of the strong coupling constant \( \alpha_S \) is also essential to the precision physics program, as a basic input in the global electroweak fits. At present lattice calculations are driving the uncertainty on the average. The recent calculation of the \( pp \rightarrow t\bar{t} \) cross section at NNLO opens a very interesting possibility to get an \( O(1)\% \) measurement at high \( Q^2 \). In future perspective, the most promising measurement can be provided by the study of \( R_l = \Gamma(Z \rightarrow \text{hadrons})/\Gamma(Z \rightarrow ll) \) at lepton colliders. ILC or ee-FCC could reach precisions of 0.0004 or 0.0001 respectively, exploiting the current precise knowledge of \( m_H \).

\[ \begin{array}{c|c|c|c|c}
\text{LH}, \text{Tevat.} & \text{ILC} & \text{ILC} & \text{ILC} & \text{ILC} \\
\text{10^{-4}} & 10^{-3} & 10^{-2} & 6g & a \\
\text{10^{-3}} & 10^{-2} & 10^{-1} & 6h & a \\
\text{LEP} & \text{Tevat.} & \text{LHC} & \text{ILC} & \text{ILC} ILC ILC ILC \\
\end{array} \]

Fig. 15: Prospects for the measurements of the \( \kappa_\gamma \) and \( \lambda_\gamma \) TGC couplings at LHC and ILC, from [98].

4.3 Top quark physics
The top quark, discovered in 1995 [99, 100], is nowadays the heaviest elementary particle. It plays a crucial role in the Standard Model phenomenology and the electroweak symmetry breaking because, thanks to its large mass, it exhibits the largest Yukawa coupling with the Higgs boson. The top-quark mass \( m_t \) is a fundamental parameter of the Standard Model: even before the Higgs discovery [42, 43], it was used, together with the \( W \) mass, to constrain the Higgs mass in the global fits. With the exception of few open problems which will be discussed in the following, all measurements for both \( t\bar{t} \) and single-top production are in agreement with the Standard Model expectations. Nevertheless, top quark phenomenology will remain one of the main fields of investigation in both theoretical and experimental particle physics, at any present and future facility, i.e. both lepton and hadron colliders, as well as linear and circular accelerators. Hereafter, we shall discuss the future perspectives regarding the measurement of the top-quark properties, taking particular care about its mass, couplings and final-state kinematic distributions.
4.3.1 Top quark mass

The mass of the heaviest fermion ($m_t$) is a physical quantity of central importance at the electroweak scale; its current world average is $m_t = 173.34 \pm 0.27 \text{ (stat)} \pm 0.71 \text{ (syst)} \text{ GeV}$ [101]. The role of the top mass in precision tests has been already mentioned, besides electroweak fits the value of the top mass is used as a key input in several other important cases. As an example [102], using the current values of the measured Higgs and top masses and assuming that possible new physics interactions at the Planck scale do not affect the stability phase diagram and the electroweak vacuum lifetime, the Standard Model vacuum is found to lie on the border between stability and metastability regions ( [103] discusses the robustness of the underlying hypotheses). In addition, the top mass plays a role in inflationary universe theories and in the open issue regarding whether the inflaton can be the Higgs field or not. As discussed, e.g. in [104], in inflationary theories the running of the couplings is important and, once the the Yukawa coupling is determined from the top mass, the spectral index depends on both top and Higgs masses.

A crucial assumption employed in all calculations using the top mass as an input is that the measured mass corresponds to the top-quark pole mass. However, as will be clarified later on, the connection between the top mass measured in current analyses of experimental data and the pole mass is not straightforward and, although the two values should be reasonably close, any effort to clarify the top mass interpretation is important in order to validate or modify the outcome of electroweak fits or other studies [102].

The standard methods to measure the top mass at hadron colliders, where $t\bar{t}$ pairs are produced in $q\bar{q}$ (dominant at the Tevatron) or $gg$ (dominant at the LHC) annihilation, are based on the investigation of the properties of the final states in top decays ($t \to bW$), which, according to $W$ decays, are classified as double-lepton, leptons+jets or all jets. In all cases, there are two $b$-tagged jets, whereas the $W$ decay products are reconstructed as isolated leptons (muons or electrons) or as jets (for $W \to q\bar{q}'$ processes). After requiring energy-momentum conservation and constraining the $W$ mass, the final-state invariant-mass distribution exhibits a peak, which is interpreted as the production of a top quark.

The conventional techniques to reconstruct the top mass are likelihood-type such, e.g. the matrix-element and template methods. The matrix-element method compares the measured quantities with predictions obtained by convolving the LO $t\bar{t}$ cross section with the detector response. The template method is based on investigating several distributions of observables depending on $m_t$, under the assumption that the final state is $WbWb$ and the $W$ mass is known; the data are then confronted with Monte Carlo templates and $m_t$ is the value which minimizes the $\chi^2$. Matrix-element and template methods are those used in the world average determination, based on the updated measurements from D0, CDF, ATLAS and CMS Collaborations. The projections for the LHC run at 14 TeV, i.e. a $t\bar{t}$ cross section of about 951 pb, according to the template/matrix-element methods, foresee, for a luminosity $\mathcal{L} = 100$ fb$^{-1}$, a systematic uncertainty of 700 MeV and a statistical error of 40 MeV. With $\mathcal{L} = 300$ fb$^{-1}$, the estimated systematic and statistical errors are 600 and 30 MeV, respectively [105].

Other techniques have been developed, for example the so-called endpoint [106, 107], $J/\psi$ [108, 109] and decay-length [110] methods. These alternative techniques are presently less precise than the conventional methods, however they turn out to be very promising in view of the high statistics to be collected at LHC, because statistical uncertainties, together with several systematic uncertainties reducible with statistics, will decrease, as discussed in [107].

Generally speaking, in most analyses the experimental results are compared with simulations based on Monte Carlo generators (an exception is the endpoint method) and, strictly speaking, the reconstructed top mass cannot be precisely identified with theoretical definitions like, e.g., the pole mass.

In fact, programs like HERWIG or PYTHIA are equivalent to LO QCD calculations, with the resummation of all leading (LL) and some next-to-leading soft/collinear logarithms (NLL) [111]. In order to fix a renormalization scheme and get the pole or $\overline{\text{MS}}$ mass, one would need at least a complete NLO computation, while parton showers only contain the soft/collinear part of the NLO corrections.
Furthermore, any observable yielded by such codes depends on parameters which are to be tuned to experimental data, in particular non-perturbative quantities, such as the shower cutoff or the parameters entering in the hadronization models, namely the cluster \[\text{[112]}\] (HERWIG) or string (PYTHIA) \[\text{[113]}\] models.

In the non-perturbative phase of the event simulation, the $b$ quark in top decay hadronizes, e.g., in a meson $B^{±,0}$, by combining with a light (anti) quark $q$, which may come from final- as well as initial-state radiation. Since the $b$ quark likely radiates gluons before hadronizing, the initial colour and part of the four-momentum of the top quark may well be transferred to light-flavoured hadrons, rather than only $B$-hadrons. As a result, there is no unique way to assign the final-state particles to the initial (anti) top quark and this leads to another contribution to the uncertainty on the top mass, when reconstructed from the invariant mass of the top-decay products. As for colour flow and reconnection, a phenomenon already investigated at $e^+e^-$ colliders in $e^+e^\to W^+W^-\to 4$ jets processes, this is typically studied by varying the non-perturbative parameters of the Monte Carlo program and it leads to an uncertainty that amounts to about 300 MeV in the current top mass world average. Moreover, parton shower algorithms neglect the top width ($\Gamma_t \approx (2.0 \pm 0.5)$ GeV \[\text{[81]}\]) and top-production and decay phases factorize: although $\Gamma_t < < m_t$, for a precise mass definition with an uncertainty below 1 GeV, even width effects should be taken into account. Therefore, one often refers to the measured mass as a ‘Monte Carlo mass’, which must be related to a given theoretical definition. Indeed, definitions like the pole mass work well for leptons, which are colourless particles, whereas, in the case of coloured quarks, it exhibits a non-perturbative uncertainty, proportional to $\Lambda_{\text{QCD}}$, due to the infrared renormalons which are contained in the higher-order corrections to the heavy-quark self energy, proportional to $\alpha_s^{n+1} n!$. Nevertheless, up to this intrinsic ambiguity, since the top mass is extracted from final-state top-decay observables, relying on the on-shell kinematics of its decay products (leptons and jets), one should reasonably expect the measured mass to be close to the pole mass. In fact, calculations based on Soft Collinear Effective Theories (SCET) \[\text{[114]}\] have proved that, assuming that the Monte Carlo mass is the SCET jet mass evaluated at a scale of the order of the shower cutoff, i.e. $Q_0 \sim \mathcal{O}(1 \text{ GeV})$, it differs from the pole mass by a small amount $\sim \mathcal{O}(\alpha_s)$. A foreseen investigation, which may help to shed light on this issue, is based on the simulation of fictitious top-flavoured hadrons, e.g. $T^{±,0}$ mesons \[\text{[115]}\]. It is well known how to relate the mass of a meson to a quark mass in any renormalization scheme. Therefore, a comparison of final-state quantities with the top quark decaying before or after hadronization, and the subsequent extraction of the top mass from their Mellin moments, can be a useful benchmark to address the nature of the reconstructed $m_t$ and the uncertainty due to non-perturbative effects, such as colour reconnection. In standard top-quark events the top quark gets its colour from an initial-state quark or gluon and, after decaying, gives it to the bottom quark; on the contrary, if it forms $t$-hadrons, it is forced to create a colour-singlet.

For the reasons explained above, it is of paramount importance to verify the description of key features of the experimental data by the simulation used to derive the top mass. In addition, the uncertainty related to modelling of, e.g. colour reconnection, can potentially be reduced by comparing sensitive observables in data and Monte Carlo simulation. To this end recent studies are described in \[\text{[116, 117]}\], based on the use of several distributions differential with respect to $m_t$. The present conclusion is that the predictions of all generators are in agreement with the data and therefore there is no bias due to the choice of the generator used to describe $t\bar{t}$ events. Differential measurements are limited by statistics, therefore these techniques will profit a lot by the foreseen increase of integrated luminosity at LHC.

In order to weaken the dependence on the shower algorithms and non-perturbative corrections, other methods have been proposed to measure the top mass at the LHC. One is based on the measurement of the total $t\bar{t}$ cross section, recently computed in the NNLO+NNLL approximation using the pole mass in \[\text{[118]}\], and extract the top mass from the comparison \[\text{[119, 120]}\]. The very fact that the mass determined from the cross section is in agreement with the value yielded by the likelihood-based standard techniques, confirms the hint that the extracted top mass should be close to the the pole mass, used in
the NNLO+NNLL calculation of [118]. Another possible strategy consists of using the $t\bar{t}$ invariant mass in events with a hard jet ($j$), since it is an observable more sensitive to the top mass than the inclusive cross section [121,122]. These methods, together with the alternative and differential techniques mentioned above, will be used to exploit the large statistics to be collected in the future LHC runs, and eventually provide a robust assessment of the uncertainty and possible biases in our determination of $m_t$.

Future lepton facilities will be an excellent environment to measure the top mass, because of the colourless initial state. As described in Section 2, we have several proposals for lepton colliders, mainly $e^+e^-$ machines: the International Linear Collider (ILC), the Compact Linear Collider (CLIC) as well as circular colliders (FCC-ee, CEPC). The potential for top-quark physics at ILC and CLIC has been studied in depth, with simulations of the luminosity spectra and detector response. The key challenge at CLIC is the pile-up of the background due to $\gamma\gamma$ annihilation into hadrons, while at ILC there is no pile-up but there are more $\gamma\gamma \rightarrow$ hadrons processes per bunch crossing. At a circular collider such as FCC-ee similar and even higher instantaneous luminosities can be obtained with a much lower level of beamstrahlung, a narrow beam energy spectrum having a very low tail and lower $\gamma\gamma$ backgrounds.

At $e^+e^-$ colliders, top-pair production near threshold is an interesting process, where two main contrasting effects play a role: because of the strong interaction, the $t$ and the $\bar{t}$ can form a Coulomb bound state, whereas the electroweak interaction smears the peak of the cross section out. The resonance cross section, computed up to NNLO accuracy [123] by using Non Relativistic QCD, is peaked at the toponium ground state, whereas the electroweak interaction smears the peak of the cross section out. The resolution in the top mass determination is given by the mass resolution of the toponium state, which is a useful feature for combined fits of the top mass and other parameters

In order to estimate the uncertainty on the measurement of the top mass at a lepton collider, a simulation scanning the range $346 \text{ GeV} < m_t < 354 \text{ GeV}$ in steps of 1 GeV, by using the TOPPIK program [124] and assuming an integrated luminosity $L = 300 \text{ fb}^{-1}$ was carried out in [125]. The overall uncertainty is gauged to be about 100 MeV, after summing in quadrature the errors due to statistics (30 MeV), luminosity (50 MeV), beam energy (35 MeV) and on the functional form of $f(\sqrt{s_{\text{res}}}, m_t)$ (80 MeV). The luminosity spectrum of the machine affects the (statistical) uncertainty of the measurement: passing from CLIC to ILC the uncertainty on the mass should improve by 10-20%. The theoretical error, due to missing higher orders and uncertainties on the quantities entering in the calculation, such as $m_t$, $\Gamma_t$ and $\alpha_s$, is predicted to be 3% of the full error. Furthermore, a 2D template fit to the cross section can be performed as well, measuring simultaneously $m_t$, and $\alpha_s$. Through this method, one can reach an error on the pole $m_t$ of 60 MeV and on the 1S mass of 30 MeV.

At circular colliders such as FCC-ee the cross section just above threshold is essentially independent on the top mass, which is a useful feature for combined fits of the top mass and other parameters (e.g. the top width, $\alpha_s$ or the top-Higgs Yukawa coupling). This expected behaviour has been recently checked with tools developed in the context of ILC studies, where typical FCC-ee beam parameters have been included [126,127], as shown in Fig. 16.

Above threshold, the top mass can still be determined by using final-state distributions, in the same manner as at hadron colliders: with $\sqrt{s} = 500 \text{ GeV}$ and $L = 500 \text{ fb}^{-1}$, current estimates foresee an experimental uncertainty of 100 MeV [127].

### 4.3.2 Top quark couplings

The determination of the coupling of top quarks to $W$, $Z$ and Higgs bosons, as well as to photons and gluons, is an important challenge in top-quark phenomenology. These measurements are complementary to the Higgs coupling measurements previously described in this chapter. In particular, possible direct measurements of the Yukawa coupling will be a crucial test of the Standard Model and will help to shed
light on some new physics models.

The strong coupling constant $\alpha_S$ can be extracted from the measurement of the $t\bar{t}$ and $t\bar{t}j$ cross sections. Ref. [119] compared the NNLO calculation [118] with the measured $t\bar{t}$ cross section in terms of $m_t$ and $\alpha_S(m_Z)$. Once the top pole mass in the computation is fixed to the world average, one can extract the strong coupling constant from the comparison, obtaining the value $\alpha_S(m_Z) = 0.1151^{+0.0033}_{-0.0032}$, which is at present the first $\alpha_S$ determination in top-quark events and within a NNLO analysis. The experimental (about 3.5%) and theory (about 5%) errors are of similar order of magnitude and are not expected to change dramatically in the future LHC operation, namely centre-of-mass energy 13 TeV and luminosity 300 fb$^{-1}$. At future $e^+e^-$ colliders, through a threshold scan of the total cross section, it will be possible to extract $\alpha_S$ with an uncertainty smaller than 1% and the width $\Gamma_t$ with an accuracy of a few percent [125].

The coupling of the top quarks to $W$ bosons can be measured through top decays and single-top production. The helicity fractions of $W$ bosons in top decays have been calculated in the NNLO approximation in [129], and therefore the theory uncertainty is by far smaller than the experimental one (see the CMS and ATLAS measurements of the $t\bar{t}W$ cross section in [130, 131]). A higher level of precision of the measurement of such helicities, by exploiting the leptonic angular distributions, is thus mandatory in the next LHC operations, in order to test the Standard Model in the top-decay sector as well. As for single-top production, the LHC cross sections in the $s$- and $t$-channel, as well as in the $Wt$ associated-production mode, are in agreement with the Standard Model expectations. In the $t$-channel case, for example, CMS reports a cross section measurement $\sigma_{t-ch}(t) = [83.6 \pm 2.3{\text{ (stat)}} \pm 7.4{\text{ (syst.)}}] \text{ pb}$ [132], whereas the ATLAS result is $\sigma_{t-ch}(t) = [68 \pm 8] \text{ pb}$ [133]. Increasing the energy and the luminosity of the LHC will not improve too much the accuracy of this measurement, but nevertheless a precision of 5% in the determination of the single-top cross section and of 2.5% in the measurement of the CKM matrix element $V_{tb}$ is foreseen [134].

Future $e^+e^-$ colliders will be able to measure the $tWb$ coupling with an accuracy about 2%, by scanning the centre-of-mass energy between $m_t$ and $2m_t$ [135]. Furthermore, a $\gamma e$ collider is predicted
to have a reach for the $tWb$ coupling between $10^{-1}$ and $10^{-2}$ [136], while an $ep$ accelerator using the LHC facility at 1.3 TeV may aim at a sensitivity within $10^{-2}$ and $10^{-3}$ [137].

As for the top coupling to photons, although measurements of the top charge [138] and of the inclusive $t\bar{t}\gamma$ cross section [139] are available, with the results being in agreement with the Standard Model predictions, it would be desirable to determine the $t\bar{t}\gamma$ coupling with a much higher level of precision. In fact, this process suffers from large QCD backgrounds, and it is therefore necessary to set strong cuts to suppress them; the NLO calculation for $t\bar{t}\gamma$ production [140] is an important ingredient for an improved measurement at the LHC. At 14 TeV, with a luminosity of 300 fb$^{-1}$, the coupling to photons will be measured with a precision of 4%, whereas at 3000 fb$^{-1}$ the expected accuracy will be about 1%.

The $t\bar{t}Z$ production cross section, sensitive to the coupling of top quarks with $Z$ bosons, has been measured by CMS ($\sigma_{t\bar{t}Z} = [200^{+80}_{−70}(\text{stat})^{±40}_{−30}(\text{syst})] \text{ fb} [130]$) and ATLAS ($\sigma_{t\bar{t}Z} = [150^{+50}_{−50}(\text{stat})] ± 21(\text{syst}) \text{ fb} [131]$). Improving this measurement will be important for the next LHC run; another key measurement will be the detection of single top production in association with a $Z$. With 300 fb$^{-1}$ at LHC, the $t\bar{t}Z$ axial coupling can be measured with an uncertainty of about 10%, while the vector one only with an accuracy of 50%; increasing the luminosity to 3000 fb$^{-1}$ the achievable limits will improve typically by a factor 2–3 [141].

An $e^+e^−$ collider will be the ideal environment to test the coupling of top quarks with $\gamma$ and $Z$ bosons. As the $e^+e^− \rightarrow t\bar{t}$ process mixes photon and $Z$ exchanges, having polarized beams, or measuring top-quark polarization in the final state, will be important to measure independently such couplings. Ref. [86] described the reach of the linear colliders ILC and CLIC, with polarizations of electrons and positrons equal to 80% and 30%, respectively, and $\sqrt{s} = 500$ GeV, finding that the expected precision is at the level of 0.1%, namely $2 \times 10^{-3}$ for the coupling to photons and between $3 \times 10^{-3}$ and $5 \times 10^{-3}$ for $t\bar{t}Z$. FCC-ee is expected to measure the couplings with an even better sensitivity, thanks to a higher luminosity, by measuring the top polarization in the final state from angular distributions at $\sqrt{s} = 365$ GeV [142].

Top couplings are sensitive to deviations from the SM as expected in various BSM schemes like Randall-Sundrum models, Little Higgs, and composite Higgs scenarios which assume that the top carries a great deal of compositeness. These models either assume mixing between the top quark and new heavy fermions or mixing between the SM gauge bosons with new heavy vector states which couple to the top quark and can induce variations of EW couplings. In [143] a wide spectrum of predictions for deviations of the $Ztt$ couplings in various models is considered. They are expressed as deviations in the $ZtLt_L$ and $ZtRt_R$ couplings since various models predict different contributions for the two top helicity components (Table I of [143], and references therein). These deviations, for a NP scale around 1 TeV, are pictorially drawn in Fig. 17 (left-panel), where the purple points represent different BSM models and the black ones correspond to natural choices for Composite Higgs Model parameters [144]. The possibility to distinguish the left- and right-handed couplings allows to disentangle various models. The region within the dashed red lines represents the sensitivity which can be reached by the LHC (HL-LHC) with 300 fb$^{-1}$ (3000 fb$^{-1}$) [135, 145] through the $Ztt$ cross-section, while the ILC sensitivity (detailed by dashed dark-blue lines) for $\sqrt{s}=500$ GeV with 500 fb$^{-1}$ and polarized beams goes down to sub-percent level [146]. The sensitivity at FCC-ee for $\sqrt{s}=365$ GeV with 2.4 ab$^{-1}$ is represented by the solid (dashed) green ellipse, obtained with the angular and energy distributions of leptons (b-quarks) in $t\bar{t}$ events [142]. At ILC and FCC-ee all top EW couplings, including $Wtb$, could be measured allowing full separation between axial and vector couplings, and between the $Ztt$ and $\gamma tt$ couplings. These analyses are described in [142,147]; Fig. 17 (right-panel) recalls these performances in terms of the CP conserving axial and vector form factors of the $Xtt$ vertex. The ILC expected accuracies for the $t_L$ and $t_R$ couplings to the $Z$ are $\Delta(ZtLt_L)/ZtLt_L(\%) = 0.6$, $\Delta(ZtRt_R)/ZtRt_R(\%) = 1.4$.

As shown in ref. [148], the total cross-section $\sigma(e^+e^- \rightarrow t\bar{t})$, the forward-backward asymmetry $A_{FB}$, and the spin asymmetry $A_L = \frac{N(−,−)+N(−,+)+N(+,+)+N(+,−)}{N_{\text{tot}}}$ are powerful observables to dis-
Fig. 17: Left: Predicted deviations of Z couplings to $t_L$ and $t_R$ in various NP models [143, 148]. Also shown are the expected sensitivities of LHC (300 fb$^{-1}$ and 3000 fb$^{-1}$), ILC and FCC-ee. Right: Comparison of statistical precisions on CP conserving axial and vector form factors expected at LHC with 300 fb$^{-1}$ [145] at ILC500 with 500 fb$^{-1}$ [147] and at FCC-ee with 2.4 ab$^{-1}$ [142]. The FCC-ee (ILC) projections are obtained at $\sqrt{s} = 365$ GeV ($\sqrt{s} = 500$ GeV). In the case of FCC-ee lepton-angular and energy distributions are used, while ILC projections are based on the use of beam polarization.

criminate among different scenarios. In the definition of the spin asymmetry $N$ denotes the number of observed events and its first (second) argument corresponds to the helicity of the final state top (antitop), whereas $N_{tot}$ is the total number of events. Figure 18 shows the deviation from the Standard Model expectation for these observables in composite Higgs models where the Higgs is a PNGB (pseudo-Nambu Goldstone boson) of $SO(5) \rightarrow SO(4)$ breaking.

The determination of the Yukawa coupling of top quarks is clearly a crucial one, since the coupling to the Higgs boson provides the highest corrections to the Higgs mass at one loop, leading to the well known naturalness problem. In order to extract the Yukawa coupling, one would need to measure the cross section of the process $pp \rightarrow t\bar{t}H$. Though only upper limits on $pp \rightarrow t\bar{t}H$ production have been determined in the first LHC run at 7 and 8 TeV, a measurement of the top-Yukawa coupling is foreseen for the forthcoming run at 13 and 14 TeV, with an expected uncertainty of about 15% at 300 fb$^{-1}$ and then 10% at 3000 fb$^{-1}$ [46]. Even better measurements of the Yukawa coupling are among the goals of lepton colliders: for 1000 fb$^{-1}$ collected at ILC, the foreseen uncertainties are 10% at $\sqrt{s} = 500$ GeV and 4% at 1 TeV, respectively, under the assumption that the polarization rates are 80% for electrons and 30% for positrons. As for CLIC, the note [149] investigates the potential for a direct measurement of the top Yukawa coupling. The relative error scales like $0.53 \times \Delta \sigma/\sigma$, $\sigma$ being the cross section for $t\bar{t}H$ production, so that, for $e^+e^- \rightarrow t\bar{t}$ annihilation at 1.4 TeV, a precision of 4% can be achieved without beam polarization. At FCC-ee centre-of-mass energies no direct $t\bar{t}H$ production is possible, however the Yukawa coupling can be determined by a threshold scan of the $e^+e^- \rightarrow t\bar{t}$ cross section, in order to be sensitive to Higgs exchange, besides the $Z$ and photon contributions. The projected sensitivity is about 30%, thus below the expectations of ILC and CLIC [146]. A very precise measurement is expected at FCC-hh, thanks to a $t\bar{t}H$ production cross section 60 times higher than at LHC(14 TeV): a precision of 1% on value of the top-Higgs Yukawa couplings is achievable.

4.3.3 Measurements of other top quark properties

A complete discussion of measurements of top properties at present and future colliders is beyond the scope of this document. Here two additional relevant examples are briefly mentioned: the search for flavour changing neutral current (FCNC) top decays and the measurement of top dipole moments.

The fact that in the standard model top quarks decay essentially 100% to $bW$ makes $t\bar{t}$ pairs an
ideal system to detect new physics by selecting a standard top decay and searching for a rare decay mode for the other top in the pair. Similarly, single top production occurs via electroweak interaction only through the $tWb$ vertex, making the search for other production modes an unambiguous signature of physics beyond the standard model. The search for FCNC top interactions exploits both methods (top decay and single top production) and it is particularly relevant as FCNC are heavily suppressed in the standard model because of the unbroken QED and QCD symmetries and the GIM mechanism, while new-physics models often predict their enhancement. Signatures involving a $Z$ boson or a photon are particularly clean and the expected sensitivity essentially scales with statistics. The reach attainable at HL-LHC for FCNC top processes is evaluated in ref. [150] and a study of the potential of FCC-ee is given in ref. [151] and summarised in Fig. 19, left panel, showing that there is considerable potential to cover unexplored territory.

In the standard model chromomagnetic ($d_V$) and chromoelectric ($d_A$) dipole moments of the top quark are expected to be small ($\mathcal{O}(10^{-3})$), however their value can significant increase in extensions of the standard model [152]. Anomalous dipole moments modify observables related to $t\bar{t}$ production (e.g. spin correlations [153]) and they can be observed with enhanced sensitivity if high $p_T$ top candidates, in the TeV range, are selected [154]. The large integrated luminosity to be collected at the LHC can be exploited, in this respect, and even higher sensitivity can be obtained at FCC-hh because one trillion $t\bar{t}$ pairs are expected for 10 ab$^{-1}$ at a centre-of-mass energy of 100 TeV, with a large amount of pairs whose invariant mass lies in the multi-TeV range. The current bounds on the $d_V$ and $d_A$ values and the expected
reach at LHC(14 TeV) and FCC-hh are shown in Fig. 19, right panel.

4.3.4 Final-state kinematics

Studying kinematic distributions relying on top production and decay does provide important tests of the Standard Model and allows one to investigate several new physics scenarios. The complete differential process $pp \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b}$ has been computed in the NLO approximation, with [155,156] and without [157] including top width effects.

Among the observables which have been investigated, the top transverse momentum spectrum has been calculated by means of resummed calculations, carried out using standard techniques [158] and in the framework of SCET [159], wherein even the $t\bar{t}$ invariant mass $m_{t\bar{t}}$ has been computed. Although such computations generally agree with the experimental data, it was found [160], by using the NLO MCFM program [161], that the uncertainty on the $p_T$ spectrum in the boosted regime, i.e. the top decay products clustered into a single jet, is about twice larger than in the unboosted case. Such a result clearly calls for a full NNLO calculation.

An important final-state observable is the forward-backward asymmetry, which has represented for some time an open issue, since it exhibited a two-standard-deviation excess at the Tevatron [162], when compared with NLO QCD predictions. However, the recent calculation [163] of the full NNLO corrections to the asymmetry, which is also the first differential NNLO computation for $2 \rightarrow 2$ QCD processes, has shown agreement with the D0 data [164], whereas the disagreement with CDF [162] is reduced to 1.5 standard deviations.

At the LHC, such a measurement, which needs a $q\bar{q}$ initial state, is more difficult, since it is a $pp$ collider and $t\bar{t}$ production is mostly driven by $gg$ annihilation. In fact, ATLAS and CMS performed measurements of the asymmetry, in agreement with the Standard Model, but affected by large errors [164,165]. Enhancing the energy to 14 TeV will increase the production of $t\bar{t}$ pairs through $gg$ annihilation, which does not produce any forward-backward asymmetry. However, as discussed in [160], the uncertainties due to background modelling and lepton identification scale with the luminosity as $1/\sqrt{L}$ and therefore, after setting appropriate cuts on the $t\bar{t}$ invariant mass and centre-of-mass rapidity, the fraction of $q\bar{q}$ annihilation will increase, thus allowing an improved measurement of the asymmetry.
Two alternatives to the standard forward-backward asymmetry have been proposed in [166] in events $t\bar{t}+$jet: they are the energy and incline asymmetries, expressed in terms of the energy difference of $t$ and $\bar{t}$ and of the rapidity of the $t\bar{t}j$ system. After setting suitable cuts, such asymmetries should be about -12% and -4%, respectively, at the 14 TeV LHC.

At a linear collider, the main kinematic properties which are foreseen to be measured are the top production angle $\theta_t$ and the helicity angle $\theta_h$. In this way, one will be able to determine the forward-backward asymmetry and the slope of the helicity angle $\lambda_t$ with an accuracy of 2% in semileptonic events, as obtained in the simulations at $\sqrt{s} = 500$ GeV carried out in [147].

4.4 Status of Monte Carlo generators for high energy colliders

The development of high energy physics experiments carried out at colliders with increasing centre-of-mass energy, has seen a parallel development in the tools for the calculation and simulation of hard processes. In the 1980’s, calculation of collider processes were typically performed at tree level, and full simulation of the events relied upon the Leading-Log (LL) shower approximation. Next-to-leading order calculations were only available for a handful of processes.

In the last few years, prompted by the perspective of the LHC runs, a remarkable progress has taken place in several areas. Fully automated techniques have been developed for the calculation of Next-to-Leading Order (NLO) cross sections, by several collaborating and competing groups. Techniques for combining fixed order calculations with parton shower generators have appeared, and have been widely applied to collider processes. Intensive work on Next-to-Next-to-Leading Order (NNLO) calculations has been carried out by several groups, with several new NNLO results having appeared since a little more than a year. Methods for interfacing NNLO calculations to shower Monte Carlo generators have also appeared for relatively simple processes.

This section summarizes what is available at present, and illustrates what can be considered to be frontier research in this field. Although it is impossible to predict what will be available ten years from now, we believe that it may be safely assumed that current frontier research will have turned into commonly used tools by that time. Additional information on perspectives for advanced QCD calculations for future colliders can be found in [47], while a thorough discussion of Monte Carlo tools for electroweak measurements is given in section A.2.

4.4.1 Presently available results

Parton Shower Monte Carlo generators (PS) fully simulate hadronic production processes by merging together a QCD component (the Shower itself) and a model for hadron formation. The QCD component is typically given in the collinear approximation. When applied to infrared finite observables, PS generators are accurate only in the collinear and soft regions, failing to predict hard, large angle emissions even at leading order. In ref. [167] a procedure was developed for matching matrix element calculations with PS generators (ME+PS), such that the production of hard, widely separated jets could be improved to LO accuracy. This prompted the application of ME+PS techniques to various ME generation tools, like, for example in ALPGEN with the MLM matching procedure (for a list of available ME+PS generators see [168]).

In the past 10 years, considerable effort has gone in building NLO-improved PS generators (NLO+PS). Methods like Mc@NLO [169] and POWHEG [170, 171] allow to interface fixed order NLO calculations to parton shower generators like PYTHIA [172, 173] and HERWIG [174, 175]. In essence, for a given process, these techniques extend the precision of the generator to NLO level for inclusive processes, and to tree level for the given process in association with one jet. For example, an NLO+PS generator for Higgs
production (a process of order $\alpha_s^3$ at the Born level) will yield distributions accurate up to order $\alpha_s^4$. That amounts to NLO accuracy for inclusive quantities (i.e. quantities that do not depend upon the emission of associated jets, like the rapidity distribution of the Higgs, and already receive contributions at order $\alpha_s^2$), and to LO accuracy for processes involving the emission of an associated jet that start at order $\alpha_s^3$. These techniques have seen recently considerable progress, due to the appearance of computer frameworks that automatize some or all aspects of the calculation: the virtual contributions, the implementation of a subtraction framework for the real corrections, and the interface to a PS. In the \texttt{MadGraph5_aMC@NLO} framework \cite{176}, all aspects of an NLO calculation are automatized, starting from the generation of the LO and NLO matrix element, down to the event generation interfaced to a PS program. The \texttt{GoSam} \cite{177}, \texttt{Recola} \cite{178} and \texttt{Open Loops} \cite{179} frameworks deal with the automatic generation of general purpose virtual amplitudes. The Black Hat \cite{180} generator provides virtual corrections for selected processes (vector Boson in association with jets) and is capable to deal with fairly high jet multiplicities. In fact it was recently used to compute $W$ production with five associated jets at NLO \cite{181}. The \texttt{Sherpa} generator \cite{182} implements a framework for NLO calculations and for NLO+PS generation based upon a variant of the \texttt{Mc@Nlo} method. The so-called MatchBox framework \cite{183} implements NLO+PS generators within the \texttt{Herwig++} \cite{175} PS generator. The \texttt{POWHEG BOX} framework automatizes all aspects of the NLO calculation interfaced to a PS generator, except for the computation of the matrix elements. For these it relies upon other programs, like \texttt{MadGraph} and \texttt{GOSAM}.

Electroweak corrections are not presently included in publicly available automatic NLO calculators. It is however clear that the same techniques that have been applied for automated NLO QCD can be extended to the full Standard Model, as well as to any renormalizable model. Interfacing calculations including Electro-Weak corrections to Shower Monte Carlo requires the ability to handle together QED and QCD collinear showers, but it does not present new conceptual problems with respect to QCD corrections alone. In fact, in few simple cases NLO calculation matched with Shower generators have appeared in the literature \cite{184,185}.

### 4.4.2 NNLO calculations

Next-to-next-to-Leading Order calculations (NNLO) for collider processes have first appeared in 1990 for the Drell-Yan process \cite{186}, followed more than ten years later by the NNLO computation of the total Higgs cross section in gluon fusion \cite{187–189}, and of the Higgs differential distributions in \cite{190,191}. We have witnessed since then a steady increase in the complexity of the processes for which NNLO calculations have become available: 3 jet cross sections in $e^+e^-$ annihilation \cite{192}, $W H$ and $ZH$ production \cite{193,194}, $\gamma\gamma$ production \cite{195}. In a little more than a year from now, several new results for complex $2 \to 2$ processes have become available: Higgs production in association with a jet \cite{196}, $tt$ production \cite{118}, a partial result on inclusive jets production \cite{197}, $Z/W + \gamma$ production \cite{198}, $ZZ$ production \cite{199}, $W^+W^-$ production \cite{200} and $t$-channel single top production \cite{201}. Important results have also been obtained for decay processes \cite{202}.

There are several components that make up a NNLO calculation, besides the two loop corrections. One must also supply the square of 1-loop contribution (double virtual), the virtual correction to one real emission (real-real) and the two-real-emission contributions. Each contribution contains soft and collinear divergences, that must cancel in the sum. This also constitutes a challenging aspect of NNLO calculations. There are several techniques currently developed for implementing these cancellations. The $q_T$ subtraction method \cite{191} has been used for Higgs production, Drell-Yan, $\gamma\gamma$, $WH$, $ZH$ and $ZZ$ processes. It is particularly useful for processes where the final state is a colour neutral system. The Antenna subtraction method \cite{203} has been used for the computation of $e^+e^- \to 3\text{jets}$ and for dijets, and is presently also used in an effort to compute fully differential $tt$ production at NNLO \cite{204}. The so-called STRIPPER method (Sector Improved Phase sPaCe for real Radiation) \cite{205,206} has been used for $t\bar{t}$, $H + j$ and $t$-channel single top production. Another method being developed is described in a sequel of publications (see \cite{207} and references therein).
The computation of the double virtual contribution is very demanding. Recent progress with integrals including massive particles [208–210] have opened the possibility of computing NNLO corrections to pairs of massive vector bosons. In general, it seems that today two loop virtual corrections to generic $2 \rightarrow 2$ processes are feasible. A recent groundbreaking technique introduced by Henn [211] is among the developments that have made this possible.

4.4.3 Current developments: NLO+PS merging and NNLO+PS generators

NLO+PS merging deals with the merging of NLO+PS generators of different associated jet multiplicity. Consider for example Higgs production in gluon fusion, a process of order $\alpha_3^2$ at the Born level. Let us call $H$, $HJ$ and $HJJ$ the NLO+PS generators for the production of a Higgs, of the Higgs in association with a jet, and of the Higgs in association with two jets respectively. The $H$ generator will yield $\alpha_3^2$ accuracy; that is to say NLO accuracy for inclusive observable, like the Higgs rapidity distribution, that include terms of order $\alpha_3^2$ (LO terms) plus terms of order $\alpha_3^3$ (NLO terms), and LO accuracy for observables requiring an associated jet, that are given at the lowest order by terms of order $\alpha_3^3$. Observable requiring more than two associated jets will be generated by the Shower Monte Carlo in the collinear approximation. The $HJ$ generator is capable of yielding NLO accuracy (i.e., $\alpha_4^3$ accuracy) for observables involving the Higgs plus one jet, and LO accuracy for those requiring two jets. It would be however unpredictable for fully inclusive observables. A merged $H$-$HJ$ generator would have in addition NLO (i.e. $\alpha_3^3$) accuracy for fully inclusive observables. In general one may ask to merge even more NLO+PS generators, for example $H+HJ+HJJ$, in order to have NLO accuracy (i.e. $\alpha_3^5$) accuracy also for observables involving two associated jets, and thus LO accuracy for those involving three associated jets.

Notice that NLO+PS merging can be seen as an intermediate step in the construction of an NNLO+PS generators. Thus, for example, if we have an $H+HJ$ merged generator, we know that it is already accurate at the $\alpha_4^2$ level for all observable, except those that are totally inclusive in the emission of associated partons, where the accuracy is instead $\alpha_3^3$. If we could reach $\alpha_3^5$ accuracy for inclusive observables, we would have full NNLO accuracy.

Several methods have been proposed for NLO+PS merging, although the accuracy that they really meet is still a debated matter [212–217]. In particular, in refs. [212,213], carried out in the frameworks of the SHERPA and MC@NLO collaborations respectively, merging is performed using a merging scale. One clusters the event using some jet clustering procedure, characterized by a merging scale $Q_0$, and uses the generator with the appropriate number of jets. In [213], stability under variations of the merging scale is interpreted as an indication of accuracy. In ref. [215], NLO accuracy is adjusted by forcing the inclusive distribution to agree with the NLO one, by subtracting appropriate terms, with a procedure dubbed UNLOPS (standing for “Unitary” NLOPS). In ref. [216] (within the so-called GENEVA collaboration) the merging scale is defined in such a way that resummation can be carried out up to the NNLL level. In refs. [218] (the MiNLO method) a method was proposed to improve the accuracy of a generator in such a way that it becomes reliable also after integrating out a radiated parton. In ref. [219] it was also shown that in certain simple cases the MiNLO method applied to generators for a Boson (Higgs, Z or W) plus one jet, can be refined in such a way that it becomes NLO accurate also for inclusive quantities.

In ref. [220] a fast NNLOPS accurate generator for Higgs production in gluon fusion was presented, based upon the MiNLO procedure of ref. [219]. The same method discussed above was also applied recently to the Drell-Yan process [221]. In refs. [222,223] NNLOPS generators were built for the Drell-Yan process and for Higgs production respectively.

In ref. [224], a general strategy for NNLOPS generators based upon the GENEVA framework was outlined. No complete application of this method to physical processes has been published, although preliminary results on the Drell-Yan process have been shown to conferences [225].
4.4.4 Conclusions

In conclusion, at present generators for NLO calculations matched to shower are obtainable with a certain ease for processes with up to four particles in the final state. It is conceivable to imagine that automated generators for electroweak corrections for generic processes may become available soon. While generators for merged mutlijet samples (i.e. for processes with an arbitrary number of associated jets), with LO accuracy, have been available for quite some time, NLO accurate merged generators are now beginning to appear, and are still subject of debate. NNLO calculation for processes with up to two particles in the final state have recently appeared for a considerable number of processes, and NNLO calculation matched to shower generators have appeared only for Higgs production in gluon fusion and Drell-Yan processes. It is conceivable that within the next decade NNLO calculations matched with shower will become generally available, and that the problem of merging for NLO generators will be solved.