$\gamma$ spectroscopy of the exotic nucleus $^{133}\text{Sb}$ produced by cold neutron induced fission on $^{235}\text{U}$ and $^{241}\text{Pu}$ targets

RELATORE: Prof.ssa Silvia Leoni
CORRELATORE: Prof.ssa Angela Bracco

Tesi di Laurea di
Beatrice Belvito
Matricola 827365

Codice PACS: 23.20.-g

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To my family
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Introduction

One of the main issues in nuclear physics concerns the interplay between single particle and collective degrees of freedom, which is a manifestation of the many body nature of the atomic nucleus. The nucleus under examination, $^{133}$Sb, has a proton outside the doubly-magic $^{132}$Sn core and it represents a good system in which particle-phonon coupled states can be investigated, since the collective vibrations are expected to be stronger around shell closures. In order to study the nature of these states, their lifetime has to be measured experimentally with a good precision.

In this thesis a lifetime analysis of the $^{133}$Sb nuclear excited states, performed with fast scintillator, is presented.

The $^{133}$Sb exotic nucleus was populated by $^{235}$U and $^{241}$Pu thermal neutron induced fission in an experiment conducted at Institut Laue-Langevin (ILL) of Grenoble. A complete explanation of fission mechanism is presented in Chapter 1. The experimental setup was made by 8 segmented clover HPGe detectors (EXILL), each equipped with an anti-Compton shield, coupled to an array of 16 LaBr$_3$(Ce) scintillator detectors (FATIMA) in 4π configuration. The combination of these two different types of detectors made possible to take advantage of the very good energy resolution (0.1% at 1332 keV) typical of the HPGe detectors and, at the same time, of the excellent time resolution (300 ps) of LaBr$_3$(Ce) scintillators. This experimental setup, with the detectors mode of operation, is described in Chapter 2.

The reaction channel of interest ($^{133}$Sb) was selected by gating on known transitions, whose precise identification was obtained by the excellent energy resolution of the HPGe detectors. On the other hand, the time delayed coincidence technique was used for lifetime measurements in a range from tens of picoseconds to few nanoseconds, by employing the LaBr$_3$(Ce) scintillators. This is the so-called "fast-timing technique", explained in Chapter 3. In this work we focused on two principal aspects. First, we checked the fast timing method reliability on three different time regions (tens of ps, hundreds of ps and ns). In this part of the work we studied and confirmed the lifetimes, already known in literature, of several states of three different
nuclei produced by the reaction on $^{235}$U, in particular the $^{100}$Zr, $^{132}$Sn and $^{90}$Kr nuclei (Chapter 3). In the second part of this work, the lifetimes of two $^{133}$Sb excited states, $13/2^+$ and $15/2^+$, were investigated. The results, obtained by the experimental analysis performed with the two targets, are in good agreement: a lifetime of $37(10)$ ps and a preliminary lifetime of $89(14)$ ps were measured for the $13/2^+$ and $15/2^+$ states, respectively. These analysis and results are shown in Chapter 4.

In conclusion, this study represents a starting point in the investigation of the interaction between collective excitation and single particle states. Experimental informations are provided on $^{133}$Sb, to be compared with theoretical models in order to improve the knowledge of this phenomenon and to confirm the nature of these states.
Chapter 1

Fission reaction

In this chapter we will explain the main aspects and principles of fission reactions [1, 2, 3]. A brief introduction on fission reactions studies on the $^{133}\text{Sb}$ nucleus will be given at the end of this Chapter [4, 5, 6].

1.1 Fission: theoretical aspects

The discovery of the neutron by Chadwick in 1932 led to intense investigations of the effects of exposing various nuclei to this new projectile. Of particular interest was the possibility of obtaining a transuranic element in neutron capture by uranium, the heaviest known element occurring in nature and with the highest atomic number known at the time. In fact, E. Fermi and his team soon experimentally verified that the element neptunium, Z = 93, can be produced in this way, after the capture of a neutron by uranium and emission of an electron. But the new discovery revealed a much more complex situation. Besides the presence of neptunium, activities were detected that could not be clearly attributed to any element in the neighbourhood of uranium. O. Hann and F. Strassmann showed, in 1939, after careful radiochemical study, that the observed activities were due to several elements with roughly half the mass of uranium. This led L. Meitner and O.R. Frisch to finally give the correct interpretation of the phenomenon. They proposed that, in capturing the neutron, the uranium can divide into two fragments of comparable masses and highly unstable. Meitner and Frisch called this process fission (a term borrowed from the
The same authors soon recognized, by an analysis of the binding energy of the participants, that the process releases a large amount of energy, close to 200 MeV per fission. Still in 1939, as the result of several studies, it was established that fission does not occur preferentially into two equal fragments but with masses distributed around $A = 95$ and $A = 140$.

Nuclear fission is nowadays a very established phenomenon: it results primarily from the competition between the nuclear and Coulomb forces in heavy nuclei. The total nuclear binding energy increases roughly in proportion to the mass number $A$, while the Coulomb repulsion energy of the protons increases faster, like the square of the atomic number $Z^2$. If we regard the emission of a heavy fragment as a decay process similar to a $\alpha$ decay, then we can regard heavy nuclei as residing very close to the top of the potential well, shown in Fig. 1.1, where the Coulomb barrier is very thin and penetrable.

![Smooth potential barrier opposing the spontaneous fission. In the figure the activation energy is also shown [1].](image)

Fission can thus occur spontaneously as a natural decay process, or it can be induced through the absorption of a relatively low-energy particle, such as neutron producing excited states that are high enough in energy.
1.1. Fission: theoretical aspects

to surmount or more easily penetrate the barrier. Although any nucleus will fission if we provide enough excitation energy, as a practical matter the process is important only for heavy nucleus (thorium and beyond) [1].

**Fission mechanism.** The phenomenon of fission can be understood both qualitatively and quantitatively on the basis of the liquid drop model. Qualitatively, this model assumes nuclei to be spherical. However, for very large nuclei, a spherical shape need not necessarily to be stable. Furthermore an external perturbation, such as an incident neutron, can create surface waves that can lead to a change in the shape of a liquid drop. The liquid drop can, for example, elongate as a result of the perturbation. If the produced deformation is sufficiently large, Coulomb repulsion between the elongated portions of the drop can produce a two-lobe structure that can push the lobes further apart, causing a complete split or fission of the initial nuclear drop into two droplets, as shown in Fig. 1.2.

\[
\text{Figure 1.2: Schematic representation of the fission splitting.}
\]

The drop model also provide a quantitatively description of nuclear fission if we consider the semiempirical formula for the binding energy

\[
B.E.(A,Z) = a_1 A - a_2 A^{\frac{2}{3}} - a_3 \frac{Z^2}{A^{\frac{4}{3}}} - a_4 \frac{(N-Z)^2}{A} \pm a_5 A^{-\frac{3}{4}} \tag{1.1}
\]

where \( a_1, a_2, a_3, a_4 \) and \( a_5 \) are experimental coefficients, while each term represents an energetic component deriving from shape and symmetry argumentation. This empirical formula has three classical terms that depend explicitly on the shape of the drop, namely the volume energy (the first one), the surface energy (the second) and the Coulomb energy (the third). Thus, if we assume that a spherical liquid drop of radius \( R \) deforms very slightly under some external perturbation to an ellipsoid of the same volume, with semi-major and semi-minor axes \( a \) and \( b \), respectively, we can write \( a \) and \( b \) in terms of a small parameter of deformation \( \epsilon \) as

\[
a = R(1 + \epsilon) \\
b = \frac{R}{(1 + \epsilon)^{\frac{3}{2}}} \tag{1.2}
\]
1.1. Fission: theoretical aspects

This choice of parametrization guarantees that the volume of the liquid drop remains unchanged

\[ V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi ab^2 \quad (1.3) \]

Since the volume is identical for the sphere and for the ellipsoid, the volume energy will be the same for both the original and the deformed liquid drops. However, the surface energy and the Coulomb energy will differ for the two cases. By comparing the surface area of an ellipsoid to that of a sphere, the surface and the ellipsoid energy assumes the forms respectively

\[ a_2 A^{\frac{2}{3}} \rightarrow a_2 A^{\frac{2}{3}} \left(1 + \frac{2}{5} \epsilon^2 \right) \quad (1.4) \]

\[ a_3 \frac{Z^2}{A^{\frac{1}{3}}} \rightarrow a_3 \frac{Z^2}{A^{\frac{1}{3}}} \left(1 - \frac{1}{5} \epsilon^2 \right) \quad (1.5) \]

The above deformation increases the surface energy while decreasing the Coulomb term. The stability of the droplet therefore depends on how these two terms compete with each other. The total change in binding energy due to the deformation can be written as

\[ \Delta = B.E.(\text{ellipsoid}) - B.E.(\text{sphere}) \]

\[ = \left(-\frac{2}{5} a_2 A^{\frac{2}{3}} + \frac{1}{5} a_3 \frac{Z^2}{A^{\frac{1}{3}}} \right) \epsilon^2 \quad (1.6) \]

If this energy difference is positive, we gain energy through the stretching and the more the nucleus is stretched, the more energy is gained. Such a nucleus is unstable against the stretching and will readily undergo fission. Therefore, for spontaneous fission, the condition is, considering the experimental values of \( a_2 \approx 16.8 \) and \( a_3 \approx 0.72 \),

\[ \frac{1}{5} a_3 \frac{Z^2}{A^{\frac{1}{3}}} > \frac{2}{5} a_2 A^{\frac{2}{3}} \]

or

\[ \frac{Z^2}{A} > 47 \quad (1.7) \]

This estimate must be modified somewhat to account for quantum mechanical barrier penetration, which permits spontaneous fission even when the deformation energy is negative. Furthermore, the nuclei in the region around uranium have permanent equilibrium deformation, the equilibrium shape is ellipsoidal rather than spherical. Nevertheless, the parameter \( Z^2 / A \), called
1.1. Fission: theoretical aspects

*fissionability parameter*, serves as a rough indicator of the stability of a nucleus to fission spontaneously. Fig. 1.3 shows the spontaneous fission half-life, as a function of this parameter. It is clear that the half-lives decrease toward the limiting value \( Z^2/A \approx 47 \); thus fission would occur “instantly” on the nuclear time scale for nuclei beyond the critical value of \( Z^2/A \).

![Figure 1.3: Spontaneous fission half-lives of some nuclei, considering the fissioning parameter [3].](image)

**Mass distribution of fission fragments.** The way a nucleus divides itself in a fission process is not identical for all events. A typical neutron-induced fission reaction is

\[
^{235}\text{U} + n \rightarrow ^{93}\text{Rb} + ^{141}\text{Cs} + 2n
\]

which is possible for incident neutrons of thermal energies. The fission products are not determined uniquely: there is a distribution of masses of the two fission products of the form shown in Fig. 1.4. The distribution must be symmetric about the center: for every heavy fragment, there must be a corresponding light fragment. It must be noticed, however, that fission into equal or nearly equal fragments (\( A_1 \approx A_2 \)) is less probable by a factor of about 600 relative to the maximum yield of fragments with \( A_1 \approx 95 \) and \( A_2 \approx 140 \).
1.1. Fission: theoretical aspects

Figure 1.4: Mass distribution of fission fragments from thermal fission of $^{235}$U. It can be notice the symmetry of the heavy and light distributions, even in the small variations near the maxima [3].

The reason for such behaviour is the influence of the shell structure of the nucleus. Hence the effect should be less important for large excitation energies, where the high level density has to include an average over a large number of states and not just a few isolated ones. It is in fact experimentally verified that asymmetric fission tends to the symmetric case when the projectile energy increases. This is seen in Fig. 1.5 for the case of $^{238}$U bombarded with protons from 10 MeV to 200 MeV. We see that from 150 MeV on, the minimum in the symmetric fission disappears, giving place to a plateau where the production of $A = 100$ to $A = 130$ fragments is equally probable. The behaviour of $^{238}$U is typical of a highly fissionable nuclide, as normally are the ones of very large mass.

Emitted neutrons. Fission fragments are necessarily rich in neutrons. This is expected as the fissioning nucleus has a neutron to proton ratio larger than that necessary in balancing the fragments, which have much smaller atomic number. The fission fragments shed this neutron excess through the emission of one or more neutrons at the instant of fission (within $10^{-16}$ s). These neutrons are known as prompt neutrons, and the number of prompt neutrons emitted in a given fission event will vary with the nature of the two fragments and, in case of induced fission, with the energy of the incident particle. The average number of prompt neutrons is characteristic
Figure 1.5: Cross sections for the production of a mass number $A$ fragment in $^{238}$U fission induced by protons of various monoenergetic energies [3].
of the particular fission process; for thermal neutron-induced fission, the experimentally observed values are 2.48 for $^{233}\text{U}$, 2.42 for $^{235}\text{U}$ and 2.86 for $^{239}\text{Pu}$. In Fig. 1.6 is shown the kinetic energy distribution of prompt neutrons: we can see that this distribution is wide, with a maximum around 0.5 MeV. The fitted curve

$$N(E) = \exp(-1.036E) \sinh\sqrt{2.29E}$$

well represents the experimental values from 0 to 10 MeV.

![Figure 1.6: Kinetic energy distribution of prompt neutrons for the fission of $^{239}\text{U}$ [3].](image)

In addition to prompt neutrons, *delayed neutrons* are often emitted in fission. These neutrons are emitted following the $\beta$ decay of the fission fragments with delay times quite short, usually of the order of seconds; they represent only 1% of all the neutrons emitted in fission.

**Cross section for fission.** Fig. 1.7 shows the fission cross section for $^{235}\text{U}$ and $^{238}\text{U}$ bombarded by neutrons from 0 to 10 MeV. From this figure, we can see that neutrons with energies close to zero are already able to fission $^{235}\text{U}$, which indicates that the $^{236}\text{U}$ activation energy (the energy need to
induce fission, essentially the height of the fission barrier above the ground state) is lower than the neutron binding energy. In this way the fission barrier is overcome and the compound nucleus fissions. With increase in energy, the cross section reduces until reaching, at 1 MeV, a value 600 times less than the corresponding thermal neutron value, after crossing a region of resonances, between 0.1 eV and 1 keV. On the other hand, $^{238}\text{U}$ does not fission by slow neutrons, with its fission threshold near 1 MeV. The energy gained by the binding of the incident neutron is not enough, in this case, to reach the summit of the barrier, and it is also necessary for the contribution of the neutron kinetic energy to reach the activation energy.

This difference in nuclei very close to each other is attributed to the pairing energy, which contributes with the last term to the mass formula (Eq. 1.1). This energy implies that an even-even nucleus is, on the average, more bound than its odd neighbours. When $^{235}\text{U}$ absorbs a neutron, the binding energy per nucleon increases, being converted into excitation energy; in this way the summit of the fission barrier is more easily reached. The absorption of a neutron by $^{238}\text{U}$, on the other hand, decreases the binding energy per nucleon, and part of the excitation energy will be used increasing the $^{239}\text{U}$ mass per nucleon. The difference has to be delivered by neutron incident energy and this explains why there is not fission of $^{238}\text{U}$ by neutron of energy lower than 1 MeV.

**Energy distribution.** The fission of a heavy nucleus releases about 200 MeV. The larger part of this energy ($\simeq 165$ MeV) is spent as kinetic energy

---

**Figure 1.7:** Fission cross sections of $^{235}\text{U}$ and $^{238}\text{U}$ bombarded by 0 to 10 MeV neutrons [3].
of the fragments. This is due to linear momentum conservation, which implies that the ratio between the energies of the fragments is equal \( \frac{m_1 V_1}{m_2 V_2} \) to the inverse ratio of the masses.

\[
\frac{\frac{1}{2}m_1 V_1^2}{\frac{1}{2}m_2 V_2^2} = \frac{V_1}{V_2} = \frac{m_2}{m_1}
\]

Thus the kinetic energy distribution of the fragments has the aspect shown in Fig. 1.8, where the 92.5 MeV peak corresponds to the light fragment and the 62.0 MeV peak to the heavy one.

![Figure 1.8: Kinetic energy distribution of the fragments in the fission of \( ^{235}\text{U} \). The ordinate indicates the number of counts of each experimental point \[3\].](image)

The rest of the available energy is shared by neutrons, \( \gamma \)-ray, and also the electrons of \( \beta^- \) decay. Neutrons consume about 5 MeV, that is the result of an average energy of 2 MeV multiplied by an average number of 2.5 neutrons by fission. \( \beta^- \) decay releases an approximate energy of 7 MeV, while prompt \( \gamma \) rays and the products of the fragment decay carry a total of 15 MeV. Finally the neutrinos emitted together with the electrons are responsible for 12 MeV of the total energy (however they cross all the experimental apparatus without being detected).

**Effect of shell model on fission.** In the first section we have treated fission as a collective phenomenon according to the liquid-drop model so to provide a useful mental image of the process. Nevertheless, we have to consider that shell effects also play an important and in many cases decisive
role in determining the outcome of fission. As a clue to the importance of shell structure we can consider the asymmetric mass distribution of the fragments. Fig. 1.9 shows the mass distributions for $^{236}\text{U}$ and for several other fissioning nuclei of heavier mass.

![Figure 1.9: Mass distributions of fission fragments from thermal-neutron induced fission of $^{233}\text{U}, ^{236}\text{U}, ^{240}\text{Pu}$, along with the spontaneous fission of $^{252}\text{Cf}$. Shaded areas show approximate locations of closed-shell nuclei [7].](image)

These distributions reveal an unexpected feature: the mass distribution for the heavy fragments overlap quite well, while the light fragment shows a large variation. Comparing $^{236}\text{U}$ with $^{252}\text{Cf}$, we note that $Z$, $N$ and $A$ all increase by about 8.5%, and if the liquid-drop model of fission were a complete description of the process, we should expect both the heavy and light distributions to shift by about 8.5% between $^{236}\text{U}$ and $^{252}\text{Cf}$; that is, the average masses should go from about 95 and 140 in $^{236}\text{U}$ to about 103 and 152 in $^{252}\text{Fm}$. Instead, the observed average masses in $^{252}\text{Cf}$ are about 115 and 145. Another indication of this effect is clear if we consider the
average masses of the light and heavy fragments over a mass range from 228 to 256. The average mass of the heavy fragments stays nearly constant at about 140, while the average mass of the lighter fragment increases linearly as \( A \) increases. Throughout this entire range, the added nucleons all go to the lighter fragment while in a liquid drop model fission we would expect the average masses to scale roughly with the mass of the drop. The explanation for this unusual observed behaviour lies within the shell model. Fig. 1.9 shows the regions in which we would expect to find fission fragments with shell-model *magic numbers* of protons or neutrons. That is for \( Z = 50 \) we have a stable nucleus with \( Z/A = 0.4 \) (and thus with \( A = 125 \)) and neutron-rich fission products ranging down to a minimum of about \( Z/A \approx 0.38 \) (corresponding to \( A = 132 \) and thus about 7 neutrons from stability). Just at the lower edge of the heavy fragment mass distribution there is the doubly magic nucleus \(^{132}_{50}\text{Sn}_{82}\). This exceptionally stable configuration determines the low edge of the mass distribution of the heavier fragment. No such effect occurs for the lighter fragment, and indeed the light fragment mass distribution has practically no overlap even with singly magic nuclei and is thus unaffected by shell closure.

Another and more drastic effect of shell structure occurs in the fission barrier itself. As we begin to stretch the nucleus (characterized by an eccentric parameter \( \epsilon \)), the energy increases like \( \epsilon^2 \), as in Eq. 1.6, giving an approximate parabolic dependence. The single-particle states in the now deformed nucleus vary with deformation according to the Nilsson model (see Fig. 1.10). As it is seen in the Fig. 1.10, for same states the energy increases with deformation, while for some others it decreases. If the valence nucleon are in a state that happens to have a positive slope, the net increase in energy with deformation will be a bit faster than the parabola, since the single particle energy also increases with \( \epsilon \). At some point, however, as \( \epsilon \) increases we encounter a crossing with a state of negative slope. The valence particle, choosing the state of lowest energy, now follows the new state, and the net change of energy with \( \epsilon \) is a bit below the parabola. It remains so until a new crossing with a state whose energy increases with \( \epsilon \), so that the total energy is once again above the parabola. This oscillation due to the changing behaviour of the valence particles with \( \epsilon \) is shown in Fig. 1.11, together with the evolution of the barrier.

At the point where fission begins to occur, the form of the single barrier becomes modified and the energy dependence introduced by the single-particle shell structure results in a fission barrier with two humps. The net effect is that to have a high probability to fission, we no longer need to excite the nucleus since it is close to the top of the barrier. If we excite it above the bottom of the well between the two humps, penetration of the thinner
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Figure 1.10: Nilsson diagram, showing the energy levels as a function of the deformation $\epsilon$ and also the number of nucleons at shell closure, for $\epsilon = 0$ [3].
barriers becomes more probable and fission can occur. The introduction of the double-humped barrier was necessary following the 1962 discovery of *isomeric fission*: isomeric excited states with unusually short half-lives for spontaneous fission [1]. These fission isomers are typically at excitations of 2-3 MeV (and thus far below the barrier height of 6-7 MeV), but their half-lives for spontaneous fission are in the range of $10^{-6}$-$10^{-9}$ s. It was postulated that these isomers were actually states in the intermediate potential well and could decay either by fission through a relative thin barrier, or by $\gamma$ emission back to the ground state. The ordinary ground state is usually one of the modest stable deformation, while the well corresponds to states of a much larger deformation.
1.2. $^{133}$Sb

The reactions studied in this work concern fissions on two targets, $^{235}$U and $^{241}$Pu. These reactions were induced by cold neutron (with meV energies) produced in a nuclear reactor. In the case of $^{235}$U fission, the nucleus of interest $^{133}$Sb was produced by the reactions, involving its partner Nb and from 0 to 3 evaporated neutrons.

\[
\begin{align*}
\text{n} & + ^{235}_{92} \text{U} \rightarrow ^{236}_{92} \text{U}^* \\
& \rightarrow \begin{cases} 
^{133}_{51} \text{Sb} + ^{103}_{41} \text{Nb} \\
^{133}_{51} \text{Sb} + ^{102}_{41} \text{Nb} + \text{n} \\
^{133}_{51} \text{Sb} + ^{101}_{41} \text{Nb} + 2\text{n} \\
^{133}_{51} \text{Sb} + ^{100}_{41} \text{Nb} + 3\text{n}
\end{cases}
\end{align*}
\]

Whereas, in case of $^{241}$Pu, the partner is represented by Tc following the reactions

\[
\begin{align*}
\text{n} & + ^{241}_{94} \text{Pu} \rightarrow ^{242}_{94} \text{Pu}^* \\
& \rightarrow \begin{cases} 
^{133}_{51} \text{Sb} + ^{109}_{43} \text{Tc} \\
^{133}_{51} \text{Sb} + ^{108}_{43} \text{Tc} + \text{n} \\
^{133}_{51} \text{Sb} + ^{107}_{43} \text{Tc} + 2\text{n} \\
^{133}_{51} \text{Sb} + ^{106}_{43} \text{Tc} + 3\text{n}
\end{cases}
\end{align*}
\]

The $^{133}_{51}$Sb$_{82}$ nucleus consists of the doubly magic core $^{132}_{50}$Sn$_{82}$ and one valence proton. Observing the level scheme obtained by a shell model with an harmonic oscillator potential (Fig. 1.12 [3]), we see that the valence proton occupies the $g_{7/2}$ level.
Figure 1.12: Level scheme obtained by shell model calculations with an harmonic oscillator potential, showing the break of the degeneracy by the spin-orbit interaction term and the magic number in the shell closure. The values in the first set of parentheses indicate the number of nucleons of each type that the level admits and the values in the second set of parentheses provide the total number of nucleons of each type up to that level. Finally, the numbers outside the parentheses indicate the total number of nucleons at shell-closure. The red circle indicates the positions of the last proton in $^{133}\text{Sb}$, while the blue one indicates that the neutrons of $^{133}\text{Sb}$ form a closed shell.
1.2. $^{133}$Sb

For this reason $^{133}$Sb is a key system for testing the nuclear shell model. It provides energies of single-particle proton excitations for the $^{132}$Sn region, as well as data to study couplings of a proton to excitation of the doubly magic core. In fact, as shown in Fig. 1.13, the $^{132}$Sn spectrum is characterized, around 4.5 MeV, by 3 phonon excitations, the $2^+$, $3^-$ and $4^+$, which are quite collective, having a strength of the order of 7 W.u.\(^1\). It is therefore expected that the single $g_\frac{7}{2}$ proton in $^{133}$Sb can couple to these excitations in $^{132}$Sn giving rise to a number of different multiplets of core-coupled states.

![Level scheme of $^{132}$Sn](image)

**Figure 1.13:** $^{132}$Sn level scheme. Energies indicated are in keV [6].

---

\(^1\)The Weisskopf unit (W.u.) is an estimate for a single-particle transition probability for each multipolarity.
Such multiplets of states can be seen, around 4 MeV, in the energy spectrum of $^{133}$Sb, shown in Fig. 1.14 obtained by studying the $\gamma$-decay of the 16.6(3) $\mu$s isomer at 4526 MeV [8]. In order to have a confirmation on the nature of these states, a lifetime analysis is required. Aim of this thesis is, therefore, providing a lifetime measurement of two $^{133}$Sb levels at energies of 4302 keV and 4464 keV, the $13/2^+$ and $15/2^+$; respectively.

\[\text{Figure 1.14: Partial level scheme of }^{133}\text{Sb. Excited levels and }\gamma\text{ transitions are labeled with energies given in keV [5].}\]
Chapter 2

Experimental setup

In this chapter we will describe the experimental setup used in our fission reaction [9] and we will briefly explain the working principles of the analysis software, named SOCOv2, used to elaborate the acquired data. The measurements were performed using a mixed array of high purity germanium (HPGe) detectors and Ce-doped LaBr$_3$ detectors in autumn 2012 and spring 2013. These detectors were placed around $^{235}$U and $^{241}$Pu targets, with thick backings, meaning that the fragments stopped in the material in $\sim$1 ps. Fission was induced by cold-neutrons with energies of meV from a collimated neutron guide of the reactor of the Institut Laue-Langevin (ILL) at Grenoble (France).

2.1 EXILL$\&$FATIMA array

The experimental setup consists of a collimation system producing a pencil beam of neutrons, with the capture flux of about $10^8$ s$^{-1}$ cm$^2$ at the target position, and an array of eight anti-compton shielded clover detector (composite detectors, each made of four Ge crystal), from the EXOGAM array of GANIL, and of 16 LaBr$_3$(Ce) detectors, from the FATIMA collaboration [10]. The neutrons are produced in the ILL research reactor [11] and delivered to the experimental area, where the array is placed, with a dedicated collimation system, designed to shape the beam at the end of the guide to a circular cross section with 1 cm diameter. The main concept of this collimation system consists of using a sequence of apertures made
from high neutron absorbing materials (in our case natural boroncarbide and enriched $^6$LiF). To avoid $\gamma$ ray background from boron, all according apertures are doubled with 5 cm thick lead absorbers downstream the neutron flux direction. The sequence of apertures is inserted in a round vacuum tube system, the inside walls of which are completely covered by 1 cm thick borated plastics. The boron of this plastics is supposed to absorb neutrons backscattered from the apertures. The total length of the apertures sequence is about 4 meters followed by a one meter free flight path section containing vacuum pump access. The target chamber of about one meter length itself is attached, followed by a one meter long beam dump system. The beam dump itself is a vacuum tube covered inside with 2 cm boron carbide and having a LiF absorber at the end. This collimation mechanism is shown in Fig. 2.1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig21}
\caption{Collimation system with the sequence of the apertures. At the end the free flight path (grey) and the target chamber (purple) are shown [10].}
\end{figure}

Depending on the measurement, in the target chamber two different actinide targets, sandwiched between dense backing, were placed:

- $^{235}$U: 99.7% enriched (675 $\mu$m) with a 25 $\mu$m thick Be backing. The measurement lasted from 26 February 2013 to 12 March 2013;
- $^{241}$Pu: 78.6% enriched (300 $\mu$m) with a 25 $\mu$m thick Be backing. The measurement lasted from 13 March 2013 to 22 March 2013;

As we can see from Fig. 2.2, two different configurations of the target chamber can be set depending on the target used. For $^{235}$U(n, f), and (n, $\gamma$) experiments, the target chamber was made of an aluminium pipe of 50 mm diameter and 2 mm wall thickness. It was, as already said, directly connected to the collimation and beam stop volumes, and vacuum was pumped. The beam stop consisted of a $^6$LiF absorber placed inside a long pipe section covered with B$_4$C and surrounded by a 20 cm thick Pb shielding, at about 2 m from the sample position. Samples were inserted and hold in teflon bags fixed via teflon wire to a small metal frame. The metal frame was inserted by opening the connection flange between target chamber and beam stop. If enhanced security configuration was demanding, as for $^{241}$Pu(n, f) experiment, a second inner target chamber could be inserted instead of the target holder frame. This second chamber is made of an aluminium tube of 35 mm inner diameter and 2 mm wall thickness. Along the beam axis
it was sealed 200 µm thick Zirkonium windows on entrances and exits sides to minimise neutron scattering. The outer target chamber was filled with Helium gas at 50 mbar and the inner chamber was equipped with a vacuum gauge allowing to monitor its integrity during the experiment. In both configuration, the chamber was surrounded with a 1 mm thick $^6\text{Li}$ rubber in order to absorb remaining scattered neutrons and reduce neutron induced background.

Figure 2.2: Schematic view of the double wall target chamber: in the lower part the target chamber system is shown as it was used in single wall configuration for $^{235}\text{U}$ and $(n,\gamma)$ experiments. In purple the insert frame, which holds the sample material in its teflon bag is seen. The upper part of the figure shows the double wall chamber configuration. The internal chamber is shown in green. It is a completely independent chamber, which is inserted into the outer wall chamber. The inner chamber is equipped with a vacuum gauge allowing to monitor its tightness during operation [10].

To complete the experimental setup, the detectors are positioned as close as possible around the target chamber described above. The HPGe detectors and the LaBr$_3$(Ce) scintillators were arranged in an array of rhombicuboctahedron geometry, as shown in Fig. 2.3.

As we can see from the figure, the central ring of the array around the target orthogonal to the beam direction has been equipped with the eight HPGe detectors with a target-to-detector distance of 14.5 cm. All the germanium detectors have an active BGO (bismuth germanate) Compton suppression shields so that all the signals from the BGO surrounding a Germanium detector were summed up and recorded together with those
2.1. EXILL & FATIMA array

![EXILL & FATIMA array diagram](image)

**Figure 2.3:** EXILL & FATIMA array. In the drawing the $4\pi$ configuration is shown: in the central ring the HPGe detectors are installed (yellow), while on both sides the LaBr$_3$(Ce) scintillators are placed. The beam line is also visible [12].

From the Germanium crystals. In addition, metal collimators were placed in front of the BGO detectors to reduce background and cross-talking between adjacent detectors. A dedicated automatic Nitrogen filling system was also developed for this experiment. Two crates were used to automatically fill up the detectors three times in a day. Moreover, all the internal temperature sensor were read allowing automatic emergency filling in case a pre-setted threshold is passed. On both sides of the central ring, eight 5% Ce-doped LaBr$_3$ detectors, with an energy resolution of 3.3(2)% (at 662 keV), were installed with angles of 40° and 140° relative to the beam direction. The faces of the cylindrical LaBr$_3$(Ce) crystals with diameter of 1.5 in. were placed at 8.5 cm relative to the target position and were almost touching each other. The LaBr$_3$(Ce) crystals differ only slightly in their length: eight crystals have a length of 1.5 in. while the others are 2 in. long [13].

The main features and working principles of HPGe and LaBr$_3$(Ce) detectors, used in this setup, are briefly explained in the next paragraphs [14]. Finally, in order to perform lifetimes measurements using fast timing technique, a particular electronic chain was needed. The electronic fast-timing circuit installed consists of constant fraction discriminators (CFD), that lets to precisely determine the arrival time of the signal, multi channel logic fan in/fan-out modules (FAN), that receive and send signals split or summed, time-to-amplitude converters (TAC). These latter are of fundamental impor-
2.1. EXILL & FATIMA array

Figure 2.4: Assembly of EXILL & FATIMA array. On the left the array is presented when only the HPGe detector were mounted, on the right the complete assembly is shown [12].

tance since they allow to produce signals whose amplitude is proportional to the time interval between a start and a stop signal. The electronic setup used in this experiment ensured that a TAC number $i$ could only be started by the LaBr$_3$(Ce) number $i$ and stopped by a LaBr$_3$(Ce) number $j$ with the hiring $i < j$.

2.1.1 Semiconductor detectors

The HPGe detectors used in our experiment are semiconductor detectors. This type of detectors uses the features of semiconductor materials which are tetravalent elements (usually Silicon and Germanium). In this materials the energy gap between conduction and valence band is of the order of 1 eV (at 300 K 1.115 eV for Silicon and 0.665 eV for Germanium), therefore the thermal energy can be sufficient to move some electrons from the valence band to the conduction one, leaving a hole. The electrical conductivity for a semiconductor comes both from electrons moving freely in the conduction band and from holes in the valence band. The couples are constantly produced by thermal energy but some of them combine until they reach a statistical equilibrium. In order to reduce the thermal component the detectors are cooled at 77 K by liquid Nitrogen. To improve the conductivity the crystal is doped, that is some impurities, pentavalent or trivalent materials, are inserted in crystal lattice. When a pentavalent material (As, Sb, P), called donor, is used, a n-type semiconductors is formed that is a crystal in which an electron can’t bond with an atom of the Ge or Si crystal lattice and therefore it places in an energy level near the conduction band inside the energy gap. The net effect is a situation in which the number of electrons is much greater than the number of holes. On the contrary, if a crystal is doped with trivalent impurities (Ga, B, In), called acceptors, a p-type semiconductor is created. In this case a covalent bound is not able
to form with an atom of Ge or Si and an energy level near the valence band inside the energy gap is made available creating an high concentration of holes. Using a semiconductor doped in the different ways, a n-p junction is created. Because of the different electrons and holes density in the two regions and therefore of the presence of a discontinuity in the conduction electron density, a net diffusion from regions of high concentration to those of low concentration must take place. Thus, there is some net diffusion of conduction electrons into the p-type material, where they quickly recombine with holes. The diffusion of conduction electrons out of the n-type material leaves behind immobile positive charges in the form of ionized donor impurities. Similarly, holes must also diffuse across the junction because they also see an abrupt density gradient. Each hole that is removed from the p side of the junction leaves behind an acceptor site that has picked up an extra electron and therefore represents a fixed and immobile negative charge. The combined effect is to build up a net negative space charge on the p side and a positive space charge on the n side of the junction. The accumulated space charge creates an electric field that diminished the tendency for further diffusion. The region over which the charge imbalance exists is called the depletion region and extends into both the p and n sides of the junction. When an incident radiation interacts in the depletion region, electron-hole pairs are created by ionization. Using an external field, obtained applying a reverse bias to the junction, these pairs can be collected and a signal proportional to the electron-hole pair number is formed.

High purity germanium (HPGe) detectors, as those used in our experiment (see Fig. 2.5), are semiconductor crystals manufactured from ultra pure germanium.

The reason that high level of purity in the material is desired has to do with the size of the depletion region. The depletion region is desired to be as large as possible. Considering the depletion equation

\[ d = \left( \frac{\epsilon V}{eN} \right)^{1/2} \]  

\( e \) is the electronic charge, \( \epsilon \) is the dielectric constant, \( V \) is the reverse bias voltage and \( N \) is the net impurity concentration in the bulk semiconductor material. As it follows, the lower the impurity concentration, the larger the depletion is. Germanium is chosen for the reason that current manufacturing techniques allow for Germanium to be refined such that the purity concentration is as low as \( 10^{10} \) atoms/cm\(^3\). At this level of impurity, a depletion depth of 10 mm can be obtained with a reverse bias of less than 1000 V.
2.1. Scintillator detectors

The scintillator detectors are constituted by materials that emit light if excited or ionized by incident radiation. Depending on the type of material used, the scintillators are called organic or inorganic. Organic scintillators are made of aromatic hydrocarbons materials in crystalline shape, or as a solvent in liquids or in matrices of amorphic polymers. Due to low atomic number $Z$, the organic scintillators are predominantly used for particle detection. Inorganic scintillators, instead, thanks to their higher density and $Z$ are the better choice for $\gamma$-ray spectroscopy. They are constituted of insulating crystalline material, among which the most used are NaI, BaF$_2$ and LaBr$_3$(Ce), in which a band structure and an energy gap of tens of eV exist. These detectors are based on the principle of fluorescence. The process of fluorescence is the prompt emission of visible radiation from a substance following its excitation by some means, usually indicated as scintillation. The incident radiation transfers energy sufficient to promote an electron from valence band to the conduction one, leaving a hole in the normally filled valence band. Between the two bands the forbidden band represents a region where electrons can never be found in a pure crystal. The return of the electron to the valence band by a de-excitation process causes the emission of a photon, whose energy is equal to the energy gap. Nevertheless this process is inefficient since the electron can be absorbed. To enhance the probability of visible photon emission during the de-excitation process, small amounts of an impurity are commonly added to inorganic scintillators. Such impurities, called activators, create special sites in the lattice at
which the normal energy band structure is modified from that of the pure crystal. As a result, energy states will be created within the forbidden gap, through which the electron can de-excite back to the valence band. These de-excitation sites determine the emission spectrum of the scintillator. With the correct concentration of activator, the fluorescence light passes through the lattice unperturbed. In order to create a signal that could be analyzed, it is necessary to convert the light emitted in an electric signal. For this reason a photomultiplier tube (PMT) has to be used. The PMT is an instrument that not only convert the light in an electric signal, but also amplify it. As, Fig. 2.6 shows, it is composed of three different parts:

- **Photocathode**, placed behind a glass or quartz window (transparent to scintillation light), is usually composed of materials with a high cross-section for photoelectric effect. In this way the photons can be converted into electrons by three steps: (a) the absorption of the incident photon and transfer of energy to an electron within the photoemissive material, (b) the migration of the electron to the surface and (c) the escape of the electron from the surface of the photocathode.

- **Dynodes** are electrodes placed after the photocathode in sequence of increasing voltage. Electrons from the photocathode are accelerated and caused to strike the surface of a dynode. Thanks to the increasing potential from a dynode to the next, the energy deposited by the incident electron results in the reemission of more than one electron from the same surface. Therefore, the electron number increases to each dynode of the same quantity as the potential difference is kept constant.

- **Voltage divider** is a chain of resistors used to keep each dynode at a potential 100-200 V more positive than the previous one and avoiding gain fluctuations.

A good example of scintillator detectors is the LaBr$_3$(Ce) that was used in our experiment. One of the most important characteristic of these detectors is the so called *Light Yield*. This parameter indicates the number of photons produced for unit of energy deposited by the radiation. From this value depends the energy resolution and also the time resolution. The higher is the Light Yield the better will be the detector performances. In the case of LaBr$_3$(Ce) the light yield is 63000 which is a factor of 2 higher than in the other type of scintillator material.
2.2 Data acquisition and analysis using SOCOv2

The data acquisition system, used to collect data during the EXILL campaign, processes and collects the energy signals of the LaBr$_3$(Ce), HPGe and BGO detectors and the amplitude of the TAC signals. Each signal is converted into an event which is stored trigger-less an a triple list-mode including the ID of the channel (indicating the detector acquiring the signal), the signal amplitude and its time of arrival. The full measurement was divided into many shorter measurements, called runs. For the $^{235}$U and $^{241}$Pu fission experiments, data were stored in 3590 and 2316 runs, respectively, each of 5 minutes. However, in a multidetector system, like EXILL, the events are being collected by various detectors and, for this reason, both parameters need off-line adjustment. Since the value of the signal depends on the properties of the detector and the associated electronics, before presenting on a common scale signals from various detectors, one needs to adjust them to this scales correcting for differences between various detectors and finding a function which translates these values to the units of the scale (calibration).

In order to standardise and simplify the analysis of trigger-less list-mode data a new C++ software package SOCOv2 (*Sorting code Cologne version 2*) was developed at the Institute of Nuclear Physics of Cologne [16].

2.2.1 Energy calibration and shift correction

The energy calibration is fundamental to map the binary representation of the signal, obtained with the acquisition system, back to their physical values. Usually, this is implemented using linear or quadratic polynomial. SOCOv2 generalises this and accepts calibration polynomial of arbitrary degree. Thus, it is able to provide a calibration of the form:
2.2. Data acquisition and analysis using SOCOv2

\[ C(x) = \sum_{i=0}^{N} c_i x^i \]  

(2.2)

where \( C \) represents the energy, \( c_i \) the parameters obtained up to an arbitrary number and \( x \) the channel of the signal acquired.

In our experiment the energy calibration was performed using a standard point-like \(^{152}\)Eu \( \gamma \)-ray source and the \(^{48}\)Ti(\(n_{th},\gamma\))\(^{49}\)Ti reaction on a 84 mg Ti sheet with an area of about 40 mm\(^2\). The parameters, indicated in Eq. 2.2, can be extracted starting from single (for each detector) spectra, created with the \textit{dump} tool contained in SOCOv2. HPGe detectors present a good linearity and the calibration is characterized by a first order polynomial, while the LaBr\(_3\)(Ce) scintillators need of a third order calibration polynomial. Moreover in LaBr\(_3\)(Ce) detectors, an effect that is often observed is a drift of detectors signals over time. This stems from induced changes of the signal voltage by temperature changes over time. Without addressing this issue, any calibration is only correct for a few runs and would thus induce a systematic error if used on other runs, increasing the FWHM of any peak and thus reducing the resolution. This effect is clearly indicated in Fig. 2.7, where an example of drift in LaBr\(_3\)(Ce) is shown.

![Figure 2.7: Calibration and shift correction of LaBr\(_3\)(Ce) spectra: on the left, reference (blue) and shifted spectra (red) showing a shift after 48 hours, while on the right the reference spectrum and the calibrated shift-corrected spectrum are presented.](image)

The approach SOCOv2 takes, for compensating these shifts, is to use the single spectra to track any shifts and generate corrections (the shift polynomials) by the \textit{shift-tracker} tool. The basic idea behind the shift tracking is calculating the numeric differential of that spectrum and finding the maxima by searching for zero-crossings in the differential. Unfortunately, raw spectra when numerically differentiated show a very high degree of noise which makes the analysis impossible. Therefore, this noise has to be filtered out first. To this end, the spectra are first smoothed using a triangular weighting function. Then, the smoothed spectrum is numerically differentiated and, finally, smoothed again using the Hann window function.
By correctly choosing the filter window sample size $N$, as a parameter, the zero-crossings in the differentiated spectra show a constant correlation to the true peak position. Therefore, they can be used to create polynomial fits that map each drifted peak back to the original position in the selected reference spectrum. Examples of the intermediate LaBr$_3$(Ce) data generated by smoothing the spectra using the triangular weighting function and by smoothing the differential spectra using the Hann weight are shown in Fig. 2.8. It is easily seen that the initial smoothing represents the real spectrum accurately, while being cleared of any directly observable noise. Operationally, the shift polynomial can be generated following three different steps. First of all, we have to determine the filter width until all peaks are correctly reproduced as zero-crossing, by looking at the spectrum and the differential: $N = 50$ was proven to be reasonably correct. Then, we have to narrow down the list of peaks by setting the min-slope parameter until at least the four strongest peaks are left and, finally, these peaks position have to be saved into a file so that the tool tracks the peaks through all the runs.
2.2. Data acquisition and analysis using SOCOv2

Figure 2.8: Intermediate LaBr$_3$(Ce) data, as generated by the shift-tracker, showing the original and smoothed spectrum after the triangular filter stage, the derivative stage and the smoothed derivative (the final output), used to track shifts by finding and tracking zero-crossing [16].
2.2.2 Physical event creation

When using a triggerless acquisition method [17], as in our experiment, there is no intrinsic grouping in the list of files. Instead, a set of hits which conform to a given requirement (that is a determined multiplicity) has to be grouped together by software. The main factor to determine hits of a group is the coincidence window size, which should ideally be chosen as small as possible. In order to facilitate the task of finding the right coincidence window size, SOCOv2 allows to generate a matrix (with the `multiplicity-info` command), shown in Fig. 2.9, where the number of valid events is plotted against the multiplicity. In this way we can easily see which window size should optimally be chosen in order to have the smallest window size but maintaining a good statistics. In our case we decide to set a time window of 200 ns.

![Figure 2.9: Exemplary output of the multiplicity-info tool for Europium calibration data. On the left ordinate a window range of 1 to 14 timestamps, while on the x-axis the multiplicity is shown. Colours indicating the number of valid event are presented on the right.](image)

Even the choice of the above mentioned requirements plays a fundamental role in the event creation: a compromise between multiplicity and statistics has to be found. When we put a requirement, in fact, we set that a group of hit can be considered an event when determined detectors and TAC have registered it. Clearly the valid event number decreases as the multiplicity asked increases. Fig. 2.10 shows how the valid event number changes with different requirements. From this figure one sees that a good compromise between statistic and multiplicity is obtained when we ask that at least 1 and at most 4 HPGe crystals and at least 2 LaBr$_3$(Ce) and 1 TAC...
2.2. Data acquisition and analysis using SOCOv2

registered the hit (blue curve). Therefore, by the SOCOv2 event builder command, these requirements were used in the creation of our event related to the \(^{235}\text{U}\) and \(^{241}\text{Pu}\) fission experiments.

![Graph showing different requirements and their effect on statistics](image)

**Figure 2.10:** The graph shows the different requirements which can be chosen in the data sorting and their effect on the statistics.

2.2.3 Creation of spectra and fast timing matrices

Once the events were created with the right coincidence window size and the wanted requirements, the events can be used to create the related spectra. During this step, SOCOv2 allows to take into account additional corrections. First, we have to consider one of the principal interactions of \(\gamma\)-rays within detector: the Compton scattering. Especially at high energies a photon has a sizeable probability of leaving the detector after Compton scattering. If that happen, only a partial amount of the total energy of the photon is deposited and can be detected. The scattering photons can leave the detector after one or more interactions in the detector material and they just contribute to the background. In order to eliminate Compton scattered events from the analysis, the HPGe are usually surrounded by a BGO shield so that when a coincidence between the detector and the BGO shield is detected, the signal from that detector is then discarded. SOCOv2 allows
2.2. Data acquisition and analysis using SOCOv2

to enable BGO suppression setting an individual threshold per BGO shield below which signals are ignored. Suitable thresholds can be determined by looking at the BGO projection spectra generated during the event building stage. Fig. 2.11 shows a direct comparison of clover spectra with and without enabling BGO suppression of a $^{152}$Eu source.

**Figure 2.11:** *Comparison between $^{152}$Eu clover spectra with and without enabling BGO suppression. The improved peak to background ratio, when suppression is enable is visible.*

Another effect we have to consider is the Compton scattering between detectors. In the setup of our experiment, the detectors were positioned as near as possible to the target in order to achieve the maximum solid angle coverage and minimize the time needed to collect high statistics. This naturally results in detectors being very close to each other, which lead to Compton scattering between detectors, if detectors don’t employ any method of suppression. In the present case, this becomes crucial for the LaBr$_3$(Ce) scintillators. In fact, if contaminated energy spectra are used to extract TAC spectra and centroid differences, one induces a significant systematic error. Therefore, it is essential to exclude events that contain a combination of neighbouring, unshielded detectors. SOCOv2 allows the exclusion of unwanted detector combinations by using a file which specifies the detector combination to be excluded. The effect of this correction is shown in Fig. 2.12.

Finally, the last correction performed, is related to the TAC spectra. The fast timing method, explained in the next chapter, requires TAC spectra generated by gating on the feeding energy, once on the start branch(with
2.2. Data acquisition and analysis using SOCOv2

Figure 2.12: Comparison of $^{152}$Eu spectra by LaBr$_3$(Ce) detectors showing the expected reduced background, when given combinations of neighbouring detectors are excluded.

the decay energy on the stop branch of the TAC) and vice versa. The first combination produces the so called start TAC spectrum and the second combination the stop spectrum. Due to different cable lengths and other factors, both start and stop spectra show no intrinsic alignment between different TACs. Thus, when looking at the superposition of all start and stop spectra for all detectors and TAC combinations, the result in general does not resemble a Gaussian and in most cases is not even symmetric in shape. Fortunately, all combinations are independent and therefore may be shifted individually. To establish the internal offsets, one takes a reference $\gamma$-$\gamma$ transition cascade and calculates the centroid of each combination for this cascade individually. Choosing an arbitrary position, where the centroids are to be aligned at, one then chooses the shift parameter for all combinations accordingly. These shift parameters can be stored in a file and applied during the spectra creation. An example of the different alignment and of its correction is shown in Fig. 2.13. Considering all the corrections discussed, the fast timing matrices can be created, using the SOCOv2 tool ft-matrix. In order to generate these matrices, one defines a number of gates in auxiliary detectors (germaniums, clovers, etc.) to clean up the spectrum, if needed, in order to select the feeding transition of interest, as a LaBr$_3$(Ce)-type gate. By using the assumption a TAC$_i$ can only be started by LaBr$_3$(Ce)$_i$ and stopped by LaBr$_3$(Ce)$_j$ (where $j > i$), the command can decide whether an
2.2. Data acquisition and analysis using SOCOv2

Figure 2.13: Comparison between TAC spectra for two LaBr$_3$(Ce) detectors, with (top) and without (bottom) applied detector-dependent TAC shift.
event belongs to the start or stop branch and increments the right matrix respectively. In each matrix each row represents the energy of the non-matching LaBr$_3$(Ce) detector and the columns represent the TAC value for that event. The difference in the output matrices is simply whether the matching LaBr$_3$(Ce) detector was the start or the stop signal for the TAC. By then cutting in the matrix on the decay energy, TAC projections can be extracted and analysed.
Chapter 3

Lifetime measurements

The lifetime of a nuclear excited state is one of the most important observable in nuclear structure studies. The lifetime determines the reduced electromagnetic transition probabilities which are used to be compared with predictions by theoretical nuclear structure models. In this chapter we will describe the analysis and the method used to measure nuclear lifetimes in the interval 10 ps - 1 ns, the delayed coincidence technique. In particular, we will apply them to evaluate known lifetimes of some nuclei produced in $^{235}\text{U}$ fission.

3.1 The fast-timing techniques

The principle of the delayed coincidence technique, also called fast-timing technique, is to measure the time difference between two coincidences, i.e. time correlated electronic signals. For measurements of nuclear excited states, a reference timing signal, indicating the moment of formation of the nuclear excited state, is needed. The start timing signal can be provided by a nuclear transition feeding directly the state of interest, while the second stop signal is obtained from the decay of the state of interest. In Fig. 3.1 the fast timing setup is shown. As we can see, each scintillator detector is connected to a Constant Fraction Discriminator (CFD) which minimizes the timing uncertainty due to statistical amplitude variations of the input signal.

Then, the Time to Amplitude Converter (TAC) generates an output with an amplitude proportional to the time difference between the two CFD
3.1. The fast-timing techniques

Figure 3.1: Scheme of the fast timing setup used in the experiment [18].

Fast timing signals, coupled directly to the start and delayed at the stop inputs. The time distribution obtained is a convolution of the setup Prompt Response Function (PRF) with an exponential decay, showing the typical asymmetric shape

\[ D(t) = n \lambda \int_{-\infty}^{t} P(x) \exp(-\lambda(t-x)) \, dx \quad \text{with} \quad \lambda = \frac{1}{\tau} \quad (3.1) \]

where \( n \) is the normalization factor, \( \tau \) is the lifetime of the level and \( P(t) \) represents the PRF [19]. This latter is a Gaussian distribution and its FWHM gives an estimate of the intrinsic time resolution of the setup, that can be measured using real prompt (simultaneously occurring) events or using prompt decay transition (\( \tau < 1 \) ps). As a result of statistical processes, the FWHM of the PRF is dependent on the \( \gamma \)-ray energy of both the feeding (start signal) and the decaying (stop signal) transitions. The worse time resolution is induced when the signals have smaller amplitude (energy), as the relative amplitude variation (jitter) increases with decreasing amplitudes. In Fig. 3.2 is shown the FWHM of the scintillator array used in the experiment (see Section 2.1) [20]. As we can see, for energies larger than 1.2 MeV the FWHM is 270(10) ps. The data are fitted with a function \( \Delta T(E) \) which describes the CFD timing uncertainty and time walk according to

\[ \Delta T(E) = \frac{a}{\sqrt{E + b}} + p(E) \quad (3.2) \]

where \( a \) and \( b \) are parameters, while \( p(E) \) represents a polynomial and, in this case, is a constant.
3.1. The fast-timing techniques

Figure 3.2: \( \gamma-\gamma \) timing performance of the FATIMA array, expressed as the FWHM of the Prompt Response Functions (PRFs). The energy of the data points corresponds to the smaller energy of the two \( \gamma \) rays used as feeder and decay. Data are taken with an \(^{152}\)Eu source and with \(^{48}\)Ti\( (n, \gamma) \)\(^{49}\)Ti reaction [20].

Time distributions of lifetimes which are longer than the FWHM of the corresponding PRF have a pronounced decay slope and the lifetime \( \tau \) can be determined directly using the *slope method*. In this case, the straight line of the decay obtained in a semi-logarithmic plot is fitted and its slope is directly the decay constant \( \lambda \), hence the inverse of the lifetime \( \tau \):

\[
\ln[D(t)] \propto -\frac{t}{\tau} \quad \text{for } \tau > \text{FWHM} \tag{3.3}
\]

When the lifetime is very short or comparable with the time resolution, the time spectrum suffers a displacement of the centroid with respect to the prompt peak and the slope is not so evident. The difference between the centroid of a prompt distribution and the centroid of the experimental data provides a direct value of the lifetime (*Centroid Shift Method*) [19]. In our work we consider the *Mirror Symmetric Difference Method* (MSCD) [21], based on the Centroid Shift Method, but considering the centroid difference, and not the centroid, between time distributions. In this method the time distributions obtained from a fast timing setup, delayed and anti-delayed (or start and stop), are used to determined the centroid difference \( \Delta C \) for a specific \( \gamma \) ray cascade. Consequently the energy dependent prompt response difference PRD, and not the centroid of the PRF, for that cascade, had to be evaluated and the shift between the \( \Delta C \) and PRD represents twice the lifetime:

\[
\tau = \frac{\Delta C - \text{PRD}}{2} \tag{3.4}
\]

A more specific explanation of these methods is given in Appendix A.
3.2 Correction procedures

In order to obtain precise lifetime measurements we have to take into account two important corrections: the PRD energy dependency and the background component.

3.2.1 FATIMA PRD curve calibration

The Mirror Symmetric Centroid Difference method makes it possible to precisely calibrate the energy dependency of the PRD [22, 23]. For any energy combination of a prompt $\gamma$-$\gamma$ cascade, two data points are obtained by taking advantage of the identity, from Eq. A.9,

$$PRD(\Delta E_\gamma = 0) = 0$$  \hspace{1cm} (3.5)

The two data points are transformed into the $(\Delta C, \Delta E_\gamma)$-representation according to

$$PRD(E_{feeder} - E_{decay}) = PRD(E_{feeder}) - PRD(E_{decay})$$  \hspace{1cm} (3.6)

According to Eq. A.6, also precisely known ps lifetimes can be used for the PRD-calibration procedure. The PRD curve of the FATIMA setup is presented in Fig. 3.3 [13]. The major part of PRD data were obtained using triple events with an additional EXILL gate, which resulted in almost zero background contributions. Triple $\gamma$-$\gamma$-$\gamma$ events were obtained from 40-hours measurement using a $^{152}$Eu point-like $\gamma$ ray source, for the energy range 40-1408 keV, and from a 20-hours measurement using the $^{48}$Ti($n_{th}$, $\gamma$)$^{49}$Ti reaction, for the energy range 137-6760 keV. The rest of the PRD data are from double events, where $\gamma$-$\gamma$ cascades were used which provided peak-to-background ratios larger than 20. The PRD data, adjusted for the reference energy of 342 keV, were fitted by the function

$$PRD(E) = \frac{a}{\sqrt{E+b}} + cE^2 + dE + e$$  \hspace{1cm} (3.7)

where $a$, $b$, $c$, $d$ and $e$ are parameters returned by the fit. The knowledge of these parameters allows to evaluate the PRDs at the energy combinations of interest.

3.2.2 Background correction

Another important correction to consider is the way to take into account the background [21]. The obtained time distribution is a superposition of time spectra due to full energy events and to Compton events of the same energy (i.e. the background underneath the full energy peak). The Compton time distribution of interest can be interpolated by the measurements
of Compton time spectra at different energies in the region around the transition of interest. The procedure follows the Mirror Symmetric Difference method and it is based on the determination of a linear trend of the centroid differences. In this way, it is possible to evaluate the background value at the energy of interest. Considering this value and the peak to background ratio $\pi$, the total centroid difference can be written as the sum of the true delayed centroid difference $\Delta C_{\text{true}}$ and the Compton centroid difference $\Delta C_{\text{Comp}}$

$$
\Delta C = \frac{\pi \Delta C_{\text{true}} + \Delta C_{\text{Comp}}}{1 + \pi}
$$

(3.8)

It follows that $\Delta C_{\text{true}}$ is

$$
\Delta C_{\text{true}} = \Delta C + \frac{\Delta C - \Delta C_{\text{Comp}}}{\pi}
$$

(3.9)

According to Eq. A.2, and considering the Eq. 3.9, the lifetime of an excited level can be evaluated by

$$
\tau = \frac{1}{2} \left( \frac{\Delta C - \Delta C_{\text{Comp}}}{\pi} + \Delta C - PRD \right)
$$

(3.10)
3.3 Tests

The effectiveness of the method described above and the good operation of the SOCOv2 Software, were proved considering the measurements of known lifetimes of three nuclei produced in $^{235}$U fission. In particular, we have studied the $2^+$ level of $^{100}$Zr, the $4^+$ of $^{132}$Sn and the $2^+$ of $^{90}$Kr.

3.3.1 $^{100}$Zr

The analysis of the $2^+$ level of $^{100}$Zr is structured in different steps. First of all, we need to find a transition that feeds the level and one that decays from it, the start and stop signals respectively. As we see from the partial level scheme of $^{100}$Zr, shown in Fig. 3.4, a good cascade is represented by the transitions 352-212 keV.

![Partial level scheme of $^{100}$Zr, with the $2^+$ level in red. Energies of levels and transitions are also indicated in keV.](image.png)
The additional transition in coincidence, at 497 keV, can be used as HPGe gate (i.e. we are asking that only $\gamma$s in coincidence are acquired) to clean the total fission spectrum from transitions not belonging to this cascade.

Looking at this spectrum, we understand how the operation of finding a good HPGe gate is fundamental in the analysis to be able to select the transitions of interest in the LaBr$_3$(Ce) detectors. Fig. 3.5 shows the very complicated structure of a fission spectrum, containing $\gamma$s from the majority of nuclei produced in our reaction.

![Figure 3.5: $\gamma$-ray spectrum of the $^{235}$U(n,f) reaction. The black line represents the HPGe detectors, the red one the LaBr$_3$(Ce) scintillators.](image)

Fig. 3.6 presents the spectrum gated on the 497 keV: the structure is less complicated and the transitions of interest, at 352 keV and 212 keV, are visible. A zoom of the region of interest in also shown in Fig. 3.7.
Figure 3.6: Spectrum of the $^{235}\text{U}(n,f)$ reaction gated by the 497 keV transition. The black line represents the HPGe detectors, the red one the LaBr$_3$(Ce) scintillators.

Figure 3.7: Zoom of the 497 keV gated spectrum of the $^{235}\text{U}(n,f)$ reaction. The black line represents the HPGe detectors, the red one the LaBr$_3$(Ce) scintillators.
Once having checked, in this way, that the transitions of interest are present in our reaction, the next step is the creation of the fast-timing matrices.
According to the Mirror Centroid Shift Method, we need to consider a gate (on LaBr$_3$(Ce) detectors) on a transition of the cascade considered. In this case we set the 212 keV transition as gate and we create two matrices, the delayed and the anti-delayed ones, depending on whether the gate is the start or the stop signal by the SOCOv2 Software. These are two dimensional matrices having on one axis the energy double gated spectrum and the time distribution on the other. The double gated spectra (energy axes of the matrices) are shown in Fig. 3.8. From each matrix it is now possible to extract the delayed (start) and the anti-delayed (stop) time distribution selecting a cut on the 352 keV transition. In Fig. 3.9 these time distributions are presented. As we can clearly see, they show well pronounced slopes as we expect since the value of the lifetime for the $2^+$ level is 0.85(19) ns.
A comparison between the two spectra is shown in Fig. 3.10 where the centroid shift is well evident. The centroid difference is evaluated as difference between the centres of gravity extrapolated by integration of the two spectra. In Tab. 3.1 the centroid values and the centroid difference, with their errors, are written. The error on the centroid is evaluated by

$$\sigma_C = \frac{\text{FWHM}}{2\ln 2\sqrt{A}} \quad (3.11)$$

where $A$ is the area of the time distribution, while FWHM, extracted considering the FATIMA time resolution (Fig. 3.2) i.e. 416 ps. The error on $\Delta C$, instead, is evaluated using the error propagation formula

$$\sigma_{\Delta C} = \sqrt{\left(\frac{\partial \Delta C}{\partial C_{\text{stop}}} \sigma_{C_{\text{stop}}}\right)^2 + \left(\frac{\partial \Delta C}{\partial C_{\text{start}}} \sigma_{C_{\text{start}}}\right)^2} \quad (3.12)$$

that leads to

$$\sigma_{\Delta C} = \sqrt{\sigma_{C_{\text{stop}}}^2 + \sigma_{C_{\text{start}}}^2} \quad (3.13)$$

In the previous section we have discussed about the necessity of considering some corrections about the PRD and the background. To eliminate the contributions derived from the latter we have determined the peak-to-background ratio and the $\Delta C_{\text{Compton}}$ at our energy (352 keV).

**Figure 3.10:** Comparison between the delayed (black) and the anti-delayed (red) time distributions for the $2^+$ level of $^{100}\text{Zr}$. The error bars are the square root of the counts.
Table 3.1: Centroids values obtained by integration of time distribution. The centroid difference is $\Delta C = C_{\text{start}} - C_{\text{stop}}$. The errors on the centroids are evaluated by Eq. 3.11 while the $\Delta C$ error comes from the error propagation (Eq. 3.13).

<table>
<thead>
<tr>
<th></th>
<th>Centroid [ps]</th>
<th>Error [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed $C_{\text{start}}$</td>
<td>24719.9</td>
<td>2.2</td>
</tr>
<tr>
<td>anti-delayed $C_{\text{stop}}$</td>
<td>13342.9</td>
<td>2.2</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>1377 ± 3 ps</td>
<td></td>
</tr>
</tbody>
</table>

The peak-to-background ratio was obtained by fitting the full energy peak of the 352 keV transition in both matrices and considering, as energy interval, the same used to extrapolate the time spectrum. We have also taken into account two regions of background around the peak, so that it is possible to determine the underlying background. An example of fit with indicated the background regions is shown in Fig. 3.11. The area of the peak is

![Example of peak-to-background evaluation for $^{100}$Zr. The fit of the full energy peak is shown. Around the peak, the two background regions considered are also shown. As we can see, the region of background underneath the peak is nearly represented by a trapezoid, whose area can be calculated.](image)

turned from the fit while that of the background is calculated as area of the trapezoid under the peak. The corresponding values, for both the delayed and anti-delayed double gated spectra, are presented in Tab. 3.2. The value
3.3. Tests

of the peak-to-background ratio $\pi$, considered in the lifetime determination, is the mean of the two values and its error the mean standard deviation. Considering that

$$\pi = \frac{A_{\text{peak}}}{A_{\text{backg}}}$$

(3.14)

the error on $\pi$, obtained by propagation, is

$$\sigma_\pi = \sqrt{\left(\frac{\sigma_{A_{\text{peak}}}}{A_{\text{backg}}}\right)^2 + \left(-\frac{A_{\text{peak}}\sigma_{A_{\text{backg}}}}{A_{\text{backg}}^2}\right)^2}$$

(3.15)

where

$$\sigma_{A_{\text{peak}}} = \sqrt{A_{\text{peak}}}$$

(3.16)

$$\sigma_{A_{\text{backg}}} = \sqrt{A_{\text{backg}}}$$

(3.17)

<table>
<thead>
<tr>
<th></th>
<th>$A_{\text{peak}} \pm \sigma$</th>
<th>$A_{\text{backg}} \pm \sigma$</th>
<th>$\pi \pm \sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed (start)</td>
<td>38134 ± 195</td>
<td>5110 ± 71</td>
<td>7.46 ± 0.11</td>
</tr>
<tr>
<td>anti-delayed (stop)</td>
<td>93998 ± 306</td>
<td>5048 ± 71</td>
<td>18.62 ± 0.27</td>
</tr>
<tr>
<td>$\pi$</td>
<td>13.04 ± 0.21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Values of the areas of the full energy peaks and underneath background of both the start and stop energy spectra. The errors on the areas are calculated according to Eq. 3.16 and Eq. 3.17, while the errors on $\pi$ by Eq. 3.15. The $\pi$ error is the mean standard deviation.

The Compton background correction is based, as already explained, on assuming that a linear trend can be found if we consider the centroid difference of time distributions around the transition of interest. For this purpose, we extract the time spectra from the two fast timing matrices at different energies around the peak (the energy interval is similar to that of the full energy peak).

By integration, we evaluate the centroids and, consequently, we calculate the centroid difference. A ($\Delta C_{\text{Compton}}, E_\gamma$) plot is then obtained and a linear fit is deduced. The parameters of the fit are used to calculate the $\Delta C_{\text{Compton}}$ at 352 keV. The linear fit is shown in Fig. 3.12, while the corresponding data are presented in Tab. 4.3. The linear fit obtained is described by the equation

$$\Delta C_{\text{Compt}}(E) = mE + q$$

(3.18)

with $m = -0.62 \pm 0.10$ and $q = 1174.20 \pm 42.18$, whose errors are given by the fit. The $\Delta C_{\text{Comp}}$ is then

$$\Delta C_{\text{Comp}}(352 \text{ keV}) = 955.96 \pm 54.94 \text{ ps}$$

(3.19)
Figure 3.12: $(\Delta C, E_\gamma)$ plot relative to the $2^+$ level of $^{100}$Zr. The red line, whose equation is indicated, represents the linear trend found and the error bars are calculated using Eq. 3.13. The square of the correlation coefficient $R^2$ is also shown.

The error on the $\Delta C_{\text{Comp}}$ is evaluated with the error propagation formula

$$\sigma_{\Delta C_{\text{Comp}}}(352 \text{ keV}) = \sqrt{(E\sigma_m)^2 + \sigma_q^2}$$

(3.20)

<table>
<thead>
<tr>
<th>E [keV]</th>
<th>$\Delta C \pm \sigma$ [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>315</td>
<td>969.8 ± 3.2</td>
</tr>
<tr>
<td>385</td>
<td>938.9 ± 3.2</td>
</tr>
<tr>
<td>455</td>
<td>910 ± 3.5</td>
</tr>
<tr>
<td>545</td>
<td>824.7 ± 3.7</td>
</tr>
</tbody>
</table>

Table 3.3: $\Delta C_{\text{Comp}}$ at different energies for the $2^+$ level of $^{100}$Zr. The errors are calculated using Eq. 3.11.
Finally, using the calibrated PRD curve, we are able to evaluate the PRD for the $\gamma-\gamma$ combination of interest:

$$\text{PRD}(352 \text{ keV} - 212 \text{ keV}) = -54.98 \pm 7.08 \text{ ps} \quad (3.21)$$

Regarding the PRD error, we have considered that each prompt datum has an error. As a consequence, the fit of the PRD curve reported in Fig. 3.3 can be shifted up or down accordingly to the error. For this reason, we have created two different set of data: one with the prompt data shifted up of the error and one shifted down of the same quantity. For each set of data, two new fits are acquired. Using these two fits we can evaluate two PRDs, that we called $\text{PRD}_{\text{plus}}$ and $\text{PRD}_{\text{minus}}$, at the energy of interest. Our PRD is located exactly in the middle so the standard deviation of each transition of the cascade is

$$\sigma_{\gamma} = \frac{\text{PRD}_{\text{plus}} - \text{PRD}_{\text{minus}}}{2} \quad (3.22)$$

and as the PRD of interest is exactly the difference, as expressed in Eq. 3.6, the error is given by

$$\sigma_{\text{PRD}} = \sqrt{\sigma_{\text{feeder}}^2 + \sigma_{\text{decay}}^2} \quad (3.23)$$

Therefore, the lifetime for the $2^+$ level measured with our analysis is

$$\tau_{2^+} = \frac{1}{2} \left( \frac{\Delta C - \Delta C_{\text{Comp}}}{\pi} + \Delta C - \text{PRD} \right)$$

$$= \frac{1}{2} \left( \frac{1377 - 955.96}{13.04} + 1377 + 54.98 \right)$$

$$= 0.73 \pm 0.15 \text{ ns} \quad (3.24)$$

which is in good agreement with the literature value of 0.85(19) ns. The error was obtained with the error propagation formula

$$\sigma_{\tau} = \sqrt{\left( \left( \frac{1}{2\pi} + \frac{1}{2} \right) \sigma_{\Delta C} \right)^2 + \left( \left( \frac{\Delta C - \Delta C_{\text{Comp}}}{2\pi^2} \right) \sigma_{\pi} \right)^2 + \left( -\frac{\sigma_{\Delta C_{\text{Comp}}}}{2\pi} \right)^2 + \left( -\frac{\sigma_{\text{PRD}}}{2} \right)^2} \quad (3.25)$$

A further confirmation of our value is given by the application of the slope method. In Fig. 3.13 the exponential fits of both time distributions are shown. The values obtained by the fit are presented in Tab. 3.4 The mean value with the mean standard deviation is

$$\tau_{2^+} = 0.88 \pm 0.05 \text{ ns} \quad (3.26)$$

The time distributions have also been fitted considering the convolution

$$y(t) = C \exp \left( \frac{(\sigma^2 + 2\tau x)^2}{2} - \frac{t}{\tau} \right) \sigma \sqrt{\frac{\pi}{2}} \text{erfc} \left( \frac{\sigma^2 + (x - t) \tau}{\sqrt{2}\sigma \tau} \right) \quad (3.27)$$
Figure 3.13: Anti-delayed (a) and delayed (b) time distributions for the $2^+$ level of $^{100}$Zr. The slopes were fitted with the exponential curve represented by the red line. The fit results are also indicated.

<table>
<thead>
<tr>
<th></th>
<th>$\tau \pm \sigma$ [ns]</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed</td>
<td>0.88 ± 0.05</td>
</tr>
<tr>
<td>anti-delayed</td>
<td>0.87 ± 0.02</td>
</tr>
</tbody>
</table>

Table 3.4: Lifetimes obtained by fitting the slope of the time distributions with an exponential curve. The errors are deduced from the fit.

In this expression $C$ is a constant, $\sigma$ and $x$ represent the variance and the centroid of the Gaussian distribution and $\tau$ is the lifetime of the level. The function $erfc$ is the complementary error function and is defined as

$$erfc(x) = \frac{1}{\sqrt{\pi}} \int_x^\infty \exp(-t^2)dt$$ (3.28)

The time distribution fitted with the convolutions are shown in Fig. 3.14, while the obtained lifetimes are presented in Tab. 3.5. The mean of the two values is

$$\tau_{2^+} = 0.69 \pm 0.14 \text{ ns}$$ (3.29)

<table>
<thead>
<tr>
<th></th>
<th>$\tau \pm \sigma$ [ns]</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed</td>
<td>0.71 ± 0.01</td>
</tr>
<tr>
<td>anti-delayed</td>
<td>0.66 ± 0.14</td>
</tr>
</tbody>
</table>

Table 3.5: Lifetimes obtained by fitting the time distributions with the convolution function given by Eq. 3.27. The errors are deduced from the fit.
3.3. Tests

![Graph](image)

Figure 3.14: Delayed (a) and anti-delayed (b) time distributions for the $2^+$ level of $^{100}\text{Zr}$, fitted with the convolution given by Eq. 3.27 and represented by the red line. The fit results are indicated.

3.3.2 $^{132}\text{Sn}$

The same analysis presented in the previous section, was performed to measure the lifetime of the $4^+$ level of $^{132}\text{Sn}$. In Fig. 3.15 is shown a partial level scheme of $^{132}\text{Sn}$.

![Partial level scheme](image)

Figure 3.15: Partial level scheme of $^{132}\text{Sn}$, with the $4^+$ level in red. Energies of levels and transitions are indicated in keV.
The 300-375 keV transitions are used as feeder and decay respectively, while the 4041 keV is imposed as gate on clover detectors. A zoom of the gated spectrum focusing on the transitions of interest is shown in Fig. 3.16.

![Zoom of the 4041 keV gated spectrum of the $^{235}$U(n,f) reaction.](image)

**Figure 3.16**: Zoom of the 4041 keV gated spectrum of the $^{235}$U(n,f) reaction. The black line represents the HPGe detectors, the red one the LaBr$_3$(Ce) scintillators.

Considering the 375 keV transition as gate on the LaBr$_3$(Ce) detectors, the two fast timing matrices were created and the corresponding time distributions were extracted. Time distributions are presented in Fig. 3.17.

![Delayed (a) and anti-delayed (b) time distributions for the 4$^+$ state of $^{132}$Sn.](image)

**Figure 3.17**: Delayed (a) and anti-delayed (b) time distributions for the 4$^+$ state of $^{132}$Sn. The error bars are the square root of the counts.

Following the same procedure explained for $^{100}$Zr, the centroid difference
and the peak-to-background ratio were evaluated. In the next pages, the values for the centroid difference are indicated in Tab. 3.6, while Fig. 3.18 shows the comparison between the time distributions. The peak-to-background ratio values and an example of their evaluation are reported in Tab. 3.7 and in Fig. 3.19.

![Comparison between the delayed (black) and the anti-delayed (red) time distributions for the $4^+$ state of $^{132}\text{Sn}$. The error bars are the square root of the counts.](image)

**Figure 3.18:** Comparison between the delayed (black) and the anti-delayed (red) time distributions for the $4^+$ state of $^{132}\text{Sn}$. The error bars are the square root of the counts.

<table>
<thead>
<tr>
<th></th>
<th>centroid [ps]</th>
<th>Error [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed $C_{\text{start}}$</td>
<td>2698</td>
<td>10</td>
</tr>
<tr>
<td>anti-delayed $C_{\text{stop}}$</td>
<td>21160</td>
<td>10</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>5820 ± 14 ps</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.6:** Centroids values obtained by integration of the time distributions for the $4^+$ level of $^{132}\text{Sn}$. The centroid difference is $\Delta C = C_{\text{start}} - C_{\text{stop}}$. The errors on the centroids are evaluated by Eq. 3.11, while the $\Delta C$ error comes from the error propagation (Eq. 3.13).
Figure 3.19: Example of peak-to-background evaluation in the case of the $4^+$ state of $^{132}\text{Sn}$. The fit of the full energy peak is shown. Around the peak, the two background regions considered are also shown. As one can see, the region of background underneath the peak is nearly represented by a trapezoid, whose area can be calculated.

Table 3.7: Values of the areas of the full energy peaks and underneath background of both the start and stop energy spectra for the $4^+$ level of $^{132}\text{Sn}$. The errors on the areas are calculated according to Eq. 3.16 and Eq. 3.17, while the errors on $\pi$ is given by Eq. 3.15. The $\pi$ error is the mean standard deviation.

<table>
<thead>
<tr>
<th></th>
<th>$A_{\text{peak}} \pm \sigma$</th>
<th>$A_{\text{backg}} \pm \sigma$</th>
<th>$\pi \pm \sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed (start)</td>
<td>$428 \pm 21$</td>
<td>$322 \pm 18$</td>
<td>$1.33 \pm 0.09$</td>
</tr>
<tr>
<td>anti-delayed (stop)</td>
<td>$375 \pm 19$</td>
<td>$364 \pm 19$</td>
<td>$1.03 \pm 0.09$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$1.18 \pm 0.09$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Finally the $\Delta C_{\text{Compton}}$ correction was evaluated. The values obtained for energies and centroid differences are given in Tab. 3.8, while the plot is shown in Fig. 3.20.

The Fig. 3.20 also presents the linear trend obtained for the Compton correction

$$\Delta C_{\text{Compt}}(E) = mE + q$$  \hspace{1cm} (3.30)

with $m = 0.47 \pm 0.01$ and $q = -76.27 \pm 0.64$, whose errors are given by the
### Table 3.8: \( \Delta C_{\text{Compt}} \) at different energies evaluated for the 4\(^+\) level of \(^{132}\text{Sn}\). The errors are calculated using Eq. 3.11.

<table>
<thead>
<tr>
<th>E [keV]</th>
<th>( \Delta C \pm \sigma ) [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>270</td>
<td>1 ± 35</td>
</tr>
<tr>
<td>320</td>
<td>100 ± 40.48</td>
</tr>
<tr>
<td>415</td>
<td>151 ± 42.47</td>
</tr>
<tr>
<td>605</td>
<td>189 ± 43.55</td>
</tr>
</tbody>
</table>

### Figure 3.20: \((\Delta C,E_{\gamma})\) plot for the 4\(^+\) state of \(^{132}\text{Sn}\). The red line, whose equation is indicated, represent the linear trend found, while the error bars are calculated using Eq. 3.13. The square of the correlation coefficient \( R^2 \) is also shown.

\[ y = 0.47x - 76.27 \]
\[ R^2 = 0.77 \]

By the already discussed fit, the PRD and its error are

\[ PRD(375 \text{ keV} - 300 \text{ keV}) = 23.35 \pm 4.74 \text{ ps} \]
Therefore, the lifetime for the \(4^+\) level is

\[
\tau_{4^+} = \frac{1}{2} \left( \frac{\Delta C - \Delta C_{\text{Compton}}}{\pi} + \Delta C - PRD \right) \\
= \frac{1}{2} \left( \frac{5820 - 64.73}{1.18} + 5820 - 23.35 \right) \\
= 5.34 \pm 0.18 \text{ ns} \tag{3.33}
\]

which is in good agreement with the literature value of 5.71 ns.

As in the case of \(^{100}\text{Zr}\) we can also evaluate the lifetime considering an exponential fit of the two time distribution slopes and determine the lifetime as the mean. In Fig. 3.21 the fit of the time spectra are shown, while Tab. 3.9 presents the obtained values.

![Figure 3.21](image)

**Figure 3.21**: Delayed (a) and anti-delayed (b) time distributions relative to the \(4^+\) state of the \(^{132}\text{Sn}\). The slopes were fitted with the exponential curve represented by the red line. In the graph the fit results are also given.

<table>
<thead>
<tr>
<th>(\tau \pm \sigma \text{ [ns]})</th>
<th>(\text{delay})</th>
<th>(5.78 \pm 0.44)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{antidelay})</td>
<td>(5.90 \pm 0.56)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.9**: Lifetimes of the \(4^+\) level of \(^{132}\text{Sn}\) obtained by fitting the slope of the time distributions with an exponential curve. The errors are deduced from the fit.

The mean value with the mean standard deviation is

\[
\tau_{4^+} = 5.84 \pm 0.71 \text{ ns} \tag{3.34}
\]
3.3. Tests

3.3.3 $^{90}\text{Kr}$

The last test is about the lifetime of $2^+$ level of $^{90}\text{Kr}$. The cascade considered is 1123-707 keV, shown in Fig. 3.22. In this case, there is not a good transition of the same nucleus that can be used as gate in the HPGe detectors, so we consider the partner nucleus $^{144}\text{Ba}$. This partner, formed with $^{90}\text{Kr}$ in $^{235}\text{U}$ fission with 2 neutrons evaporation, has a strong transition at 199 keV. This $\gamma$-decay gives a well defined full energy peak in the $^{235}\text{U}$ fission spectrum (Fig. 3.5) and it is a good HPGe gate, since it allows to clearly identify our transitions, as we can observe in Fig. 3.23.

Figure 3.22: Partial level scheme of $^{90}\text{Kr}$, with the $2^+$ level in red. Energies of levels and transitions are given in keV.
3.3. Tests

Figure 3.23: Zoom of the spectrum of the $^{235}\text{U}(n,f)$ reaction gated by the 199 keV transition of $^{144}\text{Ba}$. The black line represents the HPGe detectors, while the red one corresponds to the LaBr$_3$(Ce) scintillators.

The fast timing matrices were obtained by setting a second (LaBr$_3$(Ce)) gate on the 707 keV transition of $^{90}\text{Kr}$, while the time distributions, shown in Fig. 3.24, are extracted by selecting 1123 keV transition.

Figure 3.24: Delayed (a) and anti-delayed (b) time distributions of the $2^+$ level of $^{90}\text{Kr}$. The error bars are the square root of the counts.

In the next pages are presented the values obtained for centroid difference, peak-to-background ratio and $\Delta C_{Compton}$ correction.
Figure 3.25: Comparison between the delayed (black) and the anti-delayed (red) time distributions of the $2^+$ level of $^{90}$Kr. The error bars are the square root of the counts.

<table>
<thead>
<tr>
<th></th>
<th>Centroid [ps]</th>
<th>Error [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed $C_{\text{start}}$</td>
<td>24027.8</td>
<td>2.6</td>
</tr>
<tr>
<td>anti-delayed $C_{\text{stop}}$</td>
<td>24029.1</td>
<td>2.6</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>-1.3 ± 3.6 ps</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.10: Centroids values obtained by integration of the time distributions of the $2^+$ state of $^{90}$Kr. The centroid difference is $\Delta C = C_{\text{start}} - C_{\text{stop}}$. The errors on the centroids are evaluated by Eq. 3.11, while the $\Delta C$ error comes from the error propagation (Eq. 3.13).
3.3. Tests

Counts

Energy [keV]

Figure 3.26: Example of peak-to-background evaluation in the case of $^{90}$Kr, the $2^+$ state. The fit of the full energy peak is shown. Around the peak, the two background regions considered in the analysis are also shown. As one can see, the region of background underneath the peak is nearly represented by a trapezoid, whose area can be calculated.

<table>
<thead>
<tr>
<th></th>
<th>$A_{peak} \pm \sigma$</th>
<th>$A_{backg} \pm \sigma$</th>
<th>$\pi \pm \sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed (start)</td>
<td>1670 ± 41</td>
<td>2950 ± 54</td>
<td>0.57 ± 0.02</td>
</tr>
<tr>
<td>anti-delayed (stop)</td>
<td>3054 ± 55</td>
<td>2980 ± 55</td>
<td>1.02 ± 0.02</td>
</tr>
<tr>
<td>$\pi$</td>
<td></td>
<td></td>
<td>0.79 ± 0.02</td>
</tr>
</tbody>
</table>

Table 3.11: Values of the areas of the full energy peaks and underneath background of both the start and stop energy spectra, in the case of $^{90}$ Kr, $2^+$ state. The errors on the areas are calculated according to Eq. 3.16 and Eq. 3.17, while the errors on $\pi$ is obtained by Eq. 3.15. The $\pi$ error is the mean standard deviation.
The $\Delta C_{\text{Compton}}$-$E_\gamma$ plot, whose points are indicated in Tab. 3.12, is shown in Fig. 3.27. The Fig. 3.27 also presents the linear trend obtained:

$$\Delta C_{\text{Compt}}(E) = mE + q$$

with $m = -0.06 \pm 0.01$ and $q = 79.95 \pm 10.32$, whose errors are given by the fit. It follows

$$\Delta C_{\text{Compt}}(1123 \text{ keV}) = 13.36 \pm 13.81 \text{ ps}$$

![Figure 3.27: $(\Delta C, E_\gamma)$ plot relative to the $2^+$ state analysis of $^{90}\text{Kr}$. The red line, whose equation is indicated, represents the linear trend found and the error bars are calculated using Eq. 3.13. The square of the correlation coefficient $R^2$ is also shown.](image-url)
Table 3.12: $\Delta C_{\text{Compt}}$ at different energies for the $2^+ \text{ state analysis of } ^{90}\text{Kr}$. The errors are calculated using Eq. 3.11.

<table>
<thead>
<tr>
<th>E [keV]</th>
<th>$\Delta C \pm \sigma$ [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1075</td>
<td>17.8 ± 5.6</td>
</tr>
<tr>
<td>1175</td>
<td>10.6 ± 5.9</td>
</tr>
<tr>
<td>1265</td>
<td>2.7 ± 6.3</td>
</tr>
<tr>
<td>1320</td>
<td>-0.1 ± 6.6</td>
</tr>
<tr>
<td>1450</td>
<td>-3.7 ± 6.6</td>
</tr>
</tbody>
</table>

By the already discussed fit, the PRD and its error are

$$PRD(1123 \text{ keV} - 707 \text{ keV}) = -58.32 \pm 9.78 \text{ ps} \quad (3.37)$$

Therefore, the lifetime for the $2^+$ level is

$$\tau_{4^+} = \frac{1}{2} \left( \frac{\Delta C - \Delta C_{\text{Compton}}}{\pi} + \Delta C - PRD \right)$$

$$= \frac{1}{2} \left( \frac{-1.3 - 13.36}{0.79} - 1.3 + 58.32 \right)$$

$$= 19.29 \pm 9.78 \text{ ns} \quad (3.38)$$

which is in good agreement with the literature value of $15(10) \text{ ps}$ [24].
Chapter 4

$^{133}$Sb: experimental results

In this chapter, we will present the experimental results obtained from the analysis of two levels of $^{133}$Sb for both the targets $^{235}$U and $^{241}$Pu.

4.1 Analysis

The analysis was performed following the same procedure already explained in Chapter 3, but considering both the $^{235}$U and $^{241}$Pu targets used in the experiment. In this way, we have evaluated the lifetimes of two levels, the $13/2^+$ at 4302 keV and $15/2^+$ at 4464 keV, not known in literature. These levels are shown in Fig. 4.1 where a partial level scheme of $^{133}$Sb is presented.
Figure 4.1: Partial level scheme of $^{133}$Sb with the $13/2^+$ and $15/2^+$ levels of interest marked in red. Energies are in keV.
4.1 Analysis

4.1.1 13/2+

The lifetime of 13/2+ level has been evaluated considering as start and stop signals the cascade 162-1510 keV. In order to observe them clearly, the 2792 keV transition was used as a gate. In Fig. 4.2 the γs of interest are shown after setting the coincidence with the 2792 keV on the HPGe detectors.

Figure 4.2: Zoom of the energy spectrum around the γs forming the start and stop signals of the 13/2+ transition of 133Sb. The black line represents the HPGe detectors, the red line the LaBr3(Ce) scintillators.

A further gate on LaBr3(Ce) scintillators, needed to create the fast timing matrices, is set on the 1510 keV transition. In this way, the matrices of start and stop, presented in Fig. 4.3, clearly show the peak at 162 keV.

Figure 4.3: Matrices of start (a) and stop (b) created considering the 2792 keV and 1510 keV transitions, as clover and LaBr3(Ce) gate respectively.

Time distributions were extracted from these matrices by projecting the region of 162 keV on the time axis. The shift between the centroids is shown in Fig. 4.4, while the values obtained for the centroids and centroid difference are indicated in Tab. 4.1. The errors are calculated using Eq. 3.11 and 3.13.
4.1. Analysis

Counts/10 ps

Time [ps]

2000 2100 2200 2300 2400 2500 2600 2700 2800

Delayed

Anti-delayed

133Sb

Figure 4.4: Comparison between time distributions of the 13/2\(^{+}\) state of \(^{133}\)Sb.

<table>
<thead>
<tr>
<th></th>
<th>Centroid [ps]</th>
<th>Error [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed (C_{\text{start}})</td>
<td>24085.1</td>
<td>11.6</td>
</tr>
<tr>
<td>anti-delayed (C_{\text{stop}})</td>
<td>23961.3</td>
<td>11.7</td>
</tr>
<tr>
<td>(\Delta C)</td>
<td>123.8 ± 16.5 ps</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Centroids values obtained by integration of the time distributions of the 13/2\(^{+}\) state of \(^{133}\)Sb. The centroid difference is \(\Delta C = C_{\text{start}} - C_{\text{stop}}\). The errors on the centroids are evaluated by Eq. 3.11 while the \(\Delta C\) error comes from the error propagation (Eq. 3.13).

Regarding the correction needed to take into account the background, the peak-to-background ratio was evaluated as ratio of the areas found by fitting the full energy peak and the background underneath. An example of this procedure is shown in Fig. 4.5 and the corresponding values are in Tab. 4.2.
4.1. Analysis

Figure 4.5: Example of peak-to-background evaluation for the 13/2\(^+\) state of \(^{133}\)Sb. The fit of the full energy peak is shown. Around the peak, the two background regions considered in the analysis are also shown. As one can see, the region of background underneath the peak is nearly represented by a trapezoid, whose area can be calculated.

<table>
<thead>
<tr>
<th></th>
<th>(A_{\text{peak}} \pm \sigma)</th>
<th>(A_{\text{backg}} \pm \sigma)</th>
<th>(\pi \pm \sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed (start)</td>
<td>650 ± 25</td>
<td>421 ± 20</td>
<td>1.54 ± 0.09</td>
</tr>
<tr>
<td>anti-delayed (stop)</td>
<td>636 ± 25</td>
<td>435 ± 21</td>
<td>1.46 ± 0.09</td>
</tr>
<tr>
<td>(\pi)</td>
<td>1.50 ± 0.09</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Values of the areas of the full energy peaks and underneath background of both the start and stop energy spectra for the 13/2\(^+\) state of \(^{133}\)Sb. The errors on the areas are calculated according to Eq. 3.16 and Eq. 3.17, while the errors on \(\pi\) by Eq. 3.15. The \(\pi\) error is the mean standard deviation.

Finally, the Compton background correction was estimated. The values of the Compton centroid difference are indicated in Tab. 4.3 and the corresponding plot is shown in Fig. 4.6.

The Fig. 4.6 also presents the linear interpolation:

\[
\Delta C_{\text{Compt}}(E) = mE + q
\]  

(4.1)

with \(m = -1.55 \pm 0.01\) and \(q = -444.46 \pm 0.22\), whose errors are given by the fit. It follows
Figure 4.6: $(\Delta C, E_\gamma)$ plot for the $13/2^+$ state of $^{133}$Sb. The red line, whose equation is indicated, represents the linear trend found, while the error bars are calculated using Eq. 3.13. The square of correlation coefficient $R^2$ is also shown.

Table 4.3: $\Delta C_{\text{Compt}}$, at different energies, for the $13/2^+$ state analysis, in $^{133}$Sb. The errors are calculated using Eq. 3.11.

The PRD for the cascade of interest, evaluated by the fit procedure discussed in the previous chapter, is

$$PRD(162 \text{ keV} - 1510 \text{ keV}) = 260.71 \pm 7.91 \text{ ps}$$  \hspace{1cm} (4.3)
\[ \tau_{13/2^+} = \frac{1}{2} \left( \frac{\Delta C - \Delta C_{Compton}}{\pi} + \Delta C - PRD \right) \]
\[ = \frac{1}{2} \left( \frac{123.8 + 193.59}{1.5} + 123.8 - 260.71 \right) \]
\[ = 37.18 \pm 8.17 \text{ ps} \quad (4.4) \]

In order to confirm this value, the same analysis was made using data from \(^{241}\text{Pu}\) fission. We have used the same \(\gamma\)s cascade, 162-1510 keV, as start and stop signals and even the same transition at 2792 keV as HPGe gate. However, unlike the previous analysis with \(^{235}\text{U}\) fission data, we have considered the 162 keV transition as LaBr\(_3\)(Ce) gate to emphasize the 1510 keV line. Fig. 4.7 represents the two fast timing matrices created by setting the double gates 2792 keV on the HPGe detectors and 162 keV on the LaBr\(_3\)(Ce) scintillators.

![Spectra of \(^{133}\text{Sb}\) of start (a) and stop (b), for the 13/2\(^+\) state, created considering the 2792 keV and 162 keV transitions as clover and LaBr\(_3\)(Ce) gate respectively.](image)

Figure 4.7: Spectra of \(^{133}\text{Sb}\) of start (a) and stop (b), for the 13/2\(^+\) state, created considering the 2792 keV and 162 keV transitions as clover and LaBr\(_3\)(Ce) gate respectively.

Time distributions obtained by projecting on the time axis at 1510 keV are shown in Fig. 4.8 where the shift is observable. The values of the centroids and centroid difference are listed in Tab. 4.4. The peak-to-background ratio values and an example of their evaluation are given in Tab. 4.5 and in Fig. 4.9. Finally the \(\Delta C_{Compton}\) correction was evaluated. The values of energies and centroid differences here obtained are shown in Tab. 4.6, while the plot is given in Fig. 4.10.
Figure 4.8: Comparison between time distributions of $^{133}$Sb relative to the $13/2^+$ state.

<table>
<thead>
<tr>
<th></th>
<th>Centroid [ps]</th>
<th>Error [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed $C_{\text{start}}$</td>
<td>24050</td>
<td>15</td>
</tr>
<tr>
<td>anti-delayed $C_{\text{stop}}$</td>
<td>23850</td>
<td>16</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>-200 ± 22 ps</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4: Centroids values obtained by integration of the time distributions for the $13/2^+$ state of $^{133}$Sb, selected considering the 162 keV-1510 keV $\gamma-\gamma$ coincidence in the LaBr$_3$(Ce) detectors. The centroid difference is $\Delta C = C_{\text{stop}} - C_{\text{start}}$. The errors on the centroids are evaluated by Eq. 3.11 while the $\Delta C$ error comes from the error propagation (Eq. 3.13).
Figure 4.9: Example of peak-to-background evaluation for the $^{133}\text{Sb}$, $13/2^+$ state. The fit of the full energy peak at 1510 keV is shown. Around the peak, the two background regions considered are also shown. As one can see, the region of background underneath the peak is nearly represented by a trapezoid, whose area can be calculated.

<table>
<thead>
<tr>
<th></th>
<th>$A_{\text{peak}} \pm \sigma$</th>
<th>$A_{\text{backg}} \pm \sigma$</th>
<th>$\pi \pm \sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed (start)</td>
<td>332 $\pm$ 18</td>
<td>175 $\pm$ 13</td>
<td>1.89 $\pm$ 0.18</td>
</tr>
<tr>
<td>anti-delayed (stop)</td>
<td>311 $\pm$ 17</td>
<td>175 $\pm$ 13</td>
<td>1.78 $\pm$ 0.17</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.84 $\pm$ 0.17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Values of the areas of the full energy peaks at 1510 keV and underneath background of both the start and stop energy spectra. The errors on the areas are calculated according to Eq. 3.16 and Eq. 3.17, while the errors on $\pi$ by Eq. 3.15. The $\pi$ error is the mean standard deviation.
4.1. Analysis

Figure 4.10: ($\Delta C, E, \gamma$) plot in the case of the analysis of the 1510 keV line of $^{133}$Sb, on the $^{241}$Pu data. The red line, whose equation is indicated, represent the linear trend found and the error bars are calculated using Eq. 3.13. The square of the correlation coefficient $R^2$ is also shown.

Table 4.6: $\Delta C_{\text{Compt}}$ at different energies relative to the 1510 keV line of the 1510 keV line of $^{133}$Sb, on the $^{241}$Pu data. The errors are calculated using Eq. 3.11.

<table>
<thead>
<tr>
<th>E [keV]</th>
<th>$\Delta C \pm \sigma$ [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1725</td>
<td>-378 ± 17</td>
</tr>
<tr>
<td>1825</td>
<td>-426 ± 19</td>
</tr>
<tr>
<td>1925</td>
<td>-509 ± 20</td>
</tr>
</tbody>
</table>

The Fig. 4.10 presents the linear trend obtained:

$$\Delta C_{\text{Compt}}(E) = mE + q$$  \hspace{1cm} (4.5)

with $m = -0.66 \pm 0.02$ and $q = 776.22 \pm 22.32$, whose errors are given by the fit. It follows

$$\Delta C_{\text{Compt}}(1510 \text{ keV}) = -220.38 \pm 4.29 \text{ ps}$$  \hspace{1cm} (4.6)

The PRD is the same as for $^{235}$U but with opposite sign:
4.1. Analysis

\[ PRD(1510 \text{ keV} - 162 \text{ keV}) = -260.71 \pm 7.91 \text{ ps} \]  \hspace{1cm} (4.7)

Therefore, the lifetime for the 13/2^+ level is

\[
\tau_{13/2^+} = \frac{1}{2} \left( \frac{\Delta C - \Delta C_{\text{Compton}}}{\pi} + \Delta C - PRD \right)
\]

\[
= \frac{1}{2} \left( \frac{-200 + 220.38}{1.84} - 200 + 260.71 \right)
\]

\[
= 35.90 \pm 6.57 \text{ ps} \]  \hspace{1cm} (4.8)

This value is in good agreement with the lifetime found with the analysis for \(^{235}\text{U}\) target.
4.1. Analysis

4.1.2 $15/2^+$

The evaluation of $15/2^+$ lifetime was done considering the transitions 61-162 keV, as start and stop signals, respectively. With the aim of clearly observe these peaks, the 2792 keV transition was set as HPGe gate, selecting $\gamma$s in coincidence. In Fig. 4.11 the full energy peaks for the transitions of interest, after setting the 2792 keV as HPGe gate, are shown.

![Graph showing energy spectrum focusing on $\gamma$s of $^{133}$Sb forming the start and stop signals. The black line represents the HPGe detectors, the red line the LaBr$_3$(Ce) scintillators.](image)

**Figure 4.11:** Zoom of the energy spectrum focusing on $\gamma$s of $^{133}$Sb forming the start and stop signals. The black line represents the HPGe detectors, the red line the LaBr$_3$(Ce) scintillators.

The fast timing matrices were created by setting a further LaBr$_3$(Ce) gate on the transition at lowest energy, 62 keV. The projection, for both matrices, of the 162 keV full energy peak on the time axis let us to obtain the time distributions for the delayed and anti-delayed signals. Fig. 4.15 shows the time distributions extracted and the shift between them. The values for the centroids and the centroid difference obtained by the analysis of the spectra are reported in Tab. 4.10.

In the following pages the background corrections, related to peak-to-background ratio and Compton, are presented.
4.1. Analysis

Counts/10 ps

Time [ps]

Delayed

Anti-delayed

133Sb

Figure 4.12: Time distributions for the delayed (black) and the anti-delayed (red) signals relative to the analysis of the 15/2+ state of $^{133}$Sb. The error bars indicated are the root mean square of the counts.

<table>
<thead>
<tr>
<th></th>
<th>centroid [ps]</th>
<th>Error [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed $C_{\text{start}}$</td>
<td>23963</td>
<td>11</td>
</tr>
<tr>
<td>anti-delayed $C_{\text{stop}}$</td>
<td>24069</td>
<td>11</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>106 ± 15 ps</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7: Centroids values obtained by integration of time distributions in the case of the 15/2+ state of $^{133}$Sb. The centroid difference is $\Delta C = C_{\text{stop}} - C_{\text{start}}$. The errors on the centroids are evaluated by Eq. 3.11 while the $\Delta C$ error comes from the error propagation (Eq. 3.13).

The linear trend extrapolated and presented in Fig. 4.14

$$\Delta C_{\text{Compt}}(E) = m E + q$$ (4.9)

with $m = -1.50 \pm 0.01$ and $q = 91.5 \pm 0.38$, whose errors are given by the fit, let us to calculate

$$\Delta C_{\text{Compt}}(162 \text{ keV}) = -152.25 \pm 0.41 \text{ ps}$$ (4.10)

Regarding the PRD, it was calculated using the fit by which the setup was
Figure 4.13: Example of peak-to-background evaluation for the 162 line of $^{133}$Sb. The fit of the full energy peak is shown. Around the peak, the two background regions considered are also shown. As we can see, the region of background underneath the peak is nearly represented by a trapezoid, whose area can be calculated.

Table 4.8: Values of the areas of the full energy peaks and underneath background of both the start and stop energy spectra in the case of the $15/2^+$ lifetime analysis of $^{133}$Sb. The errors on the areas are calculated according to Eq. 3.16 and Eq. 3.17, while the errors on $\pi$ are given by Eq. 3.15. The $\pi$ error is the mean standard deviation.

<table>
<thead>
<tr>
<th></th>
<th>$A_{\text{peak}} \pm \sigma$</th>
<th>$A_{\text{backg}} \pm \sigma$</th>
<th>$\pi \pm \sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed (start)</td>
<td>1456 $\pm$ 38</td>
<td>1300 $\pm$ 36</td>
<td>1.12 $\pm$ 0.04</td>
</tr>
<tr>
<td>anti-delayed (stop)</td>
<td>1330 $\pm$ 36</td>
<td>1365 $\pm$ 37</td>
<td>0.97 $\pm$ 0.03</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.05 $\pm$ 0.04</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Therefore, the lifetime for the $15/2^+$ level is

$$PRD(62 \text{ keV} - 162 \text{ keV}) = 184.72 \pm 14.16 \text{ ps} \quad (4.11)$$
4.1. Analysis

Figure 4.14: \((\Delta C, E)\) plot relative to the analysis of the 162 keV line of \(^{133}\text{Sb}\). The red line, whose equation is indicated, represent the linear trend found and the error bars are calculated using Eq. 3.13. The square of the correlation coefficient \(R^2\) is also shown.

<table>
<thead>
<tr>
<th>E [keV]</th>
<th>(\Delta C \pm \sigma) [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>145</td>
<td>-100 ± 14</td>
</tr>
<tr>
<td>180</td>
<td>-224 ± 14</td>
</tr>
<tr>
<td>310</td>
<td>-335 ± 17</td>
</tr>
<tr>
<td>375</td>
<td>-493 ± 16</td>
</tr>
</tbody>
</table>

Table 4.9: \(\Delta C_{\text{Compt}}\) at different energies, as obtained by the analysis of the 162 keV line of \(^{133}\text{Sb}\). The errors are calculated using Eq. 3.11.

\[
\tau_{15/2^+} = \frac{1}{2} \left( \frac{\Delta C - \Delta C_{\text{Compton}}}{\pi} + \Delta C - \text{PRD} \right) \\
= \frac{1}{2} \left( \frac{106 + 152.25}{1.05} + 106 - 184.72 \right) \\
= 83.95 \pm 8.53 \text{ ps (4.12)}
\]
As already done for $13/2^+$ level, this analysis has been repeated using data with $^{241}$Pu target. We have considered the same transitions for start and stop signals (cascade 62-162 keV) and for the gates (2792 keV as HPGe gate and 62 keV as LaBr$_3$(Ce) gate). The values obtained for the centroid difference, peak-to-background ratio and $\Delta C_{Comp}$ are indicated in the next pages.

![Graph showing time distributions for delayed and anti-delayed signals](image)

**Figure 4.15:** Time distributions for the delayed (black) and the anti-delayed (red) signals obtained in the case of the $15/2^+$ state analysis of $^{133}$Sb on the $^{241}$Pu. The error bars indicated are the root mean square of the counts.

<table>
<thead>
<tr>
<th></th>
<th>centroid [ps]</th>
<th>Error [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed $C_{start}$</td>
<td>24009</td>
<td>14</td>
</tr>
<tr>
<td>anti-delayed $C_{stop}$</td>
<td>24049</td>
<td>13</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>40 ± 19 ps</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.10:** Centroids values obtained by integration of time distributions for the $15/2^+$ state analysis of $^{133}$Sb, on the $^{241}$Pu data. The centroid difference is $\Delta C = C_{stop} - C_{start}$. The errors on the centroids are evaluated by Eq. 3.11 while the $\Delta C$ error comes from the error propagation (Eq. 3.13).
4.1. Analysis

Counts
0 20 40 60 80 100 120 140
Energy [keV]
100 150 200 250

Figure 4.16: Example of peak-to-background evaluation for the 162 keV line of $^{133}$Sb, in the case of the $^{241}$Pu data. The fit of the full energy peak is shown. Around the peak, the two background regions considered are also shown. As we can see, the region of background underneath the peak is nearly represented by a trapezoid, whose area can be calculated.

<table>
<thead>
<tr>
<th></th>
<th>$A_{\text{peak}} \pm \sigma$</th>
<th>$A_{\text{backg}} \pm \sigma$</th>
<th>$\pi \pm \sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>delayed (start)</td>
<td>762 ± 27</td>
<td>966 ± 31</td>
<td>0.79 ± 0.04</td>
</tr>
<tr>
<td>anti-delayed (stop)</td>
<td>818 ± 29</td>
<td>931 ± 31</td>
<td>0.88 ± 0.04</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.83 ± 0.04</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.11: Values of the areas of the full energy peaks and underneath background of both the start and stop energy spectra for the 162 keV line of $^{133}$Sb, on the $^{241}$Pu data. The errors on the areas are calculated according to Eq. 3.16 and Eq. 3.17, while the errors on $\pi$ by Eq. 3.15. The $\pi$ error is the mean standard deviation.
4.1. Analysis

Figure 4.17: $(\Delta C,E_\gamma)$ plot for the 162 keV line of $^{133}$Sb, on the $^{241}$Pu data. The red line, whose equation is indicated, represent the linear trend found and the error bars are calculated using Eq. 3.13. The square of the correlation coefficient $R^2$ is also shown.

Table 4.12: $\Delta C_{\text{Compt}}$ at different energies for the 162 keV line of $^{133}$Sb, in the case of the $^{241}$Pu data. The errors are calculated using Eq. 3.11.

| E [keV] | $\Delta C \pm \sigma$ [ps] |
|---------|----------------|---|
| 180     | -246 ± 20    |
| 239     | -355 ± 20    |
| 310     | -562 ± 22    |
| 375     | -550 ± 24    |

The linear trend is described by the equation:

$$\Delta C_{\text{Compt}}(E) = mE + q \quad (4.13)$$

with $m = -1.68 \pm 0.01$ and $q = 36.74 \pm 0.42$, whose errors are given by the fit. In this way we can calculate the $\Delta C_{\text{Comp}}$ as

$$\Delta C_{\text{Compt}}(162 \text{ keV}) = -236.26 \pm 0.45 \text{ ps} \quad (4.14)$$

Finally, considering the same PRD, already extrapolated, since the transitions are the same

$$PRD(62 \text{ keV} - 162 \text{ keV}) = 184.72 \pm 14.16 \text{ ps} \quad (4.15)$$
the lifetime found is:

\[
\tau_{15/2^+} = \frac{1}{2} \left( \frac{\Delta C - \Delta C_{Compton}}{\pi} + \Delta C - PRD \right) \\
= \frac{1}{2} \left( \frac{40 + 236.26}{0.83} + 40 - 184.72 \right) \\
= 93.32 \pm 10.85 \text{ ps (4.16)}
\]

As one can see, also for this level, the results obtained show a good agreement with the \(^{235}\text{U}\) analysis.
4.2 Comparison with theoretical predictions

The lifetimes obtained in the presented analysis allow to do some considerations on the nature of these states. In particular, as explained in Chapter 1, we expect that the $13/2^+$ and $15/2^+$ states belong, with the $11/2^+$ level, to the same multiplet deriving, for example, from $g_{7/2}$ proton coupling to the $4^+$ state of the $^{132}$Sn core. A schematic representation of this coupling is shown in Fig. 4.18.

\begin{equation}
\langle RJI| M(M1) | RJI \rangle = \sqrt{\frac{3}{4\pi}} \frac{\epsilon h}{2Mc} \langle g_J \sqrt{I(I+1)(2I+1)} \rangle \delta(I, I') + \sqrt{\frac{3}{4\pi}} \frac{\epsilon h}{2Mc} (-1)^{R+J+I+1} (g_\lambda - g_J) \sqrt{2I' + 1} \times \sqrt{R(R+1)(2R+1)} \left\{ \begin{array}{ccc} R & J & I \\ I' & 1 & R \end{array} \right\} (4.17)
\end{equation}

where $R$ is the angular momentum of the vibrational excitation, $J$ is the single-particle angular momentum, $I$ and $I'$ are the angular momenta of the initial and final state, belonging to the multiplet. The factor $g_\lambda$ and $g_J$ cancel out if we consider the ratio between the transition probabilities and thus the matrices elements. Considering that this ratio is related to the lifetime ratio between the two states, a theoretical calculation led to determine that, in
order to confirm our hypothesis, this lifetimes relation should be

\[
\tau_{15/2^+} \sim 50\tau_{13/2^+}
\]  

(4.18)

However, the experimental results show that these levels can’t belong to the same multiplet, therefore further theoretical work is need to address the complex nature of these states.
Chapter 5

Conclusions

In this thesis we performed a lifetimes analysis in $^{133}\text{Sb}$. This nucleus was produced by cold neutron induced fission on $^{235}\text{U}$ and $^{241}\text{Pu}$ targets. The measurements, conducted at Institut Laue-Langevin (ILL) of Grenoble, were performed with a composite array constituted by HPGe detectors (EXILL) coupled with LaBr$_3$(Ce) scintillators (FATIMA).

In the first part of the work, the fast timing technique of the Centroid Difference Method was used to confirm excited states known lifetimes of the three nuclei $^{100}\text{Zr}$, $^{132}\text{Sn}$, $^{90}\text{Kr}$. In this way, we have proved the reliability of our analysis procedure in three different lifetime ranges (from tens of ps to ns). In the second part, lifetime analysis on $^{133}\text{Sb}$ was performed. In particular, the excited states 13/2$^+$ (at 4302 keV) and the 15/2$^+$ (at 4464 keV) were investigated considering both the $^{235}\text{U}$ and $^{241}\text{Pu}$ targets. The results obtained are in good agreement and a lifetime of 37(10) ps, for the 13/2$^+$ level, and a preliminary lifetime of 89(14) ps, for 15/2$^+$, were found. The lifetimes determination represents a starting point in the investigation on the interaction between collective excitation and single particle states. Experimental informations are provided on $^{133}\text{Sb}$, to be compared with theoretical models in order to improve the knowledge of this phenomenon and to confirm the nature of these states. It is anticipated that the experimental values indicate a different structure for these levels, being built on different $^{132}\text{Sn}$ core excitations.
Appendix A

The Centroid Shift Method

The Centroid Shift Method is essentially based on the evaluation of the centroid of time distributions [19, 21, 22, 23]. The Centroid or centre of gravity, i.e. the first moment of a delayed time distribution $D(t)$, is defined as

$$C^D = \langle t \rangle = \frac{\int t D(t) dt}{\int D(t) dt} \quad (A.1)$$

Assuming the ideal case of no background contribution, it follows that the time distribution, whose centroid is $C^D$, is displaced by the mean lifetime from the centroid $C^P$ of its corresponding Prompt Response Function (PRF):

$$\tau = \pm [C^D - C^P] \quad (A.2)$$

where the sign is negative if the decay transition of the cascade provides the start signal for the TAC (anti-delayed time spectrum). As the PRF centroid $C^P$ is energy dependent, the knowledge of the setup time-walk characteristics is crucial for the centroid shift analysis. Therefore, the time-walk characteristics, often called Prompt Curve has to be determined as precise as possible. The Prompt Curve can be determined by the measurements of prompt events at different energies, covering a wide dynamic range and calibrating empirically using a monotonic function of the energy, like the one physically derived for the time resolution expressed by Eq. 3.2. To evaluate this Prompt Curve in a fast timing experiment, a reference timing signal is needed. This is achieved by gating on a $\gamma$ ray detector on a directly populating or depopulating transition of interest. The other detector selects prompt coincident transitions of different energies to measure its prompt curve. In the off-line analysis this is done for all the detectors in order to gain statistics. In this way we have two prompt response curves: one $(C^P(E)_{\text{stop}})$ connects the centroids obtained by gating the reference energy with the stop detector.
and varying the gates of the start detector, while the other was achieved by
gating on the reference energy with the start detector and varying the gates
of the stop detector. The two calibrated Prompt Curves cross each other
at nearly the reference energy gate of the system, in fact, for equal energy
gates, it is not possible to distinguish between the $\gamma$ rays triggering either
both the timing of the start and the stop branch. Nevertheless this method
to determine the PRF centroid for an energy combination of interest is not
suitable, in fact the timing characteristics of the two branches differ in a real
setup.
The timing asymmetry is cancelled if we consider the centroid difference

$$\Delta C = C^D_{\text{stop}} - C^D_{\text{start}} \quad (A.3)$$

for a specific $\gamma$ ray cascade and consequently the energy dependent Prompt
Response Difference (PRD)

$$\text{PRD} = C^P_{\text{stop}} - C^P_{\text{start}} \quad (A.4)$$

which describes the combined $\gamma-\gamma$ timing characteristics of the setup independent of the single branch timing characteristics. Referring to Eq. A.2,
the centroid curve $C^D_{\text{stop}}$ is shifted by $+\tau$ from the corresponding prompt
curve $C^P_{\text{stop}}$, as the decay transition, in that case used as the reference timing
signal, is gated by the stop detector. If the decay transition is gated by
the start detector, the corresponding centroid curve $C^D_{\text{start}}$ is shifted by $-\tau$
from the prompt curve, as in this case the start detector is affected by the
lifetimes. Considering the centroid difference written as

$$\Delta C(\Delta E_\gamma)_{\text{decay}} = C^D_{\text{stop}} - C^D_{\text{start}} \quad (A.5)$$

where $\Delta E_\gamma = E_{\text{feeder}} - E_{\text{decay}}$ and the subscript decay indicates that the
reference timing signal of the setup is provided by a decay transition, which
in case of the $C^D_{\text{stop}}$ is gated on the stop detector. In this way, referring to
the decay transition and according to the previous equation, the centroid
difference is

$$\Delta C(\Delta E_\gamma)_{\text{decay}} = C^P(\Delta E_\gamma)_{\text{stop}} + \tau - (C^P(\Delta E_\gamma)_{\text{start}} - \tau)$$
$$= \text{PRD}(\Delta E_\gamma)_{\text{decay}} + 2\tau \quad (A.6)$$
or, if we use a feeding transition as reference energy gate,

$$\Delta C(\Delta E_\gamma)_{\text{feeder}} = C^P_{\text{start}} - C^P_{\text{stop}}$$
$$= C^P(\Delta E_\gamma)_{\text{start}} + \tau - (C^P(\Delta E_\gamma)_{\text{stop}} - \tau)$$
$$= \text{PRD}(\Delta E_\gamma)_{\text{feeder}} + 2\tau \quad (A.7)$$
If we consider a cascade of two subsequent $\gamma$ rays with different energies we can write

$$\Delta C(\Delta E_{\text{cascade}})_{\text{decay}} = \Delta C(\Delta E_{\text{cascade}})_{\text{feeder}} \quad \text{(A.8)}$$

$$PRD(\Delta E_{\text{cascade}})_{\text{decay}} = PRD(\Delta E_{\text{cascade}})_{\text{feeder}} \quad \text{(A.9)}$$

and accordingly

$$PRD = 0 \quad \text{and} \quad \Delta C = 2\tau \quad \text{for} \quad \Delta E_\gamma = 0 \quad \text{(A.10)}$$

These relations denote that for specific energy combination $\Delta E_{\text{cascade}}$, the PRD is independent of whether the reference energy gate corresponds to the energy of the feeding or the decaying transition. This is independent of the energy combination and, thus, the measurement of centroid differences cancels the timing asymmetry of the two timing branches in a real fast timing setup. This method is called **Mirror Symmetric Difference method**.

Considering a specific reference energy gate which corresponds to the energy of a feeding ($\text{ref.} = E_{\text{feeder}}$ and $\Delta E_\gamma = \text{ref} - E_\gamma$), the PRD can be written as

$$PRD(-\Delta E_\gamma)_{\text{ref}=\text{feeder}} = C^P(E_\gamma - \text{ref})_{\text{start}} - C^P(E_\gamma - \text{ref})_{\text{stop}}$$

$$= -(C^P(E_\gamma - \text{ref})_{\text{stop}} - (C^P(E_\gamma - \text{ref})_{\text{start}}$$

(A.11)

For a decay transition that is used as reference signal, $\Delta E_\gamma = E_\gamma - \text{ref.}$, and thus Eq. A.11 is equivalent to

$$PRD(\Delta E_\gamma)_{\text{ref}=E_{\text{decay}}} = -PRD(-\Delta E_\gamma)_{\text{ref}=E_{\text{feeder}}} \quad \text{(A.12)}$$

$$\Delta C(\Delta E_\gamma)_{\text{ref}=E_{\text{decay}}} = -\Delta C(-\Delta E_\gamma)_{\text{ref}=E_{\text{feeder}}} \quad \text{(A.13)}$$

These mirror symmetric relations imply that the value of the centroid difference (PRD) for any energy combination is independent of the reference energy gate of the system. This means, in particular, that the PRD for any energy combination is obtained from the PRD curve regardless of the reference energy gate used to calibrate the PRD curve.
Bibliography


