STUDY OF PARTICLE-VIBRATION COUPLED STATES IN THE NUCLEUS $^{61}$Cu

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Alla mia famiglia
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Introduction

This thesis deals with the experimental study, through gamma-ray spectroscopy, of excited states in nuclei around $A \approx 60$. In particular, the focus is on the experimental search for particle-phonon coupled states in $^{61}$Cu (which is one proton away from semimagic-nucleus $^{60}$Ni) and on their description in the framework of the Particle-Vibration-Coupling (PVC) model [1].

The understanding of the particle-phonon and phonon-phonon couplings is a very important issue, since this phenomenon is at the basis of fermionic many-body interacting systems, both in solid state and nuclear physics. In nuclear physics, the coupling between a particle/hole and vibration is a key ingredient to explain important phenomena, such as the observed reduction of spectroscopic factors, the anharmonicity of vibrational spectra, the damping of the Giant Resonances, etc.

The experiment analysed in this thesis was performed at Horia Hulubei National Institute for R&D in Physics and Nuclear Engineering (IFIN-HH) of Bucharest, in April of 2014.

The nucleus of interest $^{61}$Cu was populated by reaction $^{16}$O $+$ $^{48}$Ti, at 54 MeV of bombarding energy, which produced nuclei in the mass region between $A = 55$ and $A = 62$. In particular, the experiment focused on the analysis of $9/2^+$ state of the $^{61}$Cu at 2.72 MeV, which is supposed to arise from a weak coupling between a proton in the $2p_{3/2}$ state and the $3^-$ octupole vibration at 4.04 MeV, in the $^{60}$Ni core.

A short theoretical introduction on the reaction mechanism here employed
and on the importance of lifetime measurement is presented in Chapter 1.

The experimental setup, described in Chapter 2, was made of 14 HPGe and 11 LaBr$_3$(Ce) detectors in $4\pi$ configuration, coupled to a plunger device for lifetime measurements. The experimental analysis, reported in Chapter 3, can be divided in two different parts: the first one is devoted to the determination of the decay branching of the $9/2^+$ level at 2.72 MeV, while in the second part, using the Recoil-Distance-Doppler-Shift (RDDS) method, the lifetime of the level of interest is extracted.

After the calibration of the apparatus, the measurement of the branching ratio of the direct transition to ground state of the level $9/2^+$ at 2720 keV was performed, followed by the measurement of the lifetime of the $9/2^+$ state with the plunger.

The lifetime and the branching ratio were used to determine the reduced transition probability to the ground state, $B(E3) = 22(5)$ W.u.. This value is compatible with the experimental measurement of the $B(E3)$ in the $^{60}$Ni core nucleus, therefore is a first insight on the collectivity of the $9/2^+$ state of interest.

The last part of the thesis work, described in Chapter 4, deals with the theoretical interpretation of the obtained results. The structure of the $9/2^+$ state in $^{61}$Cu was interpreted within the framework of the particle vibration coupling (PVC) model, using the weak coupling approximation. At first, the single particle levels of the $^{60}$Ni were calculated using the Hartree-Fock-BCS formalism and employing two different forces: SkX and Sly5. Starting from these levels and from the experimental $B(E3)$ of the core nucleus, PVC calculation were performed.

The results obtained from this study show that the $9/2^+$ state of $^{61}$Cu nucleus presents a collective character and it is a good candidate to be member of the multiplet generated by coupling the $3^-$ octupole vibration and the proton single particle state $2p_{3/2}$. This collectivity is rather well described
by the PVC model.

In conclusion, this work provides an additional example of Particle - Vibration coupled states in $^{61}$Cu, improving the knowledge of this phenomenon along the Cu chain, a region where already previous studied were recently performed in $^{65,67}$Cu [2, 3].
Chapter 1

Nuclear Reaction with Heavy Ions

In this chapter we briefly recall key aspects of nuclear reaction mechanism, mainly in connection with fusion evaporation reactions which are relevant for this thesis work.

1.1 Nuclear Reactions

Nuclear reactions are a very important tool for the study of nuclear structure. The principal nuclear reactions are induced by particle coming from an accelerator or from a nuclear reactor.

Nuclear reactions can be divide in two principal categories: direct reactions and compound nucleus reactions. Direct reactions are fast, they occur on a time scale of $10^{-22}$ s and only few nucleons participate to the reaction. At this category belongs scattering reactions and transfer reactions. Compound nucleus reactions are much slower ($10^{-18} - 10^{-16}$ s) and in this case projectile and target nuclei merge together and sharing all the energy. After the fusion the evaporation of one or few nucleons usually happens.

The first parameter involved in the study of nuclear reactions is the $Q$-
1.1. Nuclear Reactions

value. This is given by

\[ Q = T_{\text{after}} - T_{\text{before}} \]  \hspace{1cm} (1.1)

where \( T_{\text{after}} \) and \( T_{\text{before}} \) are the kinetics energy of the system before and after the reaction. If \( Q > 0 \) the reaction is exo-energetic and an amount of nuclear mass or binding energy is released in the form of kinetic energy of the reaction products. If \( Q < 0 \) the reaction is endo-energetic and a part of the initial kinetic energy is converted into mass and binding energy, \( B(A, Z) \), which is defined as:

\[ B(A, Z) = Zm_p + (A-Z)m_n - M(A, Z) \]  \hspace{1cm} (1.2)

where \( m_p \) and \( m_n \) are the proton e neutron mass respectively and \( M(A, Z) \) is the mass of a nucleus with \( A \) nucleons and \( Z \) protons.

Another useful parameter for the classification of the reactions is the impact parameter \( (b) \), that is the distance at which the particle approach the nucleus along a straight line in absence of the repulsive force [4]. Figure 1.1 presents a schematic classification of some nuclear reactions. In this work we focus only on Fusion-Evaporation reactions, as described in the following Section.

1.1.1 Fusion Evaporation Reactions

In order to study high spin states, one requires a reaction which will impart the largest possible angular momentum to the nucleus of interest. Fusion-evaporation reactions are the best way of producing high spin states with large cross-sections.

In a fusion-evaporation reaction, the kinetic energy of the collision in the centre of mass frame is converted into excitation energy of the compound system. Generally, this kind of reaction is schematically expressed as:

\[ a + X \rightarrow C^* \rightarrow Y + b \]  \hspace{1cm} (1.3)
1.1. Nuclear Reactions

Figure 1.1: Schematic representation of possible nuclear reactions as a function of the energy of the nucleon and of the impact parameter.

where a is the projectile, X is the target, C* is the compound nucleus, Y is the final nucleus and b is the outgoing particle (or particles) [4]. As it might be assumed from seeing the reaction written in this form, we can consider a reaction that proceeds through the compound nucleus to be a two-step process: the formation and then the subsequent decay of the compound nucleus.

A given compound-nucleus may decay in a variety of different ways, and essential to the compound-nucleus model of nuclear reaction is the assumption that the relative probability for decay into any specific set of final products is independent of the means of formation of the compound nucleus. The decay probability depends only on the total energy given to the system. By conservation energy, the compound nucleus will be formed at an excitation energy which depends on the centre of mass kinetic energy of the collision.
1.1. Nuclear Reactions

and the Q-value for compound nucleus formation is such that

\[ E_{ex} = E_{cm} + Q_{fus} \]  \hspace{1cm} (1.4)

where \( E_{cm} \) is the kinetic energy of the collision which is transferred to the compound system.

By conservation of momentum, for beam (i.e. the projectile) and target masses \( M_B \) and \( M_T \), respectively, the velocity of the recoiling compound, \( V_R \) can be calculated using

\[ M_B V_B = (M_T + M_B) V_R \]  \hspace{1cm} (1.5)

and using the conservation of the energy

\[ E_{cm} = E_B - \frac{1}{2} (M_T + M_B) V_R^2 \]  \hspace{1cm} (1.6)

we obtain

\[ E_{cm} = E_B \left( 1 - \frac{M_B}{M_T + M_B} \right) \]  \hspace{1cm} (1.7)

The maximum angular momentum that can be transferred in a fusion evaporation reaction will occur when the two nuclei undergo a peripheral collision. This is the so-called sharp cut-off approximation which describes the nuclei as "hard spheres", without a diffuse surface. The fusion cross-section will be a sum of partial waves (depending on the size of the impact parameter). In the sharp cut off approximation, the assumption is that the transmission coefficient \( T_l \), which is defined as the probability of formation of compound nucleus with angular momentum \( l \), for nuclear penetration falls to zero for \( l > l_{max} \) and has a value of 1 for \( l \leq l_{max} \). Thus, the total fusion reaction cross-section, \( \sigma_f \) can be written as a sum of partial waves up to \( l_{max} \) such that

\[ \sigma_f = \pi \left( \frac{\lambda}{2\pi} \right)^2 \sum_{l=0}^{2l_{max}} (2l + 1) T_l \approx \pi \left( \frac{\lambda}{2\pi} \right) (l_{max} + 1)^2 \]  \hspace{1cm} (1.8)
1.1. Nuclear Reactions

where $\lambda$ is the wavelength of the entrance channel given by

$$\lambda = \frac{h}{2\sqrt{2E_{cm}\mu}} \tag{1.9}$$

where $E_{cm}$ is the kinetic energy of the collision in the centre of mass and

$$\mu = \frac{A_B A_T}{A_B + A_T}$$

is the reduced mass of the system, where $A_B$ and $A_T$ are the masses of the projectile and target nuclei, respectively.

The value of $l_{\text{max}}$, calculated using the reduced mass of the system, $\mu$, and imposing the conservation of the energy and angular momentum is given by

$$\hbar l_{\text{max}} = \mu v R \tag{1.10}$$

In the previous expression, the velocity $v$ can be calculated using the conservation of energy in terms of kinetic energy ($E_{cm}$) of the collision in the centre of mass and of the Coulomb barrier ($V_c$), by the expression

$$\frac{1}{2}\mu v^2 = E_{cm} - V_c \tag{1.11}$$

Substituting in for $v$ we obtain

$$l_{\text{max}}^2 = \frac{2\mu R^2}{\hbar^2}(E_{cm} - V_c) \tag{1.12}$$

where $R$ is the maximum nucleus-nucleus distance for which a reaction can occur and it is given (in fm) empirically by

$$R = 1.36 \left( A_B^{1/3} + A_T^{1/3} \right) + 0.5 \tag{1.13}$$

The Coulomb barrier, expressed in MeV, is given by the relation

$$V_c = 1.44 \frac{Z_B Z_T}{R} \tag{1.14}$$

It is clear that those collisions which maximize the value of the reduced mass (i.e. symmetric reactions) will have the largest input angular momentum for a given centre of mass energy.
1.2 Fusion Evaporation Reaction with $^{16}$O

The reaction used in present work is a fusion evaporation reaction with a $^{16}$O projectile. This is a stable nucleus which is often used in heavy ion reaction.

The partial cross section for formation of a compound nucleus of spin $J$ and parity $\pi$ from a projectile and a target nucleus (spins $J_P$, $J_T$) at a center of mass energy $E$ is given by [5]:

$$\sigma(J, \pi) = \pi \lambda^2 \frac{2J + 1}{(2J_P + 1)(2J_T + 1)} \sum_{S=|J-S|}^{J_P+J_T} T_L(E)$$  \hspace{1cm} (1.15)

The transmission coefficients $T_L$ are assumed to depend only on the energy and the orbital angular momentum $L$. Here $S = J_P + J_T$ is the channel spin. The summation over $L$ is restricted by the parity selection ruler $\pi = \pi_P \pi_T (-1)^L$.

For the strongly absorbed heavy ions, the transmission coefficients $T_L$ have a very simple behaviour as function of angular momentum. In the calculation they are approximated by a Fermi distribution:

$$T_L = \frac{1}{1 + \exp[1 - (L - L_0)/d]}$$  \hspace{1cm} (1.16)

with the parameters $L_0$ and $d$. Here $L_0$ is chosen so that the measured fusion cross section:

$$\sigma_{CN} = \sum_{J,\pi} \sigma(J, \pi)$$  \hspace{1cm} (1.17)

is reproduced. In this way, one corrects for the presence of direct reactions, which are usually assumed to be surface reactions.
1.3 Lifetime Measurements

The measurement of the lifetimes of excited states in nuclei is one of the most active areas of nuclear structure physics. During the last tens years it has been a major growth point providing data giving unique and vital informations and playing a major role in our understanding of nuclear structure. In order to measure the lifetime of an excited state it has to be populated using a nuclear reaction chosen to optimise the conditions required for accurate measurement. Once populated, the state will remain excited for a mean lifetime \( \tau \) which is related to the width \( \Gamma \) of the state by the relation:

\[
\Gamma \tau = \hbar
\]

As the state is an excited one there is a finite probability of decay to some state of lower energy, as shown Fig. 1.2.

This probability is proportional to \( \tau \) and is determined by the matrix
1.3. Lifetime Measurements

Element describing the mode of decay joining the initial and final states [6]. Thus:

\[ \Gamma \propto |\langle \psi_f | \hat{O}_{\text{decay}} | \psi_i \rangle|^2 \]  

(1.19)

where \( |\psi_i\rangle \) and \( |\psi_f\rangle \) are the nuclear wavefunctions describing the initial and final states, respectively, and \( \hat{O}_{\text{decay}} \) is a quantum-mechanical operator describing the mode of decay.

The point, therefore, of measuring nuclear lifetimes is to gain information about matrix elements of the type in Eq. 1.19 so that these, in turn, may be compared with the predictions of nuclear models. For this comparison it is preferable to have matrix elements where the operator \( \hat{O}_{\text{decay}} \) is well known so that uncertainties are reduced.

1.3.1 Reduced Transition Probabilities

The electromagnetic interaction operators have a well-known structure and only weakly perturb the strong nuclear interaction. Hence, it is possible to compare experimentally determined \( \gamma \)-ray reduced transition probabilities between nuclear states with the theoretical predictions of nuclear models. This is one of the most sensitive tools available to investigate the wavefunctions of nuclei. The reduced transition probabilities are given by the following relation [1]

\[ B(M^\lambda; I_i \rightarrow I_f) = (2I_i + 1)^{-1} |\langle \Psi_f | M(M^\lambda) | \Psi_i \rangle|^2 \]  

(1.20)

where \( M(M^\lambda) \) is the electric or magnetic multipole operator and \( M^\lambda \) refers to the case of an electric or a magnetic transition and its multipolarity, \( \lambda \). \( I_i \) and \( I_f \) are the spin of the initial and final states, respectively. For the reverse transition, \( I_f \rightarrow I_i \), we have

\[ B(M^\lambda; I_f \rightarrow I_i) = \frac{2I_f + 1}{2I_i + 1} B(M^\lambda; I_i \rightarrow I_f) \]  

(1.21)
1.3. Lifetime Measurements

In particular, for a $\gamma$-ray transition with a given multipolarity, it is possible to relate the reduced transition probabilities to the total decay probability $T$. For the electric transitions the relation are given by [7]

\begin{align}
T(E1) &= 1.578 \cdot 10^{15} E_{\gamma}^3 B(E_1) \\
T(E2) &= 1.223 \cdot 10^9 E_{\gamma}^5 B(E_2) \\
T(E3) &= 5.698 \cdot 10^2 E_{\gamma}^7 B(E_3) \\
T(E4) &= 1.694 \cdot 10^{-4} E_{\gamma}^9 B(E_4)
\end{align} (1.22)

while for the magnetic transition are

\begin{align}
T(M1) &= 1.779 \cdot 10^{13} E_{\gamma}^3 B(M_1) \\
T(M2) &= 1.371 \cdot 10^7 E_{\gamma}^5 B(M_2) \\
T(M3) &= 6.387 \cdot 10^0 E_{\gamma}^7 B(M_3) \\
T(M4) &= 1.899 \cdot 10^{-6} E_{\gamma}^9 B(M_4)
\end{align} (1.23)

The energy of the transition $E_{\gamma}$ is given in MeV, $B(E\lambda)$ in units of $e^2(\text{fm})^{2\lambda}$ and $B(M\lambda)$ in units of $\left(\frac{e\hbar}{2Mc}\right)^2 (\text{fm})^{2\lambda-2}$.

Experimentally it is possible to measure the lifetime of a level $\tau$. The knowledge of the branchig-ratio allow to obtain the partial lifetime $\tau_{\gamma}$ of a particular $\gamma$-decay by the relation

\[ \tau^{(M_E \lambda)} = \tau \frac{I_{\gamma \text{total}}}{I_{\gamma (M_E \lambda)}} \] (1.24)

where $\tau$ is the measured lifetime of the level of interest, $I_{\gamma}$ is the intensity of the $\gamma$-ray transition with multipolarity $\lambda$ and $I_{\gamma \text{total}}$ is the of the intensities of all the gamma transition depopulating the level. The relation between the decay probability and the lifetime is

\[ T^{(M_E \lambda)} = \frac{1}{\tau^{(M_E \lambda)}} \] (1.25)
1.4. Particle-Phonon Coupling

It is possible to use the Weisskopf unit that provide a simple estimation for a single particle transition probability. The transition probability expressed in Weisskopf, allow a simple comparison between theoretically and experimentally observed electromagnetic transition rates. The relation between these two units is

\[
B_W(E\lambda) = \frac{1.2^{2\lambda}}{4\pi} \left( \frac{3}{\lambda + 3} \right)^2 \cdot A^{2\lambda/3} \cdot e^2 (fm)^{2\lambda} \quad (1.26)
\]

\[
B_W(M\lambda) = \frac{10}{\pi} \cdot 1.2^{2\lambda-2} \left( \frac{3}{\lambda + 3} \right)^2 \cdot A^{(2\lambda)/3} \left( \frac{e\hbar}{2Mc} \right) \cdot (fm)^{2\lambda-2} \quad (1.27)
\]

It is important to observe that the Weisskopf transition probabilities are defined only for transition with \( I_i > I_f \).

1.4 Particle-Phonon Coupling

The understanding of particle-phonon and phonon-phonon couplings is a very important issue, since this phenomenon is at basis of fermionic many body interacting systems, both in solid state and nuclear physics. In nuclear physics, the coupling between a particle/hole and vibration is a key ingredient to explain important phenomena, such as the observed reduction of spectroscopic factors, the anharmonicity of vibrational spectra, the damping Giant Resonance, etc. [1]. The best systems to study particle-phonon coupled state are nuclei around shell closures, where collective vibrations are expected to be quite robust. Because of the shortage of the experimental apparatus, in the past, some incomplete study were performed around \(^{208}\)Pb.

The aim of this work is to extend the systematic studies of the Copper chain started in other two experimental campaign [2, 3] in order to study the proton shell closure at \( Z = 28 \).
Chapter 2

The experimental apparatus

In this chapter we will present the experimental apparatus and the nuclear reaction studied in this thesis. The experiment was performed at Horia Hulubei National Institute of Physics and Nuclear Engineering (IFIN-HH) in Bucharest in April 2014, using the reaction $^{16}\text{O} + ^{48}\text{Ti}$ at an energy of 50 MeV. The experimental setup was made by a combination of 25 detectors divided in 14 HPGe and 11 LaBr$_3$(Ce) in a 4$\pi$ configuration, named RO-SPHERE, coupled to a plunger device.

The good energy resolution of the HPGe detectors allowed a high selectivity on the reaction products, the excellent timing of the LaBr$_3$(Ce) scintillator allowed lifetime measurement of the order of tens picoseconds while the plunger device was used to measure lifetime of few picoseconds. The details of the timing analysis will be presented in Chapter 3.

2.1 The Tandem Accelerator

The accelerator available at Horia Hulubei National Institute is a 9 MV Pelletron Tandem [8]. The operating principle of this type of machine is very simple. The high voltage terminal of the accelerator is charged to maximum 9 million volts positive potential. The negative ions from the ion sources
are accelerated by the positive potential on the terminal after a selection through the inflection magnet. The ions reaching the terminal region are stripped of electrons using thin foils of carbon (5-10 g/cm$^2$). The ion beam becomes positive and it is accelerated away by the same positive terminal potential. The name ”Tandem” comes from these two stage acceleration process. The beam is then focused and directed to the scattering chamber trough the vacuum beam line.

2.2 ROSPHERE

The gamma spectroscopy work of this experiment was performed using the ROSPHERE (ROmanian array for SPectroscopy in HEavy ion REactions) array, presented in Fig. 2.1. This array consist of 14 HPGe detectors and 11 LaBr$_3$(Ce) scintillators detectors divided into 5 rings each with 5 available position for detectors. The 5 rings are placed at 37°, 70°, 90°, 110° and 143° respect to the beam axis [9]. In order to maintain the spherical symmetry,
2.2. ROSPHERE

Figure 2.2: Schematic representation of the experimental apparatus ROSPHERE. In figure (a) it is possible to observe the structure of the array and the position of the beam axis. In figure (b) it is presented a lateral vision of the apparatus with all possible detectors.

the frame is an irregular polyhedron and consist of 27 flanges: 2 regular pentagons (along the beam axis), 10 irregular pentagons and 15 irregular hexagons. In Fig. 2.2 the geometry of the array is presented.

In order to provide access to the target chamber, the frame was designed to open in two asymmetric hemispheres (with 10 positions in the fixed backward hemisphere and 15 positions in the mobile forward hemisphere) by sliding along the beam axis.

The support system of the array, together with the rails that allow the back hemisphere to slide are presented in Fig. 2.3. The present mixed configuration of detectors in the array allows the measurement of the lifetimes of nuclear states in the range of few ps up to a few tens of ns, by using the electronic timing technique and by coupling this apparatus
with the plunger system described in Chapter 3.

In the following section we briefly recall the main features of Ge semiconductors and scintillators detectors.

### 2.2.1 Germanium Semi-Conductor Detectors

Semiconductor detectors are most commonly used when best energy resolution is needed [10]. A semiconductor material present a valence and a conduction band. As we can see from Fig. 2.4, the energy space between two band (bandgap) is roughly 1 eV. That is lower than in an insulator material, where the bandgap is larger than 5 eV. Thanks to this small bandgap the radiation that interact with a semiconductor deposits enough energy to produce a large number of electron-hole pairs. Indeed, the semiconductor detector gives rise to a much larger number of carriers for a given incident radiation event than it is possible with any other common detector. This allow to reduce the statistical limit on the energy resolution.

Under the effect of an external electric field, electrons and holes, that...
have respectively negative and positive charge, move in opposite direction with drift velocity expressed by the following relations:

\[ \nu_h = \mu_h E \quad (2.1) \]

\[ \nu_e = \mu_e E \quad (2.2) \]

where \( \mu \) is the electric mobility and the labels \( h \) and \( e \) indicate the holes and the electrons respectively. Because the saturated velocity are of the order of \( 10^7 \) cm/s the time required to collect carriers over typical dimension of 0.1 cm or less will be under 10 ns. Due to the small energy gap between conduction and valence band, there is the possibility that some electron-hole pair would be product for effect of the thermal motion. The probability per unit time that an electron-hole is thermally generated is:

\[ P(T) = CT^{3/2} e^{-\frac{E_g}{2KT}} \quad (2.3) \]

where \( T \) is the absolute temperature in K, \( E_g \) is the energy band-gap, \( K \) is the Boltzmann constant and \( C \) is a proportional constant characteristic of the material. As we can see from Eq. 2.3 the thermal excitation probability depend strongly on the energy gap and on the temperature. In order to avoid this effect, called leakage current, that produces strongly noisy signal especially in HPGe detectors, this semiconductor detector are typically cool down at 77 K.

Another important element that has to take into account is the depletion region that represent the sensitivity region of the detector. The depth \( d \) of the depletion region is given by:

\[ d \approx \left( \frac{2eV}{\epsilon N} \right)^{1/2} \quad (2.4) \]

where \( e \) is the electric charge, \( \epsilon \) is the dielectric constant, \( V \) is the reverse voltage and \( N \) is the net impurity in the bulk semiconductor material. At
a given applied voltage, greater depletion depths can only be achieved by lowering the value of \( N \) through further reduction in the net impurity concentration. There are fabrication techniques that allow to reduce impurity up to concentration of approximately \( 10^{10} \) atoms/cm\(^3\). For example, at this impurity Eq. 2.4 predicts that in germanium material a depletion depth of 10 mm can be reached using a reverse bias voltage of less than 1000 V. Detectors that are manufactured from this ultrapure germanium are usually called *High Purity Germanium* (HPGe) detectors.

In this experiment we used 14 HPGe detectors of different dimensions. In Fig. 2.5 and in Tab. 2.1 are reported the number of the detectors and the position in the RO-SPHERE array. HPGe detectors are labelled by G\# plus the specific detector number while LaBr\(_3\) scintillators, discussed in the next paragraph, are marked by L\# plus the number of the detector.
2.2. ROSPHERE

Table 2.1: Position of HPGe and LaBr₃ detectors in the ROSPHERE array, schematically reported in Fig. 2.5. The angle 37°, 70° and 90° are reported with the letter A, B and C respectively (Fig. 2.5).

<table>
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<td>L#09</td>
<td>B5 -</td>
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</table>

2.2.2 Scintillator Detectors

Fast scintillator detectors can be employed for the fast timing measurement since they allow to determine nuclear lifetime down to the level of tens picoseconds. The principle of operation of this type of detector is based on the characteristic of some materials to produce scintillation light when radiation occurs. This process is called fluorescence.

Scintillator materials can be classified in two main categories: organic and inorganic [10]. Organic scintillators are made of aromatic hydrocarbons and due to their small atomic number (Z) are used predominantly for particle detection, in elementary particles physics and also in nuclear structure physics. On the other hand, for γ-ray spectroscopy, the inorganic halide scintillators with higher density $\rho$ and high Z are a better choice due to Z and $\rho$ dependency of the photoelectric effect [10]. The scintillation process in the inorganic scintillators depends on the energy of the state determined by the crystal lattice of the material. Fig. 2.6 shows schematically how the lower band, called valence band, is filled by electrons that are essentially bounded in the lattice. The electrons in the conduction band, having enough energy, are free to migrate through the crystal. The region between this two band is not accessible in a pure crystal. Absorption of energy can move an electron from its position of the valence band into the conduction band, leaving a
hole in the normally filled valence band. In the pure crystal, the return of the electron to the valence band, with emission of a photon, is an inefficient process since the emitted $\gamma$ can be reabsorbed from the material.

In order to increase the probability of visible photon emission during the de-excitation process, small amounts of an impurity are commonly added to inorganic scintillators. Such impurities, called activators, create special sites in the lattice, as it is shown in Fig. 2.6, at which the normal energy band structure is modified from that of the pure crystal. As a result, energy states will be created within the forbidden gap, through which the electron can de-excite back to the valence band. These de-excitation sites determine the emission spectrum of the scintillator.

The scintillator material is coupled to a photomultiplier tube. As we can see from Fig. 2.7 the light produced from the radiation reach the photocathode where it is converted in electron, thanks to the photoelectric effect. The electrons produced are then multiplied in a multi-step process by the dynodes until they reach the final electrode named anode. With this process it is possible to obtain an electrical pulse starting from scintillation light.

In the experiment discussed in this thesis, we have employed 11 LaBr$_3$(Ce) scintillator detectors. A schematic representation of the ROSPHERE setup including HPGe and scintillators detectors is given in Fig. 2.5 and in Tab. 2.1.
2.3 Characterization of the ROSPHERE array

In order to perform a correct data analysis, it is necessary to calibrate each detector of the ROSEPHERE array. It is also necessary to extract efficiency and energy resolution of the apparatus, in order to define the characteristic of the experimental setup and to control its performances.

2.3.1 Energy Calibration

The first step in the characterization of the ROSPHERE array was the energy calibration of HPGe and LaBr$_3$ detectors.

HPGe

The energy calibration of the HPGe detectors was performed using an $^{152}$Eu source and some transitions belonging to the radioactive natural background. The first calibration was performed using the source that allow to reach 1.4 MeV. The positions of the peaks (in channel) and the corresponding energy
2.3. Characterization of the ROSPHERE array

![Figure 2.8: Centroid position versus $E_\gamma$ (open circle) for the HPGe detector #00, in the case of (a) $^{152}$Eu source data, (b) back-ground lines. The solid line is the corresponding cubic fit used in the calibration.](image)

<table>
<thead>
<tr>
<th>Energy [keV]</th>
<th>Channel</th>
<th>Energy [keV]</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>121.7817</td>
<td>285.45</td>
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<td>2257.3</td>
</tr>
<tr>
<td>244.6975</td>
<td>572.8</td>
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<td>2542.84</td>
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<tr>
<td>344.2785</td>
<td>805.58</td>
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<td>2551.96</td>
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<td>2604.23</td>
</tr>
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<td>443.965</td>
<td>1038.61</td>
<td>1299.142</td>
<td>3042.83</td>
</tr>
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<td>778.9045</td>
<td>1823.37</td>
<td>1408.013</td>
<td>3297.82</td>
</tr>
<tr>
<td>867.378</td>
<td>2030.82</td>
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<td></td>
</tr>
</tbody>
</table>

Table 2.2: Energy of the peaks of the $^{152}$Eu source and the relative position, in channel, for HPGe detector #00.

The energy calibration of the HPGe detectors was performed using a third order polynomial. In Fig. 2.8 it is reported the channel and energy correlation for the detector #00.

In order to extend the calibration up to 2.6 MeV a second calibration with back-ground gamma lines was performed. In Tab. 2.3 are reported the energy and the nucleus from which it originates. The third order polynomial
2.3. Characterization of the ROSPHERE array

<table>
<thead>
<tr>
<th>Energy [keV]</th>
<th>Decay Chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>295.2</td>
<td>$^{222}<em>{86}$Rn $\rightarrow$ $^{218}</em>{84}$Po $\rightarrow$ $^{214}<em>{82}$Pb $\rightarrow$ $^{214}</em>{83}$Bi</td>
</tr>
<tr>
<td>351.6</td>
<td>$^{222}<em>{86}$Rn $\rightarrow$ $^{218}</em>{84}$Po $\rightarrow$ $^{214}<em>{82}$Pb $\rightarrow$ $^{214}</em>{83}$Bi</td>
</tr>
<tr>
<td>609.3</td>
<td>$^{214}<em>{83}$Bi $\rightarrow$ $^{214}</em>{84}$Po $\rightarrow$ $^{210}<em>{82}$Pb $\rightarrow$ $^{210}</em>{83}$Bi</td>
</tr>
<tr>
<td>1120.3</td>
<td>$^{214}<em>{83}$Bi $\rightarrow$ $^{214}</em>{84}$Po $\rightarrow$ $^{210}<em>{82}$Pb $\rightarrow$ $^{210}</em>{83}$Bi</td>
</tr>
<tr>
<td>1435.8</td>
<td>$^{138}_{57}$La</td>
</tr>
<tr>
<td>1460.8</td>
<td>$^{40}<em>{19}$K $\rightarrow$ $^{40}</em>{18}$Ar</td>
</tr>
<tr>
<td>1764.5</td>
<td>$^{214}<em>{84}$Bi $\rightarrow$ $^{214}</em>{84}$Po $\rightarrow$ $^{210}<em>{82}$Pb $\rightarrow$ $^{210}</em>{83}$Bi</td>
</tr>
<tr>
<td>2614.5</td>
<td>$^{208}<em>{83}$Bi $\rightarrow$ $^{208}</em>{82}$Pb</td>
</tr>
</tbody>
</table>

Table 2.3: Energy of the $\gamma$-transition of the natural background and corresponding decay chain through which the excited nuclei (in bold) are populated.

fit is reported in Fig. 2.8. After the calibration of all HPGe detectors, it was verified that the calibration coefficient did not change during the entire experiment.

LaBr$_3$(Ce)

A third order polynomial energy calibration was also performed for the LaBr$_3$(Ce) scintillator employing the $^{152}$Eu source. In Tab. 2.4 are reported the position of the peak (in channel) before the calibration and the corresponding energy.

Because of the instability of the calibration during the acquisition time it was necessary to make an energy alignment of the spectra of the LaBr$_3$ detectors. Thanks to the segmentation of the data acquired, divided into runs of about 2 hours of acquisition, it was possible to correct this effect. First of all, the spectra of the first run of the different detectors were aligned to a common peak, as reported in Fig. 2.9 (a). The spectra of each detector at different acquisition time (different run) were then aligned to the first one,
2.3. Characterization of the ROSPHERE array

<table>
<thead>
<tr>
<th>Energy [keV]</th>
<th>Channel</th>
<th>Energy [keV]</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>121.7817</td>
<td>139.85</td>
<td>443.965</td>
<td>510.14</td>
</tr>
<tr>
<td>244.6975</td>
<td>279.95</td>
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<td>900.31</td>
</tr>
<tr>
<td>344.2785</td>
<td>394.36</td>
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</tr>
<tr>
<td>411.1165</td>
<td>471.72</td>
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<td>1686.43</td>
</tr>
</tbody>
</table>

Table 2.4: Energy of the peaks of the $^{152}$Eu source and the relative position, in channel, for the LaBr$_3$ detector #00.

as reported in Fig. 2.9 (b).

Finally, another calibration with the $^{152}$Eu source was performed. Thanks to this calibration it was finally possible to be confident that the spectra are well calibrated and well aligned to each other. The calibrated spectra of HPGe and LaBr$_3$ are reported in Fig. 2.10.

2.3.2 Energy Resolution

A gamma ray interacting with a detector produces an electrical pulse that is proportional to the energy of the radiation. Theoretically, gamma rays of
2.3. Characterization of the ROSPHERE array

Figure 2.10: Single $\gamma$-ray spectrum of the $^{152}$Eu source for the two different types of detectors (HPGe and LaBr$_3$).

the same energy produce the same electrical pulse in the detector. Indeed there are different factors that can condition this process, such as statistical fluctuations. This uncertainty is reflected on the width of the associated peak. In particular the Full Width at Half Maximum (FWHM) is used to estimate the energy resolution. It is defined as:

$$R = \frac{\Delta E}{E}$$  \hspace{1cm} (2.5)

where $\Delta E$ is the FWHM of the peak at energy $E$.

Depending on the detector type, it is known the mean energy $w$ necessary to produce an ionization. For a fixed gamma ray energy, a different mean number of carriers, $J$, are produced in different detectors depending on the $w$ value. The number of ionization is given by the relation:

$$J = \frac{E}{w}$$  \hspace{1cm} (2.6)

Since the statistical error is proportional to the number $J$ of ionization the energy resolution defined in Eq. 2.5 is smaller when the parameter $w$ is small.
2.3. Characterization of the ROSPHERE array

It can be shown that the relation between the energy and the resolution is [10]

\[ R = 2.35 \sqrt{\frac{Fw}{E}} \]  

(2.7)

where \( F \) is the Fano factor, that for Germanium materials is roughly 0.13 and for Silicon materials is around 0.14. In order to check this behaviour for the ROSPHERE array the energy resolution for both HPGe and LaBr\(_3\) detectors was calculated. As we can see from Tab. 2.5 and Fig. 2.11, the energy resolution of the HPGe detectors is of order of 0.24\% at 1 MeV compared to 2.3\% for the scintillators. This is due to the extremely low energy necessary to produce a carrier in the semiconductor detector (\( w = 3 \text{ eV} \)), to be compared with \( w = 30 \text{ eV} \) for a LaBr\(_3\) detectors.

Figure 2.11: Resolution of the HPGe (red) and LaBr\(_3\) (blue). The energy value are referred to \(^{152}\text{Eu}\) gamma lines.
2.3. Characterization of the ROSPHERE array

<table>
<thead>
<tr>
<th>Energy [keV]</th>
<th>Resolution [%]</th>
<th>Energy [keV]</th>
<th>Resolution [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>121.74</td>
<td>1.19</td>
<td>121.83</td>
<td>7.16</td>
</tr>
<tr>
<td>244.66</td>
<td>0.63</td>
<td>244.69</td>
<td>4.76</td>
</tr>
<tr>
<td>344.19</td>
<td>0.47</td>
<td>344.25</td>
<td>3.83</td>
</tr>
<tr>
<td>410.98</td>
<td>0.40</td>
<td>411.55</td>
<td>3.75</td>
</tr>
<tr>
<td>443.79</td>
<td>0.39</td>
<td>444.18</td>
<td>3.52</td>
</tr>
<tr>
<td>778.96</td>
<td>0.27</td>
<td>777.14</td>
<td>2.82</td>
</tr>
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<td>867.49</td>
<td>0.26</td>
<td>866.02</td>
<td>2.65</td>
</tr>
<tr>
<td>964.19</td>
<td>0.24</td>
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</tr>
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<td>1086.05</td>
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<td>1090.09</td>
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</tr>
<tr>
<td>1408.09</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.5: Energy of the peaks of the $^{152}$Eu source and corresponding energy resolution for all HPGe and LaBr$_3$ detectors of the ROSPHERE array.

2.3.3 Efficiency

Another important parameter that can be used to characterized the experimental apparatus is the efficiency. Not all gamma rays emitted by a source will produce a signal in the system. We can define the efficiency of a detector as the probability to detect a gamma ray emitted by a source. The efficiency is conditioned by two principal factors: the solid angle covered by the detector and the intrinsic apparatus characteristics. Therefore it is defined an absolute efficiency as

$$
\varepsilon_{\text{abs}} = \frac{\text{number of pulses recorded}}{\text{number of radiation quanta emitted by source}}
$$

(2.8)

It depends not only on the detector properties but also on the geometry. Instead, the intrinsic efficiency is defined as

$$
\varepsilon_{\text{int}} = \frac{\text{number of pulses recorded}}{\text{number of radiation quanta incident on detector}}
$$

(2.9)
This quantity depends primarily on the detector material, on the radiation energy and on the physical thickness of the detector in the direction of the incident radiation. The relation between $\epsilon_{abs}$ and $\epsilon_{int}$ is

$$
\epsilon_{abs} = \frac{\Omega}{4\pi} \epsilon_{int}
$$

(2.10)

where $\Omega$ is the solid angle subtended by the detector. The energy dependence of the efficiency is expressed by the experimental relation:

$$
\epsilon(E) = \exp \left[ \frac{2\alpha + \beta \ln E + \gamma \ln^2 E}{\pi} \arctan(e^{\delta + \zeta \ln E} + e^{\eta \ln^3 E}) - 25 \right]
$$

(2.11)

In particular, in this work were interested in the efficiency of all HPGe detectors: this was calculated measuring the number of counts for each peak. Different peak counts depends on two factor: different emission intensities and different detection efficiency. Once it is known the total intensity of the observed gamma rays it is possible to obtain the relative efficiency curve. In particular, in our experiment, the transition employed are the most intense of the $^{152}$Eu and of the $^{61}$Ni nuclei. The first one is the source employed also for the energy calibration, the second one is the product of beta decay of $^{61}$Cu which is one of the reaction channels of the experiment.

The relation used for the efficiency calculation is

$$
\epsilon = \frac{A_{rel}}{I_\gamma}
$$

(2.12)

where $A_{rel}$ represents the area of the peak normalized to the peak of reference at 121 keV and $I_\gamma$ is the tabulated relative intensity of the source. The data were fitted by the relation given by Eq. 2.11 and are reported in Fig. 2.12.

2.4 The Plunger Device

In order to perform Recoil Distance Doppler Shift (RDDS) [11] measurement with the ROSPHERE array a plunger device was employed [9]. The RDDS
method allows to extract information about the lifetime of nuclear excited state in the pico second region by the comparison of spectra acquired at different distances between the target and the stopper. This distances are regulated in a micrometer range by a plunger device. A detailed description of this method will be presented in the next chapter. In Fig. 2.13 a schematical representation of the plunger available in Bucharest is reported. This device consists of three principal systems:

- a mounting system housed in a small chamber at the center of the array, allowing the parallel mounting of the target and stopper with micrometer precision.

- a driving control system composed by a piezo electric crystal that allows also short range correction, with precision of 0.1 μm.

- a system of concentric tube sliding bearing that transfer the movement of the motor to the target frame.

![Figure 2.12: Efficiency of the HPGe apparatus.](image-url)
2.5. The Reaction $^{16}\text{O} + ^{48}\text{Ti}$

The experiment studied in this work is based on the nuclear reaction

$$^{16}\text{O} + ^{48}\text{Ti} \rightarrow ^{64}\text{Zn}^*$$

at 50 MeV of beam energy, which is approximately twice the Coulomb barrier of system. In fact, the laboratory frame the Coulomb barrier is

$$V_c = \frac{1}{4\pi\epsilon_0} \frac{Z_1 Z_2 e^2}{R} = 1.44 \frac{Z_1 Z_2}{1.2(A_1^{1/3} + A_2^{1/3})} \frac{M_2}{M_1 + M_2} = 25.72 \text{MeV} \quad (2.13)$$
2.5. The Reaction $^{16}\text{O} + ^{48}\text{Ti}$

Figure 2.14: Relation between distance and voltage for the plunger device used in the experiment.

Figure 2.15: Quadratic fit of the relation between the inverse of the applied voltage and the distance used for the plunger in the range of short distances.
2.6 Reaction Products

A complete analysis of the reaction products produced with the $^{16}$O + $^{48}$Ti reaction has been performed. First of all, it was performed a theoretical calculation of the residual nuclei using the PACE code [12]. In Tab. 2.6 are reported the estimated cross section and the percent of production, for each residual nucleus. As we can see, the largest cross section expected at 50 MeV is for $^{61}$Cu the nucleus of interest for this analysis. Other nuclei are also expected to be populated with a good cross section, in particular $^{58-61}$Ni and $^{58}$Co.

<table>
<thead>
<tr>
<th>Z</th>
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<th>A</th>
<th>Percent</th>
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<td>32</td>
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<td>Co</td>
</tr>
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<td>15.3</td>
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<td>55</td>
<td>Mn</td>
</tr>
</tbody>
</table>

Table 2.6: PACE calculation of residual nuclei after the fusion evaporation reaction $^{16}$O + $^{48}$Ti at 50 MeV. In bold are reported the values for the nucleus of interest in the present analysis.

During the data analysis ten different nuclei were identified. They are indicated by yellow squares in the nuclide chart given in Fig. 2.16, where the
2.6. Reaction Products

nucleus of interest in the present analysis is marked in red and the compound nucleus is marked in green.

Figure 2.17 shows the total spectrum, measured with the HPGe detectors with the principle $\gamma$-line highlighted.

It is found that the nuclei observed in the data analysis are in good agreement with the expected one calculated with the PACE code.

![Nuclide Chart]

*Figure 2.16: Portion of the nuclide chart showing (in yellow) the main nuclei produced by the reaction $^{16}O + ^{48}Ti$ at 50 MeV, around the nucleus of interest, $^{61}Cu$, marked in red. The compound nucleus $^{64}Zn$ is indicated in green.*
2.6. Reaction Products

Figure 2.17: Total spectrum of all HPGe detector. Ten different nuclei were identified. The strongest transition are reported in the spectrum.
Chapter 3

Lifetime and Branching Measurement

This chapter describes the data analysis performed to extract the lifetime of the $9/2^+$ level at 2720 keV of the $^{61}$Cu nucleus and its branching ratio. As discussed in Chapter 1 the lifetime of a nuclear excited state is an important characteristic required for nuclear model description.

Different methods for lifetime measurements, covering a wide range of nuclear lifetimes ($10^{-15}$ - $10^{-5}$ s), have been developed in the past century. In Fig. 3.1 there are some methods covering different temporal ranges [6].

In this chapter we focus on the description of the Recoil Distance Doppler Shift method (RDDS) used in this work.

3.1 Recoil Distance Doppler Shift Technique

The Recoil Distance Doppler Shift method ("RDDS" or "Plunger" method) is a well established method for the determination of lifetimes in the picosecond range. The experimental setup, schematically sketched in Fig. 3.2, is composed of a thin fixed target and of a thick stopper that can be positioned at variable distance, from ten to hundreds micrometers. In our experimental

36
3.1. Recoil Distance Doppler Shift Technique

Figure 3.1: Chart showing the regions over which different lifetime measuring techniques are used [6].

setup, the target was composed of 1 mg/cm$^2$ of $^{48}$Ti, while the stopper was made by 5 mg/cm$^2$ of Au. Oxygen ions were accelerated to 54 MeV by the tandem accelerator and consequently deflected to the scattering chamber. Due to the thickness of the target, most of the reaction product have enough energy to get out from the target. The produced nuclei leaving the target can de-excite by gamma emission in flight or at rest, if they are absorbed by the stopper. If the gamma emission occurs in flight, the gamma

Figure 3.2: Schematic representation of the plunger method.
3.1. Recoil Distance Doppler Shift Technique

Energy is shifted due the Doppler effect by the relation:

\[ E'_\gamma = E_\gamma \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \theta} \approx E_\gamma (1 + \beta \cos \theta) \]  (3.1)

where \( E_\gamma \) and \( E'_\gamma \) are respectively the gamma energy at rest and in flight, \( \theta \) is the detector angle with respect to the beam axis and \( \beta = v/c \). For the determination of the lifetime, the counting rate of Doppler-shifted events (s), \( N' \), and the unshifted events (u), \( N_0 \), have to be compared.

There are two main methods to extract the lifetime from this comparison. The Decay Curve Method is based on the simple application of the Bateman equation [13]. This method gives good results in case of simple pattern decay, typically one or two step decay. The Differential Decay Curve Method (DDCM) provides better performances in the case of complex pattern decay and allows to minimize systematic errors [14, 15].

3.1.1 Decay Curve Method

The decay curve method [13] is commonly used in plunger analysis and it is based on the simple application of the Bateman equation. We can define the so called decay curve, using \( I_u(t) \) and \( I_s(t) \) respectively as the shifted and the unshifted component of the \( \gamma \)-transition:

\[ R(t) = \frac{I_u(t)}{I_u(t) + I_s(t)} \]  (3.2)

In the simple case of one step decay the decay curve reads:

\[ R(t) = e^{-t/\tau} \]  (3.3)

where \( \tau \) is the lifetime of the level. Therefore the extraction of \( \tau \) in the ideal case is quite easy. However, when more levels are involved, as it is typical for compound-fusion reactions, and we deal with a complicate level-feeding pattern (as reported in Fig. 3.3), the analysis of a lifetime measurement is
done solving the corresponding system of differential equations:

$$\frac{d}{dt} n_i(t) = -\lambda_i n_i(t) + \sum_{k=i+1}^{N} \lambda_k n_k(t) b_{ki}$$  \hspace{1cm} (3.4)

where the $k$ index denotes the numbers of level which feed the level of interest $i$, $N$ is the number of the highest feeding levels and $n_i(t)$ is the number of nuclei in levels $i$. The decay constant of the levels $j$ is denoted as $\lambda_j$ and is related to the level lifetimes by $\tau = 1/\lambda_j$. The quantities $b_{kj}$ are the branching ratios from levels $k$ to levels $j$.

Figure 3.3: Level scheme indicating the feeding and decaying transition for the level $L_i$.

The decay curve function obtained by this method is used to fit the experimental data in order to extract the level lifetime $\tau_i$. The difficulties in case of complex level scheme arise from the large number of parameters involved in the fit and from side feeding problems. As we can see from Eq. A.3, it is necessary to know the lifetime of the feeding level and its branching ratios.

3.1.2 Differential Decay Curve Method

The Differential Decay Curve Method (DDCM) is a transparent method for the analysis of RDDS data, in which the set of coupled differential equation
is replaced for each individual level by a single first order differential equation depending on quantities, which can be directly obtained from experimental data [14].

A full description of the method is reported in Appendix A. In particular, by integrating the Eq. A.3, referred to the decay pattern reported in Fig. 3.3, we can obtain the basic relation for the determination of the lifetime:

$$\tau = \frac{-N_i(t) + \sum b_{ki} N_h(t)}{\frac{dN_i(t)}{dt}}$$

(3.5)

The quantities $N_i(t)$ and $N_h(t)$ are directly proportional to the shifted and unshifted peak observed in the RDDS experiment at distance $d = vt$ being $t$ the time of flight of the recoil between target and stopper.

Figure 3.4: Level scheme of a typical decay pattern of the level $L_i$. In particular are labeled with $L_k$ and $L_h$ respectively the higher and lower energy level and with $L_i$ is indicated the level of interest.

Many problems caused by systematic errors in the analysis of RDDS experiments can be avoided by coincidence technique [14]. The measured intensities of two transitions in coincidence A and B, where B occurs before
3.2 Branching Measurement

A, are indicated with \( \{B, A\} \) and the labels S and U indicate, respectively, the shifted and the unshifted components. With this formalism, the formula in Eq. 3.5 in the case of a decay pattern reported in Fig. 3.4, becomes:

\[
\tau(t_k) = \frac{\{C_S, A_U\}(t_k) + \alpha\{C_S, B_S\}(t_k)}{\frac{d}{dt}\{C_S, A_S\}(t_k)}
\]  

(3.6)

where \( \alpha \) is the branching ratio of the intensities \( \{C, A\} \) and \( \{C, B\} \).

\[
\alpha = \frac{\{C, A\}}{\{C, B\}} = \frac{\{C_U, A\}}{\{C_U, B\}} = \frac{\{C_S, A\}}{\{C_U, A\}} + \frac{\{C_S, A\}}{\{C_U, B\}} + \frac{\{C_U, B\}}{\{C_U, B\}}
\]  

(3.7)

An important simplification of 3.6 is obtained if the energy gate is set on the direct feeder B of the level \( L_i \) [13]. With this condition the equation becomes:

\[
\tau(t_k) = \frac{\{B_S, A_U\}(t_k)}{\frac{d}{dt}\{B_S, A_S\}(t_k)}
\]  

(3.8)

The only quantities present in this relation are the intensities of the shifted and unshifted component of the transition of interest. The only critical point is the estimation of a time derivative of a measured quantity. This procedure is explained in Appendix A.

The main advantages of this method are that only observed feeding transitions are used to determine the lifetime and it is not necessary to determine the absolute distance but only the relative target to stopper distance. Another important advantage of this method is that there are different lifetime estimation for each distance presents. The final value is then a weighted mean of this different lifetime estimation. For this reasons we can say that the DDCM method gives particularly good results in coincidence analysis.

3.2 Branching Measurement

The first part of this experiment consisted in the measurement of the branching ratio of the unknown transition from the 9/2\(^+\) level of the \(^{61}\)Cu at 2720 keV to the ground state. This is crucial for the extraction of the B(E3) value.
3.2. Branching Measurement

for the $9/2^+ \rightarrow 3/2^-$ (g.s.), aim of this work, as explained in Chapter 1. For this purpose same reaction $^{16}\text{O} + ^{48}\text{Ti}$ was employed, at 50 MeV of beam energy, using a target of titanium 10 mg/cm$^2$ thick. By using such a thick target the $\gamma$ emission occurs from compound nuclei at rest in the target material, therefore no Doppler correction is needed and the width of the line is given by the intrinsic energy resolution of the HPGe detectors, as reported in Section 2.3.2.

First of all, it was performed a complete gamma spectroscopy of the $^{61}\text{Cu}$ nucleus of interest using the HPGe detectors. There were identified about 70 $\gamma$-lines belonging to the $^{61}\text{Cu}$ nucleus. The main transitions identified are reported in Fig. 3.5. In particular, it is reported in red the new transition observed by gating on the transition at 1361 keV, directly feeding the level at 2720 keV. The coincidence spectrum is reported in Fig. 3.6.

The branching $b_{2720}$ was calculated as the relative intensity respect to the most intense $\gamma$-line depopulating the level, mainly the 1409 keV was used for this purpose. It is necessary to take into account the different efficiency for different energy value. The relation used is then:

$$b_{2720} = \frac{I_{2720} \epsilon_{1409}}{I_{1409} \epsilon_{2720}}$$

(3.9)

where $I_{1409}$ and $I_{2720}$ are the number of counts of the peaks at 1409 keV and 2720 keV and $\epsilon_{1409}$ respectively and $\epsilon_{2720}$ are the corresponding efficiency, obtained by Eq. 2.11. In Tab. 3.1 are reported the number of counts for each peak and the relative branching ratio.

As given in Tab. 3.1, the results obtained for the known transitions of $^{61}\text{Cu}$ are in good agreement with the values present in literature [16]. In addition, the present experiment has enough statistics to have provide the branching ratio of the new transition at 2720 keV, with an error bar of ±15%.
Figure 3.5: Principal transitions of $^{61}$Cu observed in this experiment. In red is reported the new γ-line at 2720 keV.

$^{61}$Cu
3.3 Analysis of the Lifetime

The second part of the data analysis was focused on the measurement of the lifetime of the $9/2^+$ state of the $^{61}$Cu at 2720 keV, for this purpose the array ROSPHERE was coupled to the plunger device, discussed in Section 2.4. The beam energy was 54 MeV and a Titanium target 1 mg/cm$^2$ thick was employed.

A fusion evaporation reaction usually allows to populate nuclear states at high energy and high spin, as a consequence, the gamma decay path to
reach the low lying levels is often complex. For this reason it is not possible to use the Decay Curve Method (DCM), so it was chosen the Differential Decay Curve Method (DDCM), presented in Section 3.1.2.

The DDCM method presents two critical points that we have to take into account during the analysis. The first one is the normalization of data extracted at different target to stopper distances, the second one is the Doppler broadening and $v/c$ determination.

**Normalization**

Since information must be compared at each distance set between the target and the stopper, in order to extract the lifetime of the level, it is necessary that they refer to the same number of recoils produced in the reaction. Therefore, the intensities of the shifted and unshifted components have to be corrected for differences in running time and beam intensity for each target to stopper distance.

The common normalization factors $N(d)$ for each distance $d$ were determined by the comparison in the area of the peak at 349 keV that depopulates the level at 390 keV of the nucleus $^{62}$Cu, which is a strong reaction channel. The lifetime of this level, that is around 11 ns, is quite long that we can suppose that all gammas are emitted at rest. Therefore, the difference in the number of counts of the peak must be related only on difference in number of recoils produced.

The area of the peak and the relative normalization factor used for the normalization are reported in Tab. 3.2.

**Doppler Broadening and Recoil Velocity**

The energy of the gamma emitted in flight is shifted due to the Doppler effect, depending on the recoil velocity and on the observation angle $\theta$, as
3.3. Analysis of the Lifetime

<table>
<thead>
<tr>
<th>Distance [$\mu$m]</th>
<th>Counts</th>
<th>N(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.8</td>
<td>496886</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>440388</td>
<td>1.12</td>
</tr>
<tr>
<td>20</td>
<td>383824</td>
<td>1.29</td>
</tr>
<tr>
<td>25</td>
<td>335339</td>
<td>1.48</td>
</tr>
<tr>
<td>35</td>
<td>246518</td>
<td>2.01</td>
</tr>
</tbody>
</table>

Table 3.2: Number counts recorded in the peak at 349 keV of $^{62}$Cu.

follow

$$E'_\gamma = E_\gamma \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \theta}$$  \hspace{1cm} (3.10)

where $E_\gamma$ is the real energy of the $\gamma$-ray, $\beta = v/c$ and $\theta$ is the angular position of the detector. A consequence of this effect is that the shifted peaks are affected by an energy broadening which depends on the $\Delta \theta$ opening of the detector, according to the relation [10]

$$\Delta \text{FWHM} = 2 E_\gamma \beta \sin(\Delta \theta) \sin \theta$$  \hspace{1cm} (3.11)

Another contribution to the energy broadening is due to differences in the recoil velocity. Since the nuclei produced in the titanium target can lose some of their recoil energy before getting leaving the target, the residual recoil velocity can assume different values, being the energy loss in the target a statistical process. In our experiment this effect was very relevant and produced a strong increase in the FWHM of the shifted peaks.

The evidence of the dependence of the energy broadening on different recoil velocity is shown in Fig. 3.7. The three spectra are obtained gating on the direct feeder of the $9/2^+$ level at 2720 keV in $^{61}$Cu, as shown in the level scheme reported in Fig. 3.8 (a). In particular, the red and the black spectra were obtained imposing an energy condition of 1346-1351 keV and 1340-1345 keV respectively, as reported in Fig. 3.8 (b). The green dashed spectrum was
obtained with the gating condition of 1340-1351 keV. It is interesting to see that if we change the energy range of the gating condition it change also the position of the shifted peak of the $\gamma$-ray at 1409 keV (depopulating the level of interest) also changes.

This effect is due to different recoil velocities that are automatically selected imposing the gating condition. By gating on the most shifted component of the feeder, the most shifted peak depopulating the level is selected and vice versa. For this reason the determination of the $v/c$ value was not trivial and introduced uncertainties on the determination of the lifetime of the $9/2^+$ state of interest.

The correct recoil velocity was determined using the Doppler relation:

$$\beta = \frac{1}{\cos \theta} \left( \frac{E'_\gamma}{E_\gamma} - 1 \right)$$  \hspace{1cm} (3.12)

where $\theta$ is the angular position of the detector and $E'_\gamma$ and $E_\gamma$ were determined with a Gaussian fit of the shifted and unshifted peaks after the
3.3. Analysis of the Lifetime

![Level scheme of the $^{61}$Cu around the 9/2$^+$ level of interest (a). Spectrum at 20µm distance zoomed on the shifted and unshifted component of the $\gamma$-rays populating the 2720 keV level at nominal energy 1361 keV (b). Red and black lines represent the gating conditions used to obtain the spectra in Fig. 3.7.]

Figure 3.8: Level scheme of the $^{61}$Cu around the 9/2$^+$ level of interest (a). Spectrum at 20µm distance zoomed on the shifted and unshifted component of the $\gamma$-rays populating the 2720 keV level at nominal energy 1361 keV (b). Red and black lines represent the gating conditions used to obtain the spectra in Fig. 3.7.

determination of the gating condition. In order to improve the statistic, the energy range selected was 1340-1351 keV.

![Shifted and unshifted component of $\gamma$ transition at 1409 keV transition in $^{61}$Cu fitted to obtain the $v/c$ value.]

Figure 3.9: Shifted and unshifted component of $\gamma$ transition at 1409 keV transition in $^{61}$Cu fitted to obtain the $v/c$ value.

In Fig. 3.9 it is reported the fit of the two peaks. The obtained value of the recoil velocity is $v/c = 0.0127(30)$. 

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3.3. Analysis of the Lifetime

3.3.1 DDCM Method and Napatau Software Test

In this experiment, the plunger measurements have been performed for five different target to stopper distances. In principle, the minimum number of distances needed for a DDCM analysis is three. However, in order to have a higher precision for the lifetime determination, the number of distances taken into account are usually around ten.

Before starting on the data analysis of the lifetime, it was performed a test on the precision of the method and on the Napatau software, developed by Cologne group for DDCM analysis [17], behaviour with only five distances. The data used for this test were simulated. The simulation was performed with different decay path and using the target to stopper distances used in the experiment. One of the decay path reproduced in the simulation software is reported in the decay scheme in Fig. 3.10. The gating condition was the transition B directly feeding the level of interest 1.

The information required for the simulations are:

- the lifetime of the levels involved in the cascade. Simulations were performed with different values of the lifetime of the upper levels. The values used in one of these test are reported in Fig. 3.10.

- the target to stopper distances. Five values were used: 10 $\mu$m, 15 $\mu$m,
3.3. Analysis of the Lifetime

20 $\mu$m, 25 $\mu$m and 35 $\mu$m, in agreement with the experimental data.

- the recoil velocity. The value used was chosen to be very similar to the recoil velocity of the experiment. Tests were performed with a fixed and variable $v/c$ value.

- the number of iterations. This value was chosen in agreement with the number of recoils produced at each target-to-stopper distance.

A first test was done with a fixed recoil velocity $v/c = 0.0127$ and 10000 iterations. The number of counts obtained for shifted and unshifted peaks are reported in Tab. 3.3.

<table>
<thead>
<tr>
<th>Distance [$\mu$m]</th>
<th>$I_s$</th>
<th>$\sigma_s$</th>
<th>$I_u$</th>
<th>$\sigma_u$</th>
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<td>10</td>
<td>142</td>
<td>14</td>
<td>656</td>
<td>65</td>
</tr>
<tr>
<td>15</td>
<td>362</td>
<td>36</td>
<td>977</td>
<td>97</td>
</tr>
<tr>
<td>20</td>
<td>677</td>
<td>67</td>
<td>1232</td>
<td>120</td>
</tr>
<tr>
<td>25</td>
<td>999</td>
<td>99</td>
<td>1397</td>
<td>130</td>
</tr>
<tr>
<td>35</td>
<td>1736</td>
<td>170</td>
<td>1649</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 3.3: Number counts for the shifted and unshifted peaks, as obtained by the simulation.

As we can see from Fig. 3.11, the results obtained using Napatau are in excellent agreement with the expected value of 5 ps. Despite the big error, it was found a value in good agreement with the expected lifetime also using only 3 distances.

The same simulation was performed with a variable $v/c$ value. In particular it was assume a Gaussian distribution around $v/c = 0.0127$, with a standard deviation $\sigma = 0.003$. The obtained lifetime was $\tau = 4.6(9)$ ps. The result of this second test shows an increase in the uncertainty on the lifetime. However, the obtained results are always compatible with the expected
value of 5 ps. Therefore, it is possible to conclude that the adopted analysis method can give good results also for our experimental analysis, where 5 target-to-stopper are used.

### 3.3.2 Test Level

Once the analysis method was validated, by the simulation discussed in Section 3.3.1, it was performed a test of the entire experimental apparatus, by measuring of the known lifetime of the level $7/2^-$ of $^{61}$Ni at 1015 keV.

The first step was the identification of the $\gamma$-transition for the gating condition. In order to select the most intense feeder of the level of interest, it was necessary to performed a complete gamma spectroscopy analysis of the nucleus of interest. Through this preliminary analysis about 30 transition
3.3. Analysis of the Lifetime

Figure 3.12: Partial level scheme of $^{61}\text{Ni}$ showing the principal $\gamma$ transition populating and depopulating the $9/2^+$ level at 2121 keV. In this level scheme are reported: the most intense $\gamma$-line populating the level at 1015 keV and the $\gamma$-line depopulating this level.

belonging to $^{61}\text{Ni}$ were identified. The most intense feeder of the level at 1015 keV is the $\gamma$ transition at 1106 keV depopulating the $9/2^+$ level at 2121 keV. The partial level level scheme considered for the lifetime analysis is reported in Fig. 3.12.

After the determination of the transition for the coincidence condition, by gating on the shifted component of this 1106 keV $\gamma$-line, spectra used for the DDCM analysis were obtained, as reported in Fig. 3.13.

The values $I_s$ and $I_u$ required for the DDCM analysis were obtained by integration of the shifted and unshifted peaks. They were than normalized using the normalization factor present in Tab. 3.2. These value are reported in Fig. 3.14 (middle panel for $I_s$, bottom panel for $I_u$) with the Napatau fit.

The $v/c$ value used in this analysis was 0.0127 (30). The life time obtained for the level 9/2$^+$ at 2121 keV is $\tau = 5.2(5)$ that is compatible with the value reported in literature of 6.3(9) ps [18], obtained with a Recoil Distance
Doppler Shift technique.

The measurement of this known lifetime is an important test of the method and of the entire experimental apparatus.
3.3. Analysis of the Lifetime

Figure 3.13: Spectra of $^{61}$Ni zoomed around the 947 keV peak, obtained at different target-to-stopper distances, in coincidence with the shifted component of the $\gamma$-line at 1106 keV, depopulating the level at 2121 keV.
3.3. Analysis of the Lifetime

Figure 3.14: $\tau$-plot and intensities of the shifted $I_s$ (middle) and unshifted $I_u$ (bottom) components of the test level transition of $^{61}$Ni at 1015 keV.
3.3.3 Lifetime of the 9/2\(^+\) Level of \(^{61}\)Cu

Once the method and the apparatus were tested, as reported in Section 3.3.2, it was possible to measure the lifetime of the 9/2\(^+\) level at 2720 keV of \(^{61}\)Cu. The cascade that populates this level is reported in Fig. 3.8 (a). The \(\gamma\) selected for the coincidence condition is the 1361 keV line that depopulates the level at 4081 keV.

As we can see from Fig. 2.5, the germanium detector were positioned in two rings at 37\(^\circ\) (backward - B) and 143\(^\circ\) (forward - F). The spectra used for the determination of the lifetime were obtained with different combination of angles. The four possibility used are: gate on backward ring and analysis performed at backward (forward) rings (B-B/B-F) or gate on forward ring and analysis performed at backward (forward) rings (F-B/F-F). In Fig. 3.15 is reported the backward-forward (B-F) spectrum of the transition at 1409 keV that depopulates the level at 2720 keV.

In spite of the high intensity of the \(\gamma\) cascade, the shifted peak of the 1361 keV feeding transition is very weak, therefore the coincidence spectra have rather poor statistic. The cause of the few shifted \(\gamma\)-lines of the feeder is probably a long lifetime of the level at 5120 keV unknown in literature.

In order to improve the precision of the lifetime measurement, it was performed a DDCM method analysis for all the four combination of rings: B-F, B-B, F-B and F-F. The number of counts are reported in Tab. 3.4 and in Fig. 3.16 with the respectively lifetime values obtained.

The final value of the lifetime of the 9/2\(^+\) level at 2720 keV was obtained with a weighted mean of the four value obtained with the four angle combination. The lifetime is than \(\tau = 4.3(7)\)ps. As described in Chapter 1, with the lifetime and the branching ratio of the level 9/2\(^+\) it is possible to determine the partial lifetime

\[
\tau_{\text{partial}} = \tau \frac{I_{\text{tot}}}{I_{\gamma}} \simeq 324 \text{ ps}
\]  \hspace{1cm} (3.13)
Figure 3.15: Spectra $^{61}$Cu zoomed around 1409 keV peak, obtained at different target-to-stopper distances, in coincidence with the shifted component of the $\gamma$-line at 1409 keV depopulating the level at 4081 keV.
3.3. Analysis of the Lifetime

Figure 3.16: $\tau$-plot and intensities of the shifted $I_s$ (middle) and unshifted $I_u$ (bottom) components of the level $9/2^+$ of $^{61}$Cu at 2720 keV for each angles compination (B-F, F-B, F-F, B-B).

58
3.3. Analysis of the Lifetime

<table>
<thead>
<tr>
<th>B-F</th>
<th>F-B</th>
<th>F-F</th>
<th>B-B</th>
</tr>
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<td>55</td>
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<td>75</td>
</tr>
<tr>
<td>438.4</td>
<td>60.2</td>
<td>499.6</td>
<td>83.8</td>
</tr>
<tr>
<td>379.6</td>
<td>66.3</td>
<td>614.9</td>
<td>88.4</td>
</tr>
<tr>
<td>259.2</td>
<td>62.6</td>
<td>695.7</td>
<td>83.5</td>
</tr>
<tr>
<td>268.9</td>
<td>84.2</td>
<td>1046.9</td>
<td>119.1</td>
</tr>
</tbody>
</table>

Table 3.4: Number of counts of shifted and unshifted peaks of the 1409 keV transition of $^{61}$Cu gated on the shifted component of the $\gamma$ line 1361 keV.

The corresponding reduced transition probability is $B(E3) = 22(5)$ W.u. This value will be compared with the theoretical estimates for this state in the next chapter.
3.3.4 Other Lifetimes

The complete gamma spectroscopy of the nuclei produced in the second part of the experiment, with the plunger device, allowed to identified two lifetimes compatible with the RDDS technique. The first one is the $7/2^-$ state at 1732 keV of $^{61}\text{Cu}$ and the other one is the lifetime of the $4^+$ level at 2459 keV of $^{58}\text{Ni}$. The partial level schemes taken into account for the analysis of these two levels are reported in Fig. 3.17.

![Level Scheme Diagram](image_url)

*Figure 3.17: Left: partial level scheme of $^{58}\text{Ni}$ around the $4^+$ level at 2459 keV. Right: level scheme of $^{61}\text{Cu}$ around the $7/2^-$ state at 1732 keV.*

The DDCM analysis was performed as explained in the previous section. For the lifetime of the $^{58}\text{Ni}$ the gate condition was on the shifted component of the $\gamma$-line depopulating the level at 3620 keV and the $v/c$ value was of 0.0127(30). The obtained coincidence spectra are reported in Fig. 3.18 (left).

In Tab. 3.5 are reported the number of counts of the shifted and unshifted peaks of the 1004 keV $\gamma$-line, normalized with the coefficient reported in Tab. 3.2. The Napatau fit and the lifetime are reported in Fig. 3.19. The obtained lifetime $\tau = 10.5(5)$ is larger than the value present in literature, $\tau = 5.4(6)$, measured in [19]. However, the direct RDDS measurement is a technique more reliable than electron scattering method, which is known to
be model dependent [19]. For this reason we are confident that the obtained value is a better measurement of the lifetime of the $4^+$ level at 2459 keV of $^{58}\text{Ni}$.

The second measured lifetime is the one of to the $7/2^-$ level at 1732 keV of $^{61}\text{Cu}$. The gate condition was on the shifted peak at 879 keV depopulating the $9/2^-$ level at 2612 keV, as reported in Fig. 3.17, and the $\nu/c$ value was 0.0119(30). In Fig. 3.18 are reported the spectra used for the DDCM analysis. The number of counts of the shifted and unshifted peaks of the $\gamma$-line at 422 keV are reported in Tab. 3.5. Fig. 3.19 (left) reports the Napatau analysis and the lifetime values obtained.

<table>
<thead>
<tr>
<th>$^{58}\text{Ni}$</th>
<th>$^{61}\text{Cu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_U$ $\sigma_U$ $I_S$ $\sigma_S$</td>
<td>$I_U$ $\sigma_U$ $I_S$ $\sigma_S$</td>
</tr>
<tr>
<td>1294 61 358 66</td>
<td>598 54 290 45</td>
</tr>
<tr>
<td>1325.7 57.6 565.3 58.7</td>
<td>651.1 57.5 421.9 55.3</td>
</tr>
<tr>
<td>1399.4 62.2 740.5 62.1</td>
<td>485.4 60.8 480.3 60.8</td>
</tr>
<tr>
<td>1090.5 65.2 862.4 77.1</td>
<td>481.5 62.2 571.9 63.7</td>
</tr>
<tr>
<td>1068.3 76.6 1156.9 96.8</td>
<td>350.7 70.5 713.5 76.6</td>
</tr>
</tbody>
</table>

*Table 3.5: Number of counts of shifted and unshifted peaks of the 1004 keV transition of $^{58}\text{Ni}$ and of the 422 keV transition of $^{61}\text{Cu}$, gated respectively on the shifted component of the $\gamma$-line at 1161 keV and 879 keV (see level scheme in Fig. 3.17).*

The lifetime of the $7/2^-$ state at 1732 keV of $^{61}\text{Cu}$ was not known in literature. There was only a lower limit estimate $\tau > 2$ ps [20] obtained by the Doppler-shift attenuation method. The obtained value of 8.6(7) ps is in agreement with this limit.
3.3. Analysis of the Lifetime

Figure 3.18: Left: spectra of $^{58}$Ni, zoomed around the 1004 keV peak, at different target-to-stopper distances (given in the legend) in coincidence with the shifted component of the γ-line at 1161 keV depopulating the level at 3620 keV. Right: spectra of $^{61}$Cu, zoomed around the 422 keV peak, at different target-to-stopper distances in coincidence with the shifted component of the γ-line at 879 keV depopulating the level at 2612 keV. The shifted and unshifted peaks are reported with U and S, respectively.
3.3. Analysis of the Lifetime

Figure 3.19: Left: $\tau$-plot and intensities of the shifted $I_s$ (middle) and unshifted $I_u$ (bottom) components of the $4^+$ of $^{58}\text{Ni}$ at 2459 keV. Right: $\tau$-plot and intensities of the shifted $I_s$ (middle) and unshifted $I_u$ (bottom) components of the $7/2^-$ of $^{61}\text{Cu}$ at 1732 keV.
Chapter 4

Particle Vibration Coupling in $^{61}\text{Cu}$

Aim of this chapter is to study the nature and the properties of the $9/2^+$ state at 2.72 MeV in $^{61}\text{Cu}$. This state has been interpreted within the framework of the particle-vibration coupling model using the weak-coupling approximation, an approach originally introduced by A. Bohr and B. R. Mottelson [1]. In this way the nucleus $^{64}\text{Cu}$ can be seen as a core of $^{60}\text{Ni}$ with an extra proton outside the closed shell $Z = 28$. In this context, the interaction between the vibrational degrees of freedom of the core and the single particle levels shift the states given by their superposition. In our case, we are interested in the coupling of the single particle level $\pi 2p_{3/2}$ and the $3^{-}$ octupole vibration at 4.04 MeV of the $^{60}\text{Ni}$ core. The result of this coupling is the multiplet of states $3^{-} \otimes p_{3/2}$ with angular momenta $9/2^+$, $7/2^+$, $5/2^+$ and $3/2^+$. Therefore the $9/2^+$ level of $^{61}\text{Cu}$ at 2.72 MeV corresponds to the member of the multiplet with the highest angular momentum, leading to an E3 decay transition to the $3/2^-$ ground state. In order to describe this multiplet of states the basic ingredients are the single particle states, calculated with Hartree-Fock-(BCS) approximation, and the properties of the $3^{-}$ phonon, such as the
energy and the transition probability. The following sections briefly describes the main aspects of the calculations. Finally, it will be given a comparison with the experimental results.

4.1 Hartree-Fock Equations

The single particle states of $^{60}$Ni have been obtained by Hartree-Fock-BCS calculations [21]. This mean field theory provides a solution to the nuclear many-body problem, based on an Hamiltonian containing a suitable two-body interaction. In second quantization, this is given by

$$\hat{H} = \sum_{ij} t_{ij} \hat{c}_i^\dagger \hat{c}_j + \frac{1}{2} \sum_{ijkl} v_{ijkl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_l \hat{c}_k \tag{4.1}$$

where $t$ is the kinetic energy operator and $v_{ijkl} = \langle i, j | v | k, l \rangle - \langle i, j | v | l, k \rangle$ represent the antisymmetric two-body interaction matrix elements, with single particle creation and annihilation operators $\hat{a}_i^\dagger \hat{a}_i$. In the Hartree-Fock theory (HF), a single Slater determinant is selected to be a many-body wave-function

$$|\Psi^{HF}\rangle = \prod_{i=1}^A \hat{c}_i^\dagger |0\rangle \tag{4.2}$$

where the index over the product corresponds to a set of single particle states with orthonormal wave-functions $\phi_i(r)$, $i = 1, 2, \ldots, A$, being $A$ the number of nucleons. These are eigenfunctions of the single particle Hamiltonian $\hat{h}$

$$\hat{h}(x)\phi(x) = \epsilon_i \phi(x) \quad \text{with} \quad i = 1, 2, \ldots, A \tag{4.3}$$

where the single-particle wave-functions, $\psi_i(x)$, are represented in coordinate space and $\epsilon_i$ are the corresponding single-particle energies. Using the variational principle with a proper normalization condition, it is possible to give an approximation of the ground state (G.S.)
4.1. Hartree-Fock Equations

\[ \delta \langle \Psi_{HF} | \hat{H} - E | \Psi_{HF} \rangle = 0 \]  \hspace{1cm} (4.4)\\

where the average single-particle Hartree-Fock potential is

\[ \hat{H}_{HF} = \sum_{i=1}^{A} \hat{h}(i) \]  \hspace{1cm} (4.5)\\

In the formalism of the second quantization, the single-particle density associated with the state \( |\Psi_{HF}\rangle \) is defined as

\[ \hat{\rho}_{ij} = \langle i | \hat{\rho} | j \rangle = \langle \Psi_{HF} | \hat{a}_i^\dagger \hat{a}_j | \Psi_{HF} \rangle \]  \hspace{1cm} (4.6)\\

The variation of the energy functional with respect to the single particle wave-functions \( \phi_i \) leads to a set of coupled, non-linear equations

\[ \hat{h}_{kl} = \hat{t}_{kl} + A \sum_{i=1}^{A} v_{kili} = \epsilon_k \delta_{kl} \]  \hspace{1cm} (4.7)\\

The Hartee-Fock equations in coordinate space are written as

\[ \epsilon_k \phi_k(\mathbf{r}, \sigma) = -\frac{\hbar^2}{2m} \nabla^2 \phi_k(\mathbf{r}, \sigma) + \left( \int d^3 r' v(\mathbf{r}, \mathbf{r}') \sum_{j=1}^{A} |\phi_j(\mathbf{r}')|^2 \right) \phi_k(\mathbf{r}, \sigma) + \left( \int d^3 r' \sum_{\sigma'} v(\mathbf{r}, \sigma, \mathbf{r}', \sigma') \phi_j^*(\mathbf{r}, \sigma) \phi_k(\mathbf{r}, \sigma) \right) \]  \hspace{1cm} (4.8)\\

In this integro-differential equation, the integral in brakets on the second term is the mean field, and the third term is called the exchange (Fock) term. The Hartree-Fock equations present a self-consistent problem, since the mean field and the exchange terms depend on the single-particle wave-functions of the solution of the single-particle eigenvalue problem. It is usually solved by iteration methods. In order to solve the equation one has to choose an
effective nucleon-nucleon interaction $v(r, r')$. One of the most popular, as well as the one used in this work, is the *Skyrme interaction* [22]. It is a zero-range momentum-dependent two-body force which has the expression

$$v_{sk}(r, r') = t_0(1 + x_0 P_\sigma) +$$

$$+ \frac{1}{2} t_1 (1 + x_1 P_\sigma) [\delta(r - r') k^2 + \delta(r - r') k^2] +$$

$$+ t_2 (1 + x_2 P_\sigma) k^2 \delta(r - r') k +$$

$$+ \frac{1}{6} t_3 (1 + x_3 P_\sigma) \delta(r - r') \rho^\alpha \left( \frac{r + r'}{2} \right) +$$

$$+ i W_0 (\sigma_1 + \sigma_2) k^2 \delta(r - r') k$$

where $k = \frac{1}{2i} (\nabla_1 - \nabla_2)$ and $P_\sigma = \frac{1}{2} (1 + \sigma_1 \sigma_2)$ with $\sigma$ the Pauli matrices. The ten parameters $t_i$, $x_i$, $\alpha$ and $W_0$ can assume different values and each set defines the type of Skyrme interaction. In this work we have used the so called *SkX* [23] and *Sly5* [24] forces.

### 4.2 Bardeen-Cooper-Schrieffer (BCS) Theory

$^{60}\text{Ni}$ is a semi-magic nucleus: it is a closed shell in protons ($Z = 28$), and an open shell in neutrons ($N = 32$). For this reason the HF theory, as described above, is incomplete and superfluidity has to be taken into account. In order to describe this nucleus the residual pairing interaction has to be included in the calculation. The model used is therefore the Hartree-Fock-BCS theory, which is an approximation of the well known Hartree-Fock-Bogoliubov [25]. The starting point is the generalization of the ground state wavefunction. In the BCS formalism this can be done introducing

$$|BCS\rangle = \prod_{k>0} \langle u_k + v_k \hat{a}_k \hat{a}_k^{\dagger} \rangle \langle 0 \rangle$$

(4.10)
4.2. Bardeen-Cooper-Schrieffer (BCS) Theory

where \( v_k \) is the probability that the pair \((k, -k)\) is occupied. The normalization of BCS state gives the condition for the coefficients \( v_k, u_k \)

\[
u_k^2 + u_k^2 = 1 \tag{4.11}
\]

From this choice it follows that the number of particles is not conserved. The best that it can be done is to conserve the mean value. Defining \( \hat{N} \) the operator number of particles it follows that

\[
\langle BCS| \hat{N} |BCS \rangle := \sum_{k>0} 2v_k^2 = N \tag{4.12}
\]

Therefore, the best wave-function can be found using the variational principle

\[
\delta \langle BCS| \hat{H} - \lambda \hat{N} |BCS \rangle \tag{4.13}
\]

The expression for the pairing interaction employed in this work is

\[
V_p(r_1, r_2) = V_0 \left[ 1 + \eta \left( \frac{\rho((r_1 + r_2)/2)}{\rho_0} \right)^\gamma \right] \delta(r_1 - r_2) \tag{4.14}
\]

where \( \eta, \gamma \) and \( \rho_0 \) are parameters chosen in order to fit experimental data. In this way the pairing problem reduces to solve the system of equations

\[
\begin{align*}
\Delta_k &= -\frac{1}{2} \sum_{k' > 0} \frac{\langle k'\bar{k} \mid V_p \mid k\bar{k} \rangle \Delta_{k'}}{\sqrt{(\epsilon_{k'} - \lambda)^2 + \Delta_{k'}^2}} \quad \text{(gap equation)} \\
2 \sum_{k>0} v_k^2 &= N \quad \text{(number equation)}
\end{align*} \tag{4.15}
\]

where

\[
\Delta_k = -\sum_{k' > 0} \langle k'\bar{k} \mid V_p \mid k\bar{k} \rangle v_{k'} u_{k'} \tag{4.16}
\]

is the pairing gap. As shown in Fig. 4.1, one of the effects of the pairing interaction is the modification of the probabilities of occupation within a energy range of \( 2\Delta \) around the Fermi surface.
4.3 Particle Vibration Coupling

The Particle-Vibration Coupling model here discussed was originally developed by Bohr and Mottelson [1] and later extended by Hamamoto [26] in order to predict both the energies and the ground state transition probabilities for the members of the multiplet of states arising from the coupling scheme. The model is based on the initial assumption that when we have a vibration, the density variations associated with the vibrational motion make corresponding variations in the average nuclear potential. The distortion of the average potential gives a coupling between the vibrational degrees of freedom and those of the individual particles. One may consider various effects which arise from the coupling: for example, the re-normalization of the properties of both the particles and the vibrations, the effective interactions between those elementary modes of excitation, and so on. In this work it has been considered the so called weak coupling, that is a particular condition that occurs in the treatment of nuclei near close shell. In this context, the effects arising from the coupling can be treated with the perturbation theory. The leading order particle-vibration coupling is linear in the vibrational
4.3. Particle Vibration Coupling

amplitude $\alpha$. Such a coupling can be expressed by the Hamiltonian

$$H' = k \alpha F$$

(4.17)

where $F$ is a one-particle field and $k$ takes into account the strength of the coupling. In the following we confine ourselves to shape oscillations, as an example of possible vibrations. Considering a collective vibration of the nucleus it is possible to associate a variation of the average nuclear static potential. A density variation of multipole order $\lambda$ produces a variation of the nuclear potential so that the coupling can be written as

$$\delta V = H' = -k_\lambda(r) \sum_\mu Y^*_{\lambda\mu}(\theta, \varphi) \alpha_{\lambda\mu} = (-1)^{\lambda+1}(2\lambda + 1)^{1/2}k_\lambda(r)(Y_{\lambda}\alpha_\lambda)_0$$

(4.18)

where

$$k_\lambda(r) = R_0 \frac{\partial V}{\partial r}$$

(4.19)

This coupling produces a scattering of the particle with the emission or absorption of a quantum, and the square of the matrix element for this process is given by

$$\hbar^2(j, j_1, \lambda) = \frac{1}{2j + 1} \left( \int dr \ u_{j_1} u_j \ k_\lambda(r) \langle j_1 | Y_{\lambda} | j \rangle \sqrt{\frac{\hbar\omega}{2C}} \right)^2$$

(4.20)

where $\sqrt{\frac{\hbar\omega}{2C}} = \alpha_0$ is the zero-point amplitude which is related to the reduced transition probability $B(E\lambda; n_\lambda = 0 \rightarrow n_\lambda = 1)$ by

$$B(E\lambda; n_\lambda = 0 \rightarrow n_\lambda = 1) = (2\lambda + 1) \left( \frac{3}{4\pi} ZeR_\lambda \right)^2 \frac{\hbar\omega}{2C}$$

(4.21)

4.3.1 Energy Shift

At the first order, the particle-vibration coupling generates an excited state at an energy given by the sum of the excitation energy of the single-particle state
4.3. Particle Vibration Coupling

$j_1$ and the vibrational frequency $\hbar \omega$. The lowest-order non-zero contributions to the energy shifts come from the second-order in $H'$. There are four terms, corresponding to the different types of intermediate states

$$V = V^{(a)} + V^{(b)} + V^{(c)} + V^{(d)}$$  \hspace{1cm} (4.22)

expressed by the four diagrams shown in Fig. 4.2. When a system exploits spherical symmetry, as in the present case, we can adopt the angular momenta formalism. For angular momentum-coupled states $(j_1 \lambda)I$, it is seen that the matrix elements illustrated by Figs. 4.2 (a) and 4.2 (b) are proportional to $\delta(j, I)$. The matrix elements shown in Figs. 4.2 (c) and 4.2 (d) can be evaluated by performing a recoupling in the intermediate states. Thus, for the energy splitting within the multiplet, we obtain

$$\langle j_1, n\lambda; IM | V^{(a)} + V^{(b)} | j_1, n\lambda; IM \rangle = \sum_j \frac{\hbar^2(j_1)}{\epsilon(j_1) - \epsilon(j) - \hbar \omega} \delta(j, I)$$  \hspace{1cm} (4.23)

Figure 4.2: Second-order contributions to the energy shifts of the multiplet member $(j_1 \lambda)I \rightarrow j_1$. 

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\[ \langle j_1, n_\lambda = 1; IM | V^{(c)} + V^{(d)} | j_1, n_\lambda = 1; IM \rangle = \sum_j \frac{\hbar^2 (j_1, j, \lambda)}{\varepsilon(j) - \varepsilon(j_1) - \hbar \omega} (2j_1 + 1) \left\{ \begin{array}{ccc} \lambda & j_1 & j \\ \lambda & j_1 & I \end{array} \right\} \] (4.24)

4.3.2 Decay Scheme

We consider the decay scheme of the multiplet members only to the lowerlying states of which the predominant component consists of a single-particle coupled to the ground state of the core [26]. In order to discuss the decay scheme of the octupole vibrational state in the nuclei consisting of one nucleon plus a core (such as the nucleus \(^{60}\text{Ni}\)), it is sufficient to consider only these kind of decays.

The following operator which act on the particle line is used for the electric operator, in the case that no low-lying collective state corresponding to the degree of freedom of the operator is found in the core

\[ M_{\nu \mu}^E = e^{(n)}_{\nu \mu} r^{(n)} Y_{\nu \mu} (\theta, \varphi) \] (4.25)
4.4 Study of the Particle Vibration Coupling (PVC) in $^{61}$Cu

Figure 4.3: The unperturbed collective $E_\lambda$ transition, $(j\lambda)I \rightarrow j_1$ \cite{26}.

The contribution from the graphs 4.4 (a) and 4.4 (b) to the reduced matrix element of the transition $M_\nu$ can be written in the same form

$$\langle j_2 | M_\nu | j \rangle \frac{j|H'|(j\lambda)I\rangle}{\epsilon_{j_1} - \epsilon_j + \hbar \omega} \delta(j_2, I)$$  \hspace{1cm} (4.27)

while the contribution arising from the diagram 4.4 (c) and 4.4 (d) can be written in the form

$$\langle j | M_\nu | j_1 \rangle \frac{j_2|H'|(j\lambda)j_2\rangle}{\epsilon_{j_2} - \epsilon_j + \hbar \omega} \sqrt{2j_2 + 1}\sqrt{2I + 1} W(j\lambda\nu I ; j_2j_1)$$ \hspace{1cm} (4.28)

The sum of these two expressions over all the single-particle orbits with angular momentum $j$, gives the total reduced transition matrix element in the first-order of $H'$.

4.4 Study of the Particle Vibration Coupling (PVC) in $^{61}$Cu

Nuclei around a shell or double shell closure have low-energy levels that are candidate to be described with the formalism of the Particle-Vibration-Coupling (PVC). In this thesis we have studied the near closed shell nucleus $^{61}$Cu.
4.4. Study of the Particle Vibration Coupling (PVC) in $^{61}$Cu

Figure 4.4: First-order contributions to the electromagnetic ($E_\nu$) transition matrix elements between the single-particle state $j_2$ and the member of the multiplet $(j_1\lambda)I \rightarrow j_1$.

$^{61}$Cu. This nucleus can be interpreted as a core of $^{60}$Ni with an extra proton. Aim of this work is the description of the first $9/2^+$ excited state at 2.72 MeV, as the coupling between the octupolar vibration of the core and the single particle level $\pi 2p_{3/2}$. Results are compared with the experimental value obtained from the analysis of the lifetime of the $9/2^+$ state, discussed in Section 3.3.3. In order to convert the measured lifetime in a reduced transition probability are uses the relation

$$\Gamma(\sigma\lambda; I_i \rightarrow I_f) = \frac{\hbar}{\tau} = \frac{8\pi(\lambda + 1)}{\lambda[(2\lambda + 1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda + 1} B(\sigma\lambda; I_i \rightarrow I_f) \quad (4.29)$$

which gives a value of $B(E3) = 22 \pm 5$ W.u. for the transition $9/2^+ \rightarrow 3/2^-$ of the $^{61}$Cu, already discussed in 3.3.3. A more complete treatment of the transition probability is reported in Section 1.3.1.

The first part of the analysis is concentrated on a systematic study of the nucleus $^{60}$Ni. Looking into the literature one finds what is known experimentally for the B(E3) transition at energy of 4.04 MeV [16]. If the hypothesis of coupling is reasonable, one aspects to find a value of B(E3) of the core
very close to that of the $9/2^+$ state of interest.

In Fig. 4.5 it is presented a comparison between all the B(E3) values of $^{60}\text{Ni}$, known from literature [27, 28, 29], and that obtained from the analysis of $^{61}\text{Cu}$. Considering the weighted mean of the experimental values, which is equal to 18.2(4) W.u. for $^{61}\text{Ni}$, it is possible to observe a rather good agreement between the two nuclei. As reported in the previous sections, close values of B(E3) represent a first evidence of the collectivity of the core+particle nuclear system.

Once it was verified that the hypothesis of the coupling is reasonable, it was performed a theoretical calculation of the energy level and of the B(E3) of the $9/2^+$ level of $^{61}\text{Cu}$. At first it was performed a calculation of the single particle levels of $^{60}\text{Ni}$ (see Section 4.4.1) and than it was performed a PVC calculation in order to obtain the quantity of interest, as discussed in Section 4.4.2.
4.4. Study of the Particle Vibration Coupling (PVC) in $^{61}$Cu

![Figure 4.6: The neutron pairing gap $\Delta_n$ for even mass Ni isotopes with $A=58-68$. The "3 points formula" was used to plot the experimental data. The theoretical calculations for the two effective interactions SkX and Sly5 was calculated.](image)

4.4.1 Computational Hatree-Fock-BCS Calculations

The single particle levels of $^{60}$Ni were obtained with the Hartree-Fock-BCS formalism previously described. In order to determinate the value of $V_0$ present in Eq. 4.14, the experimental neutron pairing gap in the Ni isotopic chain was calculated. The experimental values were obtained by the tabulated binding energies $B(Z,N)$ [16] by means of the three different formulas [1]

$$\Delta_n = \frac{1}{2} \{ B(Z, N + 2) - 2B(Z, N + 1) + B(Z, N) \}$$ \hspace{1cm} (4.30)

$$\Delta_n = \frac{1}{2} \{ B(Z, N + 1) - 2B(Z, N + 1) + B(Z, N + 1) \}$$ \hspace{1cm} (4.31)

$$\Delta_n = \frac{1}{4} \{ B(Z, N + 1) + 3B(Z, N + 1) - 3B(Z, N - 1) + B(Z, N - 2) \}$$ \hspace{1cm} (4.32)
The experimental data, together with the theoretical calculations performed with the two Skyrme interactions, are shown in Fig. 4.6. The values \( V_0 = 825 \text{ MeVfm}^3 \) was obtained for the SkX and \( V_0 = 930 \text{ MeVfm}^3 \) for Sly5. After the determination of the best value for the parameter \( V_0 \), the single particle levels were calculated for the neutron part and the proton part in the HF-BCS formalism, using two different effective interaction. Figure 4.7 shows the single particle levels and the potential are shown.

4.4.2 Particle Vibration Coupling (PVC) Calculations

Once the single particle levels of the \(^{60}\text{Ni}\) were calculated it was perform a PVC calculation for \(^{61}\text{Cu}\). Following the formalism already described, the calculation was performed using two different forces (SkX and Sly5). As a consequence of the weak coupling assumption, the perturbation theory was used for the calculation of the energy and of the reduced transition probability. It was also calculated the B(E3) strength assuming for the 3\(^{-}\) phonon of \(^{60}\text{Ni}\) the experimental weighted average \(18.2(4) \text{ W.u.} \) (reported in Fig. 4.5). The obtained results are reported in Fig. 4.8.

As it is possible to observe from the plot, the PVC model reproduces very well the energy of the transition of the 9/2\(^{+}\) level of \(^{61}\text{Cu}\) which is the highest spin member of the \(\pi 2p_{3/2}\) multiplet (studied in the present work). However, the calculated reduced transition probability for the transition 9/2\(^{+}\) \(\rightarrow\) 3/2\(^{-}\) is reduced by \(\approx 30\%\) respect to the experimental one. The theoretical value is obtained with a perturbation theory starting from the experimental B(E3) of the core nucleus and for this reason it is strongly infulenced by it. Because the transition probability of the core is a weighted mean of three different value not compatible each other, Fig. 4.5, there is a big uncertain on the theoretical value.

Figure 4.9 summarises the results obtained for the 9/2\(^{+}\) along the Cu isotope chain in a series of experiment performed at IFIN-HH laboratory of
Bucharest [2, 3]. It is found that the present result on \( ^{61}\text{Cu} \), together with the one of \( ^{65}\text{Cu} \), follows the trend expected by the theoretical PVC model, supporting the validity of the weak-coupling approach, at least up to \( ^{65}\text{Cu} \). For heavier Cu isotopes the theory largely underestimates the experimental results pointing to structural changes, such as shape evolution, in more neutron rich Cu systems, as reported in Ref. [30, 31].

This analysis work demonstrates the importance of the accurate lifetime measurements along isotopic chains to investigate the evolution of the shell structure moving away from the stability valley.
4.4. Study of the Particle Vibration Coupling (PVC) in $^{61}$Cu

![Diagram showing single particle energies of the bound states in $^{60}$Ni obtained via HF-BCS calculation using two different Skyrme interactions, SkX (up) and SLy5 (down). Both proton (left) and neutron (right) levels are reported. The potentials employed are shown by black solid lines.](image-url)

Figure 4.7: Single particle energies of the bound states in $^{60}$Ni obtained via HF-BCS calculation using two different Skyrme interactions, SkX (up) and SLy5 (down). Both proton (left) and neutron (right) levels are reported. The potentials employed are shown by black solid lines.
4.4. Study of the Particle Vibration Coupling (PVC) in $^{61}$Cu

Figure 4.8: Level scheme of $^{61}$Cu arising from the coupling between the single particle level $\pi 2p_{3/2}$ of $^{61}$Cu and the octupole vibration $3^-$ at 4.04 MeV of the $^{60}$Ni core. Calculations are performed for two different Skyrme interactions (SkX and Sly5), assuming a $B(E3)$ value for $^{60}$Ni equal to 18.2(4) W.u., corresponding to the experimental weighted average [16].
4.4. Study of the Particle Vibration Coupling (PVC) in $^{61}$Cu

![Graph showing comparison of experimental transition probability and theoretical calculation of B(E3) for $^{61,65,67}$Cu. Data for $^{65}$Cu and $^{67}$Cu are taken from Ref. [2, 3].](image)

Figure 4.9: Comparison of the experimental transition probability and theoretical calculation of B(E3) for the nuclei $^{61,65,67}$Cu. Data for $^{65}$Cu and $^{67}$Cu are taken from Ref. [2, 3].
Chapter 5

Conclusion

This thesis deals with the experimental study, through gamma-ray spectroscopy, of excited states in $^{61}\text{Cu}$. This nucleus was produced using the reaction $^{16}\text{O} + ^{48}\text{Ti}$ at 54 MeV, performed at Horia Hulubei National Institute for R&D in Physics and Nuclear Engineering (IFIN-HH) of Bucharest, in April of 2014.

The experimental analysis performed in this work can be divided into two different parts: the first one was devoted to the determination of the decay branching of the $9/2^+$ level at 2.72 MeV, while in the second part, using the Recoil-Distance-Doppler-Shift (RDDS) method, the lifetime of the level of interest was extracted. The obtained lifetime of 4.3(7) ps corresponds to a transition probability $B(E3) = 22(5)$ to the ground state.

The last part of the thesis work was focused on the theoretical interpretation of the obtained results. The $9/2^+$ excited state of the nucleus $^{61}\text{Cu}$ is supposed to originate from the coupling of a particle to the $3^-$ phonon of the $^{60}\text{Ni}$. Starting from the calculated energy level of the $^{60}\text{Ni}$ and from its experimental $B(E3)$ value, a PVC calculation was performed employing two different forces: SkX and Sly5.

The results obtained from this study show that the $9/2^+$ state of $^{61}\text{Cu}$ nucleus presents a collective character and it is a good candidate to be mem-
ber of the multiplet generated by coupling the $3^-$ octupole vibration and the proton single particle state $p_{3/2}$. This collectivity is rather well described by the PVC model.

This work provides an additional example of Particle - Vibration coupled states in $^{61}$Cu, improving the knowledge of this phenomenon along the Cu chain, a region where already previous studied were recently performed in $^{65,67}$Cu [2, 3].
Appendix A

Differential Decay Curve Method

The Differential Decay Curve Method (DDCM) is used to measure the lifetime of an excited nuclear state [13, 14]. This analysis is done by measuring with a plunger device the intensity of a $\gamma$-transition depopulating the state of interest, as a function of the target to stopper distance.

The time evolution of a level of interest is described by the equation:

$$n_i(t) = n_0 e^{-\lambda_i t} + \sum_j n_{0j}(1 - e^{-\lambda_j t})$$  \hspace{1cm} (A.1)

where $j$ indicates the feeders level, $n_0$ is the number of nuclei in the state of interest with constant decay $\lambda_i$ and $n_{0j}$ are the number of nuclei in the feeder level $j$ with constant decay $\lambda_j$. It is convenient to rewrite:

$$n_{0j} = \frac{n_{0j}}{n_{tot}} n_{tot} = b_{hi} n_{tot}$$  \hspace{1cm} (A.2)

where $n_{tot}$ is the total number of nuclei in a $j$-state and $b_{hi}$ is the relative intensity of the transition from the $h$-level to the $i$-level. Differentiating the relation in Eq. A.1 one obtains:
\begin{equation}
\frac{dn_i(t)}{dt} = -\lambda_i n(t) + \sum_h \lambda_h n_h(t) b_{hi}
\tag{A.3}
\end{equation}

Integration of A.3 gives:
\begin{equation}
\int_t^\infty \frac{dn_i(t)}{dt} \, dt = - \int_t^\infty \lambda_i n_i(t) \, dt + \sum_h \int_t^\infty \lambda_h n_h(t) b_{hi} \, dt
\tag{A.4}
\end{equation}

if one calls:
\begin{equation}
N_i(t) = \int_t^\infty \lambda_i n_i(t) \, dt \quad N_{hi}(t) = \int_t^\infty \lambda_h n_h(t) b_{hi} \, dt
\tag{A.5}
\end{equation}

one obtains:
\begin{equation}
n_i(\infty) - n_i(t) = -N_i(t) + \sum_h N_{hi}(t)
\tag{A.6}
\end{equation}

If the lifetime \( \tau_i \) of the level \( L_i \) and of the feeding levels are finite, \( n_i(\infty) \) vanishes and one obtains:
\begin{equation}
-n_i(t) = -N_i(t) + \sum_h N_{hi}(t)
\tag{A.7}
\end{equation}

Differentiating the first of the Eq. A.5 the relation:
\begin{equation}
\frac{dN_i(t)}{dt} = -\lambda_i n_i(t)
\tag{A.8}
\end{equation}

is obtained. Therefore Eq. A.7 can be rewrite as:
\begin{equation}
\frac{N_i(t)}{dt} = -\lambda_i [N_i(t) + \sum_h N_{hi}(t)]
\tag{A.9}
\end{equation}

Considering the identity \( N_i b_{ij} = N_{ij} \) one obtains:
\begin{equation}
\frac{N_{ij}(t)}{dt} = -\lambda_i [N_{ij}(t) + b_{ij} \sum_h N_{hi}(t)]
\tag{A.10}
\end{equation}

Finally the relation for the life time \( \tau_i \) is written as:
\begin{equation}
\tau_i = - \frac{N_{ij}(t) + b_{ij} \sum_h N_{hi}(t)}{\frac{N_{ij}(t)}{dt}}
\tag{A.11}
\end{equation}

where \( \tau_i \) is the inverse of \( \lambda_i \).
A.1 Coincidence Analysis

With the advent of large detector arrays it became possible to have plunger-\(\gamma\)-\(\gamma\)-coincidence experiment. This allow to avoid many problems caused by systematic errors in the analysis of singles plunger experiments. The feeding pattern of the level of interest is in fact simplified by gating on a feeding transition. With this condition Eq. A.11 can be rewrites:

\[
\tau_i = -\frac{N_{ij}(t)}{N_{ij}(t) dt}\quad (A.12)
\]

At this point it is convenient to introduce the following formalism. The measured intensities of two transitions in coincidence, \(Y\) and \(X\), are indicated \(\{Y, X\}\). Using the label S and U for the Shifted and Unshifted components of the transition, the coincidence intensities \(Y, X\) can be written as:

\[
\{Y, X\} = \{Y_U, X_U\} + \{Y_S, X_U\} + \{Y_S, X_S\} + \{Y_U, X_S\}\quad (A.13)
\]

Due to the time order of the transition, \(\{Y_S, X_U\} = 0\). Therefore, considering a level scheme as reported in Fig. 3.4, one can rewrite Eq. A.12:

\[
\tau_i = \frac{\{B_S, A_U\}}{\frac{d}{dx}\{B_S, A_S\}}\quad (A.14)
\]

For practical purpose in the analysis, Eq. A.14 can be written as:

\[
\tau_i = \frac{\{B_S, A_U\}(x)}{\frac{d}{dx}\{B_S, A_S\}(x) v}\quad (A.15)
\]

It is important to observe that one can extract from the data the quantity in the numerator, while one has to use approximated methods to estimate the derivative of the observable quantity which is at the denominator.

For example one can estimate the lifetime of the level of interest by gating on the shifted component of the transition that feeds the level directly in order to determine the shifted and unshifted component of the gamma rays that depopulates the level. One can write the relation:

\[
\tau = \frac{A_S(x)}{A_S(x + \Delta x) + A_S(x - \Delta x)} \quad (A.16)
\]
where $v$ is the recoil velocity. The quantities measured at the three different
distances have to be normalized to the total number of reactions.

Another way to estimate the lifetime is to plot the shifted and unshifted
component of the depopulating transition, as for example reported in Fig. 3.4,
and to observe that the unshifted component is proportional to the derivative
in the denominator of Eq. A.14. In particular one has:

$$\{B_S, A_U\} = \tau_i \frac{d}{dt} \{B_S, A_S\}$$

(A.17)

The derivative can be estimated by fitting the intensity $\{B_S, A_U\}$ with a
second order polynomial.

As one can see from the Eq. A.12, the lifetime $\tau_i$ is obtained from the
ratio of two values. If we suppose to have similar errors for both quantities
and for all flight times, a larger error is obtained for small values of the
denominator. For this reason it is used to define a sensitive region, which
has its limits where the slope of the $N_{ij}(t)$ curve is at half of the maximum
value.

### A.2 The Napatau Analysis Program

Napatau is a lifetime analysis tool, developed by the Cologne group [17],
which has a graphical user interface. It supports the DDCM analysis of
direct and indirect gated spectra, measured in RDDS measurement. In the
DDCM analysis, the time derivative of the Doppler shifted intensities, $\frac{d}{dt}I^{sh}$,
has to be determined from the measured intensities $I^{sh}$. Indicating with $I^{sh}$
and $I^{us}$, it is possible to rewrite Eq. A.14 as follow:

$$\tau_i = \frac{I^{us}_i}{\frac{d}{dt}I^{sh}_i}$$

(A.18)

Making the assumption that $\tau_i = \dot{t}$ for all target to stopper distances one can
write:

$$I^{us}_i = \dot{t} \frac{d}{dt} I^{sh}_i$$

(A.19)
Napatau implements an algorithm that allows to estimate the time derivative of a measured quantity. This time derivative is evaluated in the case of the DDCM method by fitting a second order polynomial piecewise, at the intensities $I_i^{sh}$ measured at every target to stopper distance. This program allow to fit the data by minimizing the $\chi^2$ function in Eq. A.20 for a fit function $f(t)$ which depends from the free parameters $a_1, ..., a_n$, where $\Delta I_i^{sh}$ is the statistical error of $I_i^{sh}$.

$$\chi^2 = \sum_i \left[ \frac{I_i^{sh} - f(a_1, ..., a_n)(t_i)}{\Delta I_i^{sh}} \right]^2$$  \hspace{1cm} (A.20)

The life time $\tilde{t}_{hyp}$ is introduced in the $\chi^2$ function through the relation in Eq. A.19:

$$\chi^2 = \sum_i \left[ \left( \frac{I_i^{sh} - f(a_1, ..., a_n)(t_i)}{\Delta I_i^{sh}} \right)^2 + w \left( \frac{I_i^{sh} - \tilde{t}_{hyp} \frac{df(a_1, ..., a_n)(t_i)}{dt}}{\Delta I_i^{us}} \right)^2 \right]$$  \hspace{1cm} (A.21)

Through the minimization of this relation it is possible to obtain, $f_{opt}(t)$, the fit function for which the fit is better (the complete calculation is reported in [17]). For each target to stopper distance the value of $\tau_i$ is:

$$\tau_i = \frac{I_i^{us}}{\frac{d}{dt} I_i^{sh}} = \frac{I_i^{us}}{\frac{d}{dt} f_{opt}(t_i)}$$  \hspace{1cm} (A.22)

where $t_i$ denotes the mean time of flight corresponding to target to stopper distance. The final value of the lifetime $\tau_{final}$ is obtained through a weighted mean of all the calculated $\tau_i$. 
Bibliography


