DEVELOPMENT OF A RING LASER GYRO:
ACTIVE STABILIZATION AND SENSITIVITY ANALYSIS

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Abstract

Ring laser gyroscopes are inertial sensor based on the Sagnac effect: rotation causes the frequency of the two counter-propagating beam in the ring cavity to be shifted by an amount proportional to the angular velocity. This shift (the Sagnac frequency) can be easily measured letting the two beams beat. Applications of ring laser gyros range from inertial navigation system to geodesy, to test of fundamentals physics. Several large laser gyros with high sensitivity have been developed in the last years.

This thesis presents the work done with a square ring laser with a side length of 1.40 m in the contest of the experiment G-Pisa of the INFN. This experiment may help improve the performance of the mirrors suspension of the gravitational wave antenna Virgo. The laser design is based on instruments developed by the joint ring laser working group in New Zealand and Germany. It is a helium-neon laser working on the 474 THz line of neon. Earth rotation is enough to bias the Sagnac frequency to 110 Hz. The capacitive-coupled discharge exciting the laser, whose stabilization was specially designed, is the flagship of this experiment.

In the first apparatus the operation was quite unstable with the laser gyro often blind to rotation. Long term operation of the laser was limited by the contamination of the gas.

The laser have been extensively tested. Diagnosis of the discharge shows the prominent role of hydrogen as contaminant, leading to the installation of getter pumps. Ring-down time was measured to check the quality factor of the laser cavity. Investigation of the mode structure proved an invaluable tool to understand the behaviour of the laser. The laser usually works single mode, but unusually good Sagnac signal can be achieved with the laser working multimode. These analysis shows that the gyro is mostly limited by split modes: independent mode jump of counter-propagating beams causing the gyro to be intermittently blind to rotation.

Stabilization of the beam intensity has been implemented to obtain more stable operations. Active stabilization of the optical frequency is designed, developed and tested in order to avoid mode jump and improve the performance of the gyro. A piezoelectric transducer is used to move one of the four mirrors, acting on the perimeter of the cavity. This kind of stabilization brings the laser against the subtle effect of backscattering of light of each beam in the other. A simple theoretical treatment of the backscattering and of its influence on the Sagnac effect is reported to explain acquired data, followed by some numerical simulations. Results obtained with this setup are reported: achieved optical frequency stability (30 kHz with an integration time of 100s), prolonged operation (till now of several hours), consequences on the Sagnac frequency (periodic pulling) and rotation sensitivity of the order $5 \times 10^{-9} \text{ (rad/s)} / \sqrt{\text{Hz}}$ at 0.5 Hz.
I would like to acknowledge the help, support and patience of several people during this work. I would like to thank my supervisor Prof. Nicolò Beverini and the following people: Jacopo Belfi, Angela Di Virgilio, Giorgio Carelli, Fiodor Sorrentino, and Mario and Francesco Francesconi.
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RING LASER gyroscopes are able to detect the rotation rate of their cavity relative to an inertial frame. Their principle of operation exploits the Sagnac effect: rotation causes the length of the cavity as seen by the two counter-propagating running waves in the laser to be slightly different. In the ring laser gyroscope this difference translates directly in a optical frequency shift between the two beams. This frequency difference is proportional to the rotation rate accordingly to

\[ \Delta \nu = \frac{4A \cdot \Omega}{\lambda L} \]

\( \nu \) where \( \Omega \) is the angular velocity vector, \( L \) is the perimeter of the ring cavity, \( A \) is the area vector (with direction perpendicular to the surface of the ring) and \( \lambda \) is the wavelength of the laser radiation (in absence of rotation). This shift (the Sagnac frequency) can be easily measured letting the two counter-propagating beams beat [5, 10, 32].

Compared to conventional spinning gyroscopes, ring laser gyros shows several advantages: they have large dynamic range, high precision, small size, they do not require any moving mechanical part and they are insensitive to translational accelerations. Thanks to these features laser gyros acquire a prominent role in many applications, ranging from inertial navigation system on commercial airliners, ships and spacecraft to geodesy and geophysics, to test of fundamental physics [30, 32].

As suggested by the first equation, the larger the ring, the easier the detection of the Sagnac frequency. Large size also mitigates the effects of lock-in, a major problem with active ring laser, especially the small one when measuring very low angular velocity. Lock-in is the tendency (typical of coupled oscillators with similar frequency) of the counter-propagating laser
beams to lock to one or the other frequency, practically blinding the ring laser as rotation sensor [5, 10, 21]. The coupling arises in ring laser usually because of backscattering: part of radiation of both beams scattered in the counter-rotating direction. Unlike the small ring lasers used for navigation systems, large gyros easily detect Earth’s rotation, which provides a nearly constant background rotation rate. Earth’s contribution is enough to bias the Sagnac frequency of large laser gyros so to avoid lock-in. Even without actual locking, associated pulling of the Sagnac frequency may cause serious systematic errors [25, 33]. A large gyro gives a Sagnac frequency biased further from zero and less dependant on backscattering [16].

1.1 Ring laser gyroscopes around the world

Several large laser gyros with high sensitivity have been developed in the last years [24], in particular by a collaboration among New Zealand (G.E. Stedman and coworkers, University of Christchurch) and Germany (U.K. Schreiber, Technische Universität München and Fundamentalstation Wetzell).

Most of the large ring laser gyroscopes used to monitor Earth’s rotation, but also of the small rings used for navigation, are helium-neon lasers. He-Ne was chosen because this is well known and established and optical components designed to operate at the 633 nm wavelength are largely and ready available. Moreover, the gas can be made fill all the laser cavity, avoiding optical surfaces (e.g., Brewster windows) where imperfections and dirt can cause backscatter. (Solid state devices should be made monolithic to avoid strong backscattering in a ring configuration.) At last, when two neon isotopes are used, $^{20}$Ne and $^{22}$Ne in equal part, this system shows good multi-directional capabilities and the two counter-rotating beams are practically uncoupled [4].

Table I summarizes the characteristics of some large frame noteworthy laser gyros around the world. The C-II laser is 1 m × 1 m square laser constructed from a monolithic block of Zerodur [28]. As the other ring lasers listed, it uses very high reflectivity mirrors (that deserve the name supermirror), granting an high quality factor of the cavity and little backscatter. The C-II laser was basically a prototype for a larger ring, the “Gross Ring” or G, located in Wetzell, Germany. G is a state-of-the-art 4 m × 4 m ring laser, is constructed from a monolithic block of Zerodur (the largest available) and kept in an underground controlled room to achieve perimeter stability. Up to now it is the most stable and sensitive large ring laser gyro. After the experience of G, a cheaper ring called GEOsensor has been developed, designed to be a compromise between the stability and sensitivity of G and the flexibility of smaller instruments [35]. It is currently located in the United States. UG-2, a rectangle 40 m × 21 m, is the largest ring, it is located underground
at Cashmere Cavern, New Zealand, an environment with excellent thermal stability [17]. An American, 39 m perimeter, triangular He-Ne ring laser has been developed in Arkansas, which, unlike the Canterbury and German ring lasers, employs a sealed gain section with Brewster windows.

### 1.2 Motivation and thesis outline

This thesis presents the work done with a square ring laser with a side length of 1.40 m in the contest of the experiment G-Pisa of the INFN. It is a helium-neon laser working on the red 474 THz line of neon. Earth rotation is enough to bias the Sagnac frequency to 110 Hz.
G-Pisa’s gyro is based on the model of GEOsensor, designed by U.K. Schreiber.

G-Pisa experiment was born to test, study and take confidence with this kind of instruments. The experiment aims to improve the performance of the mirrors suspension of the gravitational wave antenna Virgo. Virgo is the French-Italian interferometer with 3 km arms for the detection of gravitational waves emitted by astrophysical sources. The sensitivity strongly depends on the mirrors suspensions, which have to reduce the seismic noise, provide the forces on the mirrors to keep the interferometer aligned and do not introduce any kind of noise higher than the unavoidable thermal noise. Each suspension is equipped with controls to contain the low frequency, below 1 Hz, motion of the test masses. The most important control of the suspension is the so called Inverted Pendulum (IP) control, which is active on top of the Virgo suspensions. It is based on linear accelerometers, able to control four degrees of freedom, the two tilts motions are missed. At the moment the lack of such tilt control reduces the sensitivity of the antenna during severe weather and sea conditions. An upgrade of the IP control using inertial angular sensors, as the gyro lasers, can extend the control to six degrees of freedom. Moreover, the use of inertial sensors could be extended to the control of the mirror itself, building a special stage of the suspension equipped with gyro lasers.

The gyro laser has been studied extensively: results gathered are reported in this work.

Chapter 2 introduces the theory of ring laser gyroscopes, starting with a classical derivation of the Sagnac effect. Attention is given mostly to the treatment of backscattering and to its dependency with the geometry of the cavity. Then we shall discuss the features of the He-Ne laser systems important in the gyro case.

Chapter 3 presents the experimental setup of the gyroscope.

Chapter 4 presents the preliminary measures characterizing the system. Diagnosis of the discharge shows the prominent role of hydrogen as contaminant, leading to the installation of getter pumps. The quality of the mirrors is an important factor in the performance of the laser gyro and we have tested it with a ring-down time measurement. Investigation of the mode structure proved an invaluable tool to understand the behaviour of the laser. The laser usually works with only a mode in each direction, but good Sagnac signal can be achieved with the laser working multimode. These analysis shows that the gyro is mostly limited by split modes: independent mode jumps of counter-propagating beams that cause the gyro to be intermittently blind to rotation.

This quite unstable operation was a matter of concern: the resolution of this problem is the main topic of this thesis. In order to avoid mode jumps and improve the performance of the gyro we designed, developed and tested active stabilization of the optical frequency and of the intensity of the laser.
Frequency stabilization is obtained using a piezoelectric transducer to move one of the four mirrors, acting on the perimeter of the cavity. However this kind of stabilization brings the laser against the subtle effect of the backscattering, as we shall discuss in chapter 5.

In chapter 6 we present our conclusion and the study of the sensitivity of the gyro laser obtained so far.
CHAPTER 2

A bit of theory

In this chapter we review a bit of the theory beyond ring laser gyros. We start with a classical derivation of the Sagnac effect. Then we concentrate our effort in the development of a simple theory of the backscattering coupling. We conclude the chapter describing briefly the He-Ne laser system.

2.1 Sagnac effect

The basic principle of operation of a ring laser is based on the so-called Sagnac effect. This was first discussed by Sagnac in 1913 when he considered the use of a ring interferometer to sense rotation [27]. Dealing with non-inertial rotating frame, the rigorous treatment of the Sagnac effect involves the general theory of relativity. However, a classical result can be obtained, correct at the first order in the velocity \( v/c \), where \( c \) is the speed of light. Both derivations can be found in the work by Chow et al. [10]. Here the classical effect is presented, following [29].

2.1.1 The Sagnac interferometer

The Sagnac effect can be easily understood considering an ideal circular interferometer of radius \( R \), depicted in Fig. 2.1. Light enters the interferometer in point A and is divided by a beam-splitter in two counter-propagating beam. Each beam travels, by some mean, along the ring, clockwise (CW) or counter-clockwise (CCW). If the interferometer is not rotating, the light beams recombine at the beam-splitter in A after a time \( t = 2\pi R/c \). Instead, when the system is rotating, with angular velocity \( \Omega \), about an axis through the center and perpendicular to the plane of the interferometer, the two
beams reencounter the beam-splitter at different times. In fact, the clock-
wise (in the same direction of $\Omega$) beam must travel slightly more of $2\pi R$ to
complete a round-trip, since the beam-splitter has moved a bit during the
round-trip transit time. So the transit time $t_+$ for the clockwise beam is
given by the equation

$$t_+ = \frac{2\pi R + R\Omega t_+}{c},$$

that is

$$t_+ = \frac{2\pi R}{c} \left(1 - \frac{R\Omega}{c}\right)^{-1}.$$

Similarly, the counter-clockwise beam transit time $t_-$ is

$$t_- = \frac{2\pi R - R\Omega t_-}{c},$$

$$t_- = \frac{2\pi R}{c} \left(1 + \frac{R\Omega}{c}\right)^{-1}.$$

The difference between the two times is then

$$\Delta t := t_+ - t_- = \frac{4\pi R^2 \Omega}{c^2 - (R\Omega)^2} \approx \frac{4\pi R^2 \Omega}{c^2}. \tag{2.1}$$

where the approximation arises from $(R\Omega) \ll c$, surely valid for reasonable
values of $\Omega$ and $R$. This corresponds to a difference in round-trip optical
path

$$\Delta L = c \Delta t = \frac{4\pi R^2 \Omega}{c^2}. \tag{2.2}$$

The last equation can be re-casted in a general form, for arbitrary shape
and configuration (see, for example, [5]):

$$\Delta L = \frac{4\Omega \cdot A}{c}, \tag{2.3}$$

where $\Omega$ is the angular velocity vector and $A$ is the area vector, with mod-
ulus equal to the area enclosed by the light and direction perpendicular to
the surface of the interferometer. The optical path difference is directly pro-
portional to the angular velocity of the system (or, better, to the component
perpendicular to the surface of the interferometer).

The efficiency of the Sagnac interferometer is limited, except for high
rotation rate, because the optical path difference given by Eq. (2.3) is much
less than any typical optical wavelength. (For example, for typical value $R =$
1 m, $\Omega = 15^\circ/h \approx 7.3 \times 10^{-5}$ rad/s, approximately the rate of rotation of
Earth, then $\Delta L \approx 3 \times 10^{-12}$ m.) Different schemes can be used to overcome
this difficulty.
2.1.2 Fiber optic gyro

Fiber optic gyroscopes (or, briefly, FOG) directly exploit the Sagnac effect to sense rotation. They can increase sensitivity using a kilometre-long optical fiber as cavity, arranged in coils, greatly increasing the round-trip path of light. That is, generalising equation (2.2) for an arbitrary number of circular loop path, we can write the phase shift between counter-rotating beams as

$$\Delta \phi = \frac{2\pi \Delta L}{\lambda} = \frac{8\pi^2 R^2 N \Omega}{c \lambda},$$

where $\lambda$ is the wavelength of the radiation and $N$ is the number of coils. Since the length of the fiber scales as $N$, the phase shift due to rotation in a fiber optic gyroscope is directly proportional to the length of the fiber.

Fiber optic gyroscopes are commercially available and are widely used in navigation systems of planes and ships. However, they are limited by calibration and stability of the scale factor (the constant factor between the rotation rate and the measured phase shift in the last equation).

2.1.3 Active laser gyro

A different approach, the one we are interested in this work, involves the introduction of an active laser medium in the ring cavity, as shown in Fig. 2.2. That is a so-called active ring laser gyro (also known as RLG or, in this contest, simply as gyro). This way the optical path difference between the two counter-propagating beams of equation (2.3) translates directly into a frequency shift. In fact, the resonator can sustain laser action only if an integer
number of wavelength $\lambda$ fits in the (optical) length $L$ of the cavity. For a linear cavity, this modes are standing waves. In the ring laser, modes are oppositely directed travelling waves, that can lase at different amplitudes and frequencies. That is, the resonator frequency $\nu$ is given by

$$L = m\lambda = \frac{mc}{\nu}; \quad \nu = \frac{mc}{L},$$

with $m$ is an integer. In the ring laser gyro case, the frequencies of the two counter-rotating beams are

$$\nu_\pm = \frac{mc}{L_\pm}, \quad (2.4)$$

where $L_+$ and $L_-$ are the effective cavity length as seen by the CW and CCW beam respectively, $L_\pm = ct_\pm$. We have assumed that both beams lase on the same longitudinal mode (i.e., with the same integer $m$), as usually, but not always, happens. The frequency difference can be written as

$$\Delta\nu := \nu_+ - \nu_- = \frac{mc}{L_+} - \frac{mc}{L_-} = \frac{mc\Delta L}{L^2} = \nu\frac{\Delta L}{L},$$

where $\Delta L = L_+ - L_-$ and $L_+L_- = L^2$. Using equation (2.3) we obtain

$$\Delta\nu = \frac{4A \cdot \Omega}{\lambda L} \quad (2.5)$$

that we call the Sagnac frequency for an active ring laser gyro.

The Sagnac frequency can be easily measured observing the beat between the two counter-propagating beams, called Sagnac signal. The measure of a frequency is the biggest difference between active ring lasers and fiber optic gyros, that detect a difference of phase. (Note that a path with more turns does not increase the Sagnac frequency. This way there is no need to increase length of the cavity doing more coils.)

Equation (2.5) is sometimes re-written as

$$\Delta\nu = \frac{4A \cos \theta}{\lambda L} \Omega = S \Omega, \quad (2.6)$$

where $\theta$ is the angle between $A$ and $\Omega$. The role of the so-called scale factor

$$S = \frac{4A \cos \theta}{\lambda L} \quad (2.7)$$

is emphasized as conversion factor between rotation rate and Sagnac frequency.

Substituting appropriated values for G-Pisa and Earth rotation, or $\Omega = 15.0^\circ/h \approx 7.3 \times 10^{-5} \text{ rad/s}$, $A = 1.96 \text{ m}^2$, $L = 5.60 \text{ m}$, $\lambda = 633 \text{ nm}$ and considering the co-latitude in Pisa $\theta = 46.3^\circ$ (approximately the angle between $\Omega$ and $A$, since the laser is set on an horizontal plane), we expect a Sagnac frequency

$$\Delta\nu = 111 \text{ Hz}, \quad (2.8)$$
Figure 2.2: Basic schematics of an active ring laser gyroscope, as in the G-Pisa experiment. The Sagnac signal is obtained simply superimposing the two counter-rotating beams.

easily detectable and measurable by heterodyne techniques, even if it is less than $10^{-12}$ of the value of the optical frequency (red He-Ne lasers work at 474 THz).

2.1.4 Passive gyro

One last option exploiting the Sagnac effect is to inject an external laser in a ring cavity. The external laser is split in two counter-rotating beams whose frequencies can be shifted by acousto-optic mean. The cavity path length difference and thus the rotation rate, is determined locking the resonant frequencies of the cavity to the shifted frequency of the injected laser. Even this schemes measure a frequency and can be used to avoid some problem of RLG, as backscattering (see next section).

2.2 Further approximations

The elementary derivation of the Sagnac effect in the previous section fails to describe important features of ring laser gyros. We are concerned in particular to one thing, the source of any evil regarding not enough large RLG’s: backscattering. In any ring laser, part of the radiation of both beams is back-reflected in the counter-rotating direction, coupling the two waves. Backscattering may arise from dust, contaminant, and imperfection on surfaces of mirrors and optical elements (e.g., Brewster windows) or from dust in the air. This effect is enhanced by the presence of the active
Figure 2.3: Typical response of a ring laser gyro to rotation rate. The dashed line is the ideal performance. Real performance shows a blind lock-in region at low rotation rate. Similar figures can be found in the works cited in the text.

medium: if the difference between the two opposite side frequency resonance is much smaller than the gain linewidth, the backscattering can dominate the stimulated amplification.

For example, one of the most evident side effect of backscattering is the so-called lock-in effect. When the rotation rate causing the Sagnac effect is smaller than a critical value $\Omega_L$, the lock-in threshold, counter-rotating beams lase at the same frequency (are locked) and the gyro is blind to rotation. It should be noted that the lock-in threshold is usually given as a Sagnac frequency $l = S\Omega_L$: in fact this effect arises because of laser dynamics, no matter the biasing mechanism (rotation or other).

Moreover backscattering cause the departure of the observed Sagnac frequency from the ideal case of Eq. (2.5). A typical situation shows pulling of the Sagnac frequency, as in Fig. 2.3, that agrees well with the relation

$$\Delta \nu_{\text{obs}} = \sqrt{f^2 - l^2},$$

where $f$ is the ideal Sagnac frequency given by Eq. (2.5) and $l$ is the lock-in threshold.

Some papers dealing with these problems are, at different level of approximation and mentioning only a few, that of Aronowitz [4–6], Chow et al. [10], Menegozzi and Lamb [21], Stedman [32], Stedman et al. [33].

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1The term “lock-in” is well established in the literature of ring lasers, starting from [5]. It has nothing to do with lock-in amplifiers. We could generally speak of “locking”.

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Backscattering should be minimized. This is why RLG’s avoid interfaces at all costs: in the design of the G-Pisa and other high performance gyro lasers, for example, the mixture of helium and neon fills all the cavity (see chapter 3) and we do not use Brewster angle windows.

The next step should be considering the dispersion effects in the plasma of the discharge. Because the real and imaginary part of the dielectric function of the gas are related by the Kramers-Kronig relations \[19\], the (frequency-dependant) gain in the active medium is always accompanied by dispersion. However the effects of the backscattering are the dominant ones, at least in our experiment.

### 2.2.1 The backscattering equations

Consider the (complex) electric field in the cavity \(\tilde{E}_{\pm} = E_{\pm}(t)\exp\{i\varphi_{\pm}\) with

\[\varphi_{\pm} = \omega_{\pm}(t \pm z/c) + \phi_{\pm}(t)\]

where \(E_{\pm}\) and \(\phi_{\pm}\) are slowly varying function of the time (respect to \(\omega_{\pm}\)) and \(z\) is the position around the optical path. The frequencies \(\omega_{\pm}\) are given by Eq. (2.4) in term of the optical cavity lengths \(\omega_{\pm} = 2\pi mc/L_{\pm}\). When we are not interested in the small difference between \(\omega_{+}\) and \(\omega_{-}\), we refer to the optical frequency of the laser simply as \(\omega\), in our case, for a red He-Ne laser, 474 THz. The ideal Sagnac frequency \(f\) is given by Eq. (2.5),

\[2\pi f = \omega_{+} - \omega_{-} = \frac{8\pi A \cdot \Omega}{\lambda L},\]

It is expected to be, in our experiment, 111 Hz.

Phasors \(\tilde{E}_{\pm}\) obey an equation parametrized by the laser gain \(a\), saturation \(\beta\) and cross saturation \(\xi\). We add the contribution of backscattering through complex coefficients \(R_{\pm} = r_{\pm} \exp\{i\varepsilon_{\pm}\); each beam is back-reflected on the other at a rate \(\partial\tilde{E}_{\pm}/\partial t|_{bs} = R_{\pm}\tilde{E}_{\mp}\). Hence the equation for the phasor is

\[\frac{d\tilde{E}_{\pm}}{dt} = \left(i\omega_{\pm} + \pi a - \beta E_{\pm}^2 - \xi E_{\mp}^2\right)\tilde{E}_{\pm} + R_{\pm}\tilde{E}_{\mp}e^{i\varepsilon_{\mp}}. \tag{2.9}\]

The terms of pump \(a\), saturation \(\beta\) and cross saturation \(\xi\) are analogous to those introduced by Lamb for linear lasers [20]. They do depend in complex manner from the optical frequencies \(\omega_{\pm}\), in particular respect to the resonance of the active medium, giving dispersion corrections (e.g., frequency pulling and pushing). Expression for these parameters can be found in the work by Menegozzi and Lamb [21]. We suppose the laser dynamics to be dominated by backscattering and we neglect these complications. For example we expect, since the counter-rotating beams have different frequencies, that we should correct for the different dispersion (both normal
or anomalous near the atomic resonance) or for different gain. However the difference $f$ is much smaller not only of the optical frequency $\omega$, but also of the Doppler width of the atomic resonance and so could be neglected, at least in first approximation.

Taking the real and imaginary part of Eq. (2.9) gives the equations for the amplitudes and the phases (cf. [21, 33]):

$$\frac{1}{E_{\mp}} \frac{dE_{\mp}}{dt} = \left( \pi a - \beta E_{\mp}^2 - \xi E_{\mp}^2 \right) + \rho_{\mp} \cos (\psi \mp \zeta), \quad (2.10)$$

$$\frac{d\varphi_{\mp}}{dt} = \omega_{\mp} \mp \rho_{\mp} \sin (\psi \mp \zeta), \quad (2.11)$$

where

$$\rho_{\pm} := r_{\pm} E_{\mp}/E_{\mp}, \quad (2.12)$$

$$\zeta := (\varepsilon_+ + \varepsilon_-)/2, \quad (2.13)$$

are the backscattering amplitude and the net backscatter phase respectively, and

$$\psi := \varphi_+ - \varphi_- + (\varepsilon_+ - \varepsilon_-)/2 \quad (2.14)$$

is the relative beam phase (aside from a constant). The phase $\psi$ solves, simply using Eq. (2.11),

$$\frac{d\psi}{dt} = 2\pi f - \rho_- \sin (\psi - \zeta) - \rho_+ \sin (\psi + \zeta). \quad (2.15)$$

Extended discussion on the solutions of these equations can be found in [33]. For our purpose it is sufficient to make the approximation

$$\rho_+ = \rho_- =: \rho = \text{constant}, \quad (2.16)$$

albeit the amplitudes $E_{\pm}$ are themselves modulated at the Sagnac frequency. In practice, we are considering the phase dynamics independent from the amplitudes one. We obtain the simplified equation (cf. [5, 10, 21, 25, 33])

$$\frac{1}{2\pi} \frac{d\psi}{dt} = f - l \sin \psi, \quad (2.17)$$

$$l = \frac{\rho}{\pi} \cos \zeta. \quad (2.18)$$

Note that $\dot{\psi}$ is just the beat note between the counter-propagating beams, the quantity we are going to read experimentally.² This equation is typical of coupled oscillators and it is referred as Adler equation [1]. Albeit approximated, Eqs. (2.10), (2.17) and (2.18) are enough to explain most of the backscattering effects in ring lasers.

²We are still neglecting amplitude dynamics that, of course, contributes partially to the beat signal. See, for example, [33].
In absence of backscattering, $\rho = 0$ and Eq. (2.17) gives the same result of the ideal performance of Eq. (2.5). Otherwise, lock-in or pulling of the Sagnac frequency can be deduced. In fact the behaviour of the solution of Eq. (2.17) depends critically on whether $f < l$ or $f > l$. If $f < l$ a stationary solution exists, with $\psi = 0$, given by:

$$\sin \psi = \frac{f}{l}$$

(2.19)

In this case, since the phase is stationary and fixed, there is no beat note even if the rotation rate $\Omega \neq 0$. Moreover, since $-1 < \sin \psi < 1$, a stationary solution can exists only for $f < l$, hence, the frequency $l$ is the lock-in threshold.

When $f > l$ oscillatory solutions exist. Integration gives

$$\tan \frac{\psi}{2} = \frac{l}{f} - \sqrt{\frac{f^2 - l^2}{f}} \tan \left[ \pi(t - t_0) \sqrt{f^2 - l^2} \right],$$

(2.20)

where $t_0$ is a constant of integration. Thus the instant frequency $\dot{\psi}$ is oscillating, modulated by backscattering: the Sagnac signal is not simply sinusoidal. That is, the deformation of the signal reveals backscattering. However the average frequency, i.e., the Sagnac frequency we are going to measure, should be the frequency of the term $\tan \left[ \pi(t - t_0) \sqrt{f^2 - l^2} \right]$ of Eq. (2.20). This gives a pulled Sagnac frequency

$$p = \sqrt{f^2 - l^2},$$

(2.21)

as anticipated in Fig. 2.3.

This simple model predict only pulling of the Sagnac frequency (a observed frequency less than the expected one). More detailed analysis shows that, in some cases, also pushing (a increase in frequency) is possible [6].

Th backscattering also affects the amplitude of the two counter-rotating beams, through Eq. (2.10). It can be shown that the single amplitudes are modulated at the same pulled Sagnac frequency [33]. This intensity modulation of the single beam (often referred as monobeam modulation) is one of the most evident sign of backscattering. The phase between the modulation of the two beams is related to the phase $\zeta$.

Finally, we rewrite the last equations dropping the request $\rho_+ = \rho_-$ (but keeping the required constancy of both values). This does not add any new physics, but gives equations easier to use in numerical calculations. We can write, with a bit of trigonometry

$$\frac{1}{2\pi} \frac{d\psi}{dt} = f - l \sin(\psi + \gamma),$$

(2.22)

with

$$l = \frac{1}{2\pi} \left[ \rho_+^2 + \rho_-^2 + 2\rho_+\rho_- \cos(2\zeta) \right]^{1/2}$$

(2.23)
and the (inessential) phase $\gamma$ given by

$$\tan \gamma = \frac{\rho_+ - \rho_-}{\rho_+ + \rho_-} \tan \zeta.$$  

These last equations are very similar to the ones reported by Rodloff [25].

### 2.2.2 Interference between backscatterers

Up to this point we have not considered the physical origin of the backscatter coefficients $R_{\pm}$. Equations (2.12), (2.18) and (2.21) let us calculate the pulled Sagnac frequency, once we have $R_{\pm}$. Indeed the backscattering coefficient underlies some nontrivial physical mechanism.

The backscatter coefficients depend from all the various backscatter centers. In our experiment these backscatterers are practically only the mirrors since there are not any other optical elements in the cavity. Consider the contribution of the backscattering at some point in the ring, say $P$. The scattering center are labeled with $n$ and are at position $z_n$ from $P$. At the scatterer position, each beam is reflected on the other with a factor $A_{n\pm}$ (in general complex) and, in addition, a position-dependent phase shift $\exp \mp i\omega z_n/c$. This should propagate back to $P$, adding a further factor $\exp \mp i\omega z_n/c$. To a very good approximation we can use the average value $\omega$ evaluating these phases. The total phase shift is then $\exp \mp 2i\omega z_n/c$ and the total effect of backscattering is [28]

$$R_{\pm} = r_{\pm} e^{i\epsilon_{\pm}} = \sum_n A_{n\pm} e^{\mp 2i\omega z_n/c}.$$  

(2.24)

It should be noted that the phase part sees a change in sign between $R_+$ and $R_-$, contrary to what was reported in both [25, 33], and how can be deduced by the more rigorous procedure given by Menegozzi and Lamb [21]. This does not influence the theoretical considerations, but, for example, should be cared in numerical analysis.

The coefficients $R_{\pm}$ strongly depend on the distance between the scatterers. Basically we have interference, possibly constructive or destructive, between the reflected waves at various scatterers. This way the resonator geometry and the distribution of backscatterers is a strong source of systematic errors. As noted by Rodloff [25] this interference plays a tremendous role if the perimeter of the ring is stabilized moving a piezo; moreover, this mechanism explains why the performance of laser gyros often differs, even between instruments of the same manufacturer, under similar conditions.

It should be noted that the lock-in frequency $l = (\rho/\pi) \cos \zeta$, given in Eq. (2.18) seems to depend heavily on the phase $\zeta$. For example it can disappear when $\zeta = \pi/2$. However in this case a more complete treatment is needed and only a reduction of the lock-in threshold is expected [33].
Moreover most of the variation of $l$ with the $z_n$ comes from the interference term arising taking the absolute value of Eq. (2.24),

$$r_\pm = \left| \sum_n A_n \pm e^{\mp 2i\omega z_n/c} \right|,$$

even with $\zeta = \text{constant}$.

### 2.3 The helium-neon laser system

The He-Ne laser system is the most used in ring laser gyros. This choice arises mostly because of usability and ready availability of optical components at the 633 nm wavelength. There are other advantages. A ring laser can be easily designed so that the He-Ne gas fills all the cavity avoiding the need of interfaces or optical surfaces that can cause backscatter. Then, the plasma discharge are usually limited to one small part of the perimeter, in a narrow discharge tube. Another benefit can be obtained using an isotopic mixture of neon ($^{20}\text{Ne}$ and $^{22}\text{Ne}$ in equal concentration): the presence of both isotopes prevents competition between counter-propagating modes, assuring the good multi-directional capabilities needed for operating the gyro [4].

A simple explanation of the He-Ne laser system can be given following [34]. The famous transition at 633 nm or 474 THz corresponds to population inversion established between the neon 5s and 3p levels (for simplicity, we indicate the electron configuration of only the excited electron). The pumping mechanism, (in our case, free electrons accelerated by a radio-frequency electric field) bring helium atoms to a long lived metastable energy level, $2^1 S$ (for helium, we indicate principal quantum number, total spin, and total orbital angular momentum). The state He($^21S$) is nearly resonant with the Ne(5s), so there is a very efficient collisional pump mechanism on the neon 5s level. This is the dominant pump mechanism, albeit collision between neon atoms and electrons can contributes. After stimulated emission, the neon atom decay from the 3p state to the ground state mostly because collision with the walls of the discharge tube.

The gain-versus-frequency curve of He-Ne laser is determined by both homogeneous and inhomogeneous broadening. By the combination of the two depends many of the properties of He-Ne laser, as we shall briefly consider here.

#### 2.3.1 Gain profile

The homogeneous, complex susceptibility of an atom with resonant frequency $\omega_a$ for light at frequency $\omega$ is proportional to the Lorentzian profile

$$\mathcal{D}(\omega, \omega_a) = -\frac{\gamma_{ab}}{\omega - \omega_a + i\gamma_{ab}},$$  \hspace{1cm} (2.25)
where we denoted the half-width at half-maximum of the lorentzian atomic transition $\gamma_{ab}$. (The correct derivation of this equation is of course quantum-mechanical, but this result can be obtained also classically, see, for example, [19].) This homogeneous broadening comprehends natural and, mostly, collision broadening (due to pressure in the gas; see, for example, [8]).

However the gain curve of He-Ne lasers is dominated by the Doppler broadening. We can assume that the atoms of the active medium have a (normalized) Maxwellian speed distribution (along the perimeter of the ring)

$$W(v) = \frac{1}{u\sqrt{\pi}} e^{-v^2/u^2}, \quad (2.26)$$

where the parameter $u$ denotes the standard deviation and can be related to an effective temperature $T$ by the equation

$$\frac{1}{2}mu^2 = k_BT, \quad (2.27)$$

where $m$ is the atomic mass and $k_B$ is the Boltzmann constant. For our purpose we can neglect the small difference in width from the two isotopes arising from the slightly different masses of $^{20}\text{Ne}$ and $^{22}\text{Ne}$ when the effective temperature is fixed.

Because of Doppler effect, when an atom moving with velocity $v$ interacts with an electro-magnetic wave with frequency $\omega$ in the cavity, the frequency of the wave as seen by the atom will be the Doppler-shifted frequency $\omega' = (1 \pm v/c)\omega$, where the sign depends on the direction of propagation of light. Resonance occurs when the frequency seen by the atom $\omega'$ equals resonance frequency of the atom $\omega_0$.

Alternatively, the resonance frequency of an atom moving with velocity $v$ measured in the laboratory frame (as seen by the electro-magnetic wave) will be the shifted frequency

$$\omega_a = \omega_0 \left(1 \pm \frac{v}{c}\right) = \omega_0 \pm kv, \quad (2.28)$$

where, again, sign depends from the direction of propagation of light and $k = \omega_0/c$ (note that in a laser this is nearly the same as the wavenumber of the light). This way, the velocity distribution $W(v)$ translates directly in the Doppler, inhomogeneous, broadened distribution of the shifted resonance frequency

$$g(\omega_a) = \frac{1}{ku\sqrt{\pi}} e^{-(\omega_a-\omega_0)^2/(ku)^2}. \quad (2.29)$$

The field interacts only with atoms whose resonant frequency $\omega_0$ is properly Doppler-shifted to the field frequency; that is with atoms whose velocity is such that $\omega = \omega_0 \pm kv$, where the sign depends on the direction of propagation of light. The ensemble of atoms with a given such Doppler-shifted resonant
frequency $\omega_a$, plus or minus roughly one homogeneous width $\gamma_{ab}$ is the so-called spectral packet [31].

Another source of inhomogeneous broadening is the presence of two neon isotopes, with different resonant frequency.

Since both inhomogeneous and homogeneous broadening are equally present, the gain profile can be described in terms of the plasma dispersion function or Voight profile. Moreover, the competition effect between modes of the laser can be understand easily in terms of a hole-burning model (see Bennet [8]; see also the discussion by Lamb [20]). In the steady state of the laser the gain at the frequency of oscillation $\omega$ should equal the losses of the cavity. In an inhomogeneously broadened laser this condition is satisfied “burning a hole” in the gain curve. The width of the hole is given by the homogeneous width $\gamma_{ab}$. A cautionary remark, talking about two isotopes, should be made: the population-vs-velocity curve of each isotope is Doppler broadened and we can take them to be Maxwellian (gaussian) distribution with zero mean; of course, there is no meaning in the sum of the two, while we can sum the gain-vs-frequency curves (that have different centers). It should be noted that the hole are in first instance burnt in the population-vs-velocity curve: only atoms in the spectral packet whose resonant frequency is the same as the frequency of the electro-magnetic field interact, that is, only atoms with a given velocity. It is the saturation of the population inversion of the packet interacting with the field that give rise to the hole in the curve of gain versus frequency.

To be concrete, in a typical He-Ne ring laser gyro the Doppler width, for each isotope, is roughly $ku \approx 2\pi \times 1.5\, \text{GHz}$. Pressure and homogeneous broadening are slightly higher in ring laser gyro than that of commercially available He-Ne lasers: the homogeneous one could be around $\gamma_{ab} \approx 2\pi \times 100\, \text{MHz}$, say due to pressure ranging from 5 hPa to 6 hPa. However, is not terrible to consider the case of “Doppler limit,” i.e., $\gamma_{ab} \ll ku$. (This assumption let us uses the results obtained with this common approximation found in several papers [4, 8, 20, 21].) A large homogeneous broadening facilitate the laser working single mode. The homogeneous broadening could be greater that the free spectral range (FSR) of the ring, $\gamma_{ab} > \Delta \omega_{\text{FSR}}$. This way the hole burned by one mode reduce heavily the gain also for the nearest adjacent one and thus there is a strong competition. This is the desired behaviour because it helps single-mode operation [16]: laser gyro are usually designed to operate with just one mode in each direction. This is the picture described in the previous sections. High pressure is essential, since the FSR is usually much shorter (we are interested in large gyro) than most common He-Ne lasers and mode cleaners would be source of backscatter. The desired single mode operation is achieved simply working at low power levels, near laser threshold.
2.3.2 Effects of two isotopes

The separation of the resonant frequencies \( \omega_0 \) of \( ^{20}\text{Ne} \) and \( \omega'_0 \) of \( ^{22}\text{Ne} \) is \( \omega'_0 - \omega_0 \approx 2\pi \times 850 \text{ MHz} \). The total gain profile is the sum of the gain-versus-frequency curves of the two isotopes, weighted with the concentration of each neon specie. Thus, with 50:50 isotopic composition and considering the Doppler width as above \( ku \approx 2\pi \times 1.5 \text{ GHz} \), the gain profile is single-peaked, but appears rather flat.

With only one isotope present, lasing occurs near the maximum of the gain curve, that is near the resonant frequency of the sole isotope. In this situation, electro-magnetic waves moving in both directions interact with the spectral packet with zero velocity, and thus they heavily compete. This competition may suppress one of the waves, eliminating the Sagnac signal and making the laser unable to sense rotation.

On the other hand, when both isotopes are present, lasing occurs approximately halfway of the resonant frequencies of the two species. A schematic representation of gain-vs-frequency curves for both beams and of population-vs-velocity for the two isotopes is shown in Fig. 2.4. So, for example, the counter-clockwise (CCW) beam interacts with \( ^{20}\text{Ne} \) atoms moving clockwise, and with \( ^{22}\text{Ne} \) atoms moving counter-clockwise because of Doppler effect. For the clockwise (CW) beam is just the opposite, i.e., the two beams interact with different velocity packets. Atoms with zero-velocity do not lase. It should be noted that each beam do burn holes in the gain curve of the counter-propagating one, symmetrical respect to the transition frequencies \( \omega_0 \) and \( \omega'_0 \). These holes are on the tail of the curve and do not affect lasin action. This way the competition between the counter-rotating beams is very weak.

In this picture the Sagnac shift \( f = 110 \text{ Hz} \) between the counter-rotating beam plays no role, since the Sagnac frequency is much smaller not only of the optical frequency but also of the homogeneous and inhomogeneous broadening. Basically we can write \( 2\pi f \ll \gamma_{ab} \ll ku \ll \omega \).
Figure 2.4: Gain profiles and velocity distributions with holes burnt, two isotopes case, both CW and CCW beam single mode. Blue: holes burnt by the CCW beam. Red: holes burnt by the CW beam. Doppler width and separation of resonant frequency are realistic; not so the hole width that is much smaller in the figure for clarity. Also the depths of the holes are exaggerated.
Experimental work done with a square ring laser gyro with side 1.40 m in the context of the experiment G-Pisa of the INFN is presented in this and in the next chapters. The laser is located in a basement laboratory at the INFN of Pisa. Stainless steel tubes form the square cavity. G-Pisa gyro is a helium-neon laser working on the red 474 THz (632.8 nm) line of neon; the plasma discharge is excited in a small portion of the perimeter, a small Pyrex tube halfway one of the side. Earth rotation is enough to bias the Sagnac signal of the gyro to 111 Hz, beyond lock-in threshold. Figure 3.1 shows pictures of the experiment and of the discharge tube. Table II summarizes some parameters of the gyro laser.

This chapter describe the experimental setup in detail, dealing with the mechanical setup, details of the cavity and peculiarity of the discharge. The optical setup used to test several characteristics of the gyro are also described. Chapter 4 will collect some of our results with the laser free-running. Chapter 5 will describe instead our effort in the active stabilization of the gyro.

### 3.1 Mechanical setup

Four high-reflectivity mirrors (supermirrors) form a closed square beam path with side length of 1.40 m. The whole cavity is enclosed by a stainless-steel modular structure: four corner turrets contains the mirror holders and are connected by tubes along the sides (see the drawing in Fig. 3.2). This chamber is completely filled with a mixture of neon and helium. This minimizes interfaces and optical surfaces than can cause backscattering (section 2.2). The mirrors are rigidly connected to the turrets and can be tilted, to align
Figure 3.1: Photos of the G-Pisa gyro laser: (a) the square ring on the table (b) detail of the Pyrex tube and of the discharge, monitored by two optical fibers.
**Table II:** Summary of the main features (nominal values) of the G-Pisa gyro laser.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>43.7°</td>
</tr>
<tr>
<td>Configuration</td>
<td>Square 1.40 m × 1.40 m</td>
</tr>
<tr>
<td>Perimeter</td>
<td>5.60 m</td>
</tr>
<tr>
<td>Area</td>
<td>1.96 m²</td>
</tr>
<tr>
<td>Wavelength</td>
<td>632.8 nm</td>
</tr>
<tr>
<td>Sagnac freq.</td>
<td>111 Hz</td>
</tr>
<tr>
<td>Free spectral range</td>
<td>53.5 MHz</td>
</tr>
</tbody>
</table>

**Figure 3.2:** Drawing of the mechanical part of the gyro laser. On the right side is shown the discharge tube.

The cavity, by means of levers controlled by micrometric screws (each lever moves, slightly, all the corner and not only the mirror inside the structure). Bellow tubes at the connections provide the necessary freedom of movement to the structure under the small change needed in the alignment. The whole mechanic is attached to a stainless-steel optical breadboard.

Unfortunately, the floor of the laboratory is a floating structure. We employ compressed air dampeners under the table in the attempt to isolate the gyro from the floor.

Halfway of one of the side there is a Pyrex component long approximately 15 cm: a capillary of internal diameter 4 mm where the plasma discharge is excited (Fig. 3.1b). In this glass section the gas mixture is excited by two electrodes by capacitive coupling of radio-frequency (RF) power, as described later. Also this section can be adjusted a little thanks to bellow
tubes.

The corners (and the mirrors with them, of course) are numbered clockwise from 1 to 4, starting from the corner before the discharge tube. Each corner has two windows from which the clockwise (CW) and the counterclockwise (CCW) laser beams can exit. We can refer to these exits along with the number of the corner and so speak of the CCW window of the corner 1 or the CCW 1 window. Support plans are mounted along with corners 1, 2 and 3 to mount any optical elements necessary outside the ring cavity. These support plans are rigidly connected with the corner turrets. This way the alignment of the external elements are easily preserved.

The mirror number 4 can be moved by a piezoelectric transducer (PZT) (and thus misses the plans for support of optical elements). Like the levers, the piezo is external to the cavity and acts on the whole corner turret (movement allowed by the bellows). The piezo may be used to move the mirror along the diagonal of the square cavity, controlling the length of the beam path perimeter. This piezo will be used for the optical frequency stabilization of the laser, see section 5.4.

Near the corner number 4, a junction with a valve connects the gyro chamber with an external tank (the reservoir), used in the filling operations. The reservoir and the chamber can be evacuated through a oil-free membrane pump and a turbo-molecular pump or replenished by helium and neon gas cylinders. Pirani gauges monitor gas pressure in the gyro chamber and in the reservoir (between $10^{-2}$ Pa and $10^5$ Pa). During evacuation, high vacuum (less than $10^{-2}$ Pa) is monitored by a Penning gauge. Between the reservoir and the gyro there is a small auxiliary chamber housing a getter pump, a little toroid with diameter of 2 cm. The getter (SAES getter 5K0621) is used to keep the gas mixture clean from contaminants, especially hydrogen (see later section 4.2). Figures 3.3 and 3.4 show the schematics and a picture of the vacuum setup. This setup let us:

- evacuate or refill everything;

- evacuate or refill the gyro cavity with the getter chamber closed (for example during baking of the cavity);

- evacuate the getter chamber with the gyro chamber closed (during baking or reactivating of the getter);

- have the getter in communication with the gyro chamber to remove hydrogen with the reservoir and the pumps closed (standard operation).
Figure 3.3: Schematics of the vacuum setup.

Figure 3.4: Photo of the reservoirs, of the valves and of the turbomolecular pump. On the right there is corner 4.
3.2 Laser cavity

The multilayer dielectric supermirrors have a nominal reflectivity of 0.999 99 (so-called five-9s mirrors). All mirrors have a 6 m radius of curvature.

The cavity is highly astigmatic, since there is an obvious difference between the in-plane and out-of-plane directions. The beam waist is thus elliptical: the horizontal beam waist radius is 0.69 mm, the sagittal one 0.56 mm (intended at $1/e^2$ of the intensity), how can be calculated numerically by matrix optics (see, for example, [31]).

Longitudinal mode in cavity, with a perimeter length of $L = 5.60$ m, are separated by a free spectral range (FSR)

$$\Delta \nu_{\text{FSR}} = \frac{c}{L} = 53.5 \text{ MHz}.$$

A measure obtained with a Fabry-Pérot interferometer (section 3.5.2) resulted in $\Delta \nu_{\text{FSR}} = (53.66 \pm 0.07) \text{ MHz}$ in good agreement with the expected one.

Usually both beams lase only on the pure gaussian TEM$_{00}$ transverse mode. We have observed TEM$_{10}$ with the mirrors not well aligned. Higher order modes are not observed simply increasing the power (their gain is limited by the narrow capillary tube of the discharge). Running the laser gyro in TEM$_{00}$ is desirable, but not essential. For example the G ring laser located in Wetzell, Germany, works on TEM$_{20}$ mode, without particular disadvantages [16]. In the rest of this thesis we assume a single, common, transverse mode is achieved on both beams.

The laser beams are linearly polarized, with polarization perpendicular to the plane of the laser, because of the much higher reflectivity of the mirrors for out-of-plane instead of in-plane polarization. This corresponds to the well known different reflectivity for transverse electric (TE) or transverse magnetic (TM) incidence on dielectric interfaces (see, for example, [19]).

3.2.1 Alignment

Alignment of the cavity can be searched with the help of a green laser externally injected from the CCW window of corner 3. First, the green laser is aligned to pass through the centers of the inner faces of mirrors 2 and 3. Then, the mirrors are tilted to close the green laser beam path on itself. The position of the discharge tube can be adjusted to not obstruct the green beam. In the meanwhile the discharge of the gyro laser is kept switched on at high power, so, once the alignment is good enough the laser

---

1The distribution of light in a laser beam is usually described by Hermite-Gaussian functions, referred as transverse electromagnetic modes (see, for example, Siegman [31]). We indicate only the transverse part as TEM$_{nm}$, where the first index $n$ is the order of the in-plane mode, while $m$ is the order of the out-of-plane mode.
action starts. This procedure is usually needed only when the mirrors are removed and rehoused in their turrets, for example after cleaning. Once the laser is running we can optimize alignment checking the position of the red laser spots on the mirrors (ideally, they should be in the centers, a condition reached only approximately). Every day, then, the mirrors are slightly moved so to maximize the laser intensity or to minimize the laser threshold.

Since the external optical elements are fixed at support plates, in turn fixed at the corner structures moving with the mirrors, their alignment is robust and need to be adjusted only occasionally.

3.2.2 Lock-in threshold estimate

An interesting estimate of the lock-in threshold frequency \( l \) (see back section 2.2) can be obtained following [32]. In modern ring laser gyro, without any intracavity element, backscattering arises from mirrors. We assume that, at each reflection, a fraction \( s^2 \) of the intensity of the beams will be scattered. For a supermirror \( s^2 \) is expected to be of the order of \( 10^{-6} \). In a ring with perimeter \( L \) and with four mirrors these reflections occur at a rate \( 4c/L \). For simplicity sake we assume the scattering to be uniform, so that only a fraction \( d\Omega/4\pi \) is scattered into the other beam, where \( d\Omega \) is the solid angle seen by the beam. This angle can be approximated as \( d\Omega = \pi \theta^2 \), where \( \theta \) is given by diffraction \( \theta = \lambda/d \); \( d \) is the beam diameter at the waist, it is approximately 1 mm and it is weakly dependant on the dimension of the ring. So the rate of backscattering goes as \( r \sim cs\lambda/(dL) \). Then the lock-in threshold, Eq. (2.18), becomes, using appropriate values for our experiment,

\[
l \sim \frac{cs\lambda}{\pi dL} \approx 10 \text{ Hz} \tag{3.1}
\]

which is of the same order but lower than the expected Sagnac frequency \( f = 111 \text{ Hz} \) of Eq. (2.8). This estimate does not consider the interference of backscattering and should be considered an upper limit. In our experiment Earth’s rotation is enough to bias the two frequency beyond lock-in threshold.

Equation (3.1) makes evident that lock-in and backscattering affect small rings, and their effects are progressively reduced for larger rings. This fact is emphasized because the Sagnac frequency scales as the perimeter of the laser, in particular to the ratio between area and perimeter. Small laser gyro (up to a perimeter length of 60 cm) used for navigation are unable to detect Earth’s rotation and need to be dithered to avoid lock-in [10]. Backscattering is the dominant mechanism in ring lasers as large as the one studied in this thesis (with side length of about 1 m). Larger gyros are less affected as, for example, G 4 m × 4 m ring laser, that shows no monobeam modulation [35] or UG-2 (40 m × 20 m) [17].
3.2.3 Cleaning of the mirrors

Dust and contaminants on the surfaces of the mirrors increases losses, are source of backscattering and contributes to the lock-in threshold of the Sagnac frequency. All of these phenomena affect the performance of the gyro laser. It is therefore important that mirrors be cleaned, a procedure that is done with the maximum care. (It should be noted that the previous estimate is for cleaned supermirrors.)

After the mirrors are removed, they are in special boxes. Then they can be transported in clean room. Here they can be examined observing the light scattered from the surface of the mirror by a 5 mW He-Ne laser, incident at 45°. The surface can be searched for defects, scratches or scatterers with an optical microscope. To clean the surface, each mirror is washed with methanol and then rinsed with distilled water. This is followed by spray-washing with isopropyl alcohol (it removes water and prevents condensation on the surface). The mirror is then dried using a towel moistened with isopropyl alcohol, letting it adheres completely to the surface. The mirror can be rechecked again at the microscope and the procedure can be repeated as needed.

3.3 Capacitive discharge

The capacitive coupled discharge is the flagship of the G-Pisa experiment. A capacitor made by two (semi)cylindrical electrodes (length 5 cm) surrounds the Pyrex tube of the laser. This capillary has an internal diameter of 4 mm. No electrodes are required inside the tube. A radio-frequency power source (a voltage controlled oscillator or VCO) is used to power the capacitor and thus the discharge in the gas in the tube. However, the plasma itself intervenes in the capacitance of the system and thus any variation in the plasma (with temperature, gas mixture, density, power absorbed) contributes to change the response to the RF power given by the VCO [14]. To overcome this problem and to obtain impedance matching, the voltage controlled oscillator can be adjusted both in amplitude and frequency. Excitation frequency follows the self-oscillation of the system, adapting to any condition of the plasma. It is usually 130 MHz. If the plasma condition changes, the excitation of frequency changes but not the coupling and the power supplied—at least in first approximation. Amplitude control is used to vary the power of the laser. An integrated circuit takes care of amplification. The amplified VCO output is separated and send, in counter phase, to the two capacitor plates. This RF discharge is designed to be symmetrical, avoiding any non-reciprocal effects (e.g., Langmuir flow; see, for example, [5]) that can bias the Sagnac frequency. (That is why it is preferred to direct current, DC, excitation.) A water cooling system takes care of heat dissipation.

A very stable power supply (Hewlett-Packard 6543A DC power supply)
Figure 3.5: Detail of the discharge Pyrex capillary. A square plastic box surrounds the two electrodes. Two optical fibers monitor the discharge glow. The metal box under the tube contains most of the electronics.

sustains the electronics. A multimeter monitors the (rectified) voltage at one of the capacitor plates.

To control the gas quality and to check for contaminants, the spectrum of the discharge fluorescence is monitored frequently. Spectra were obtained using a Acton Research Corporation SpectraPro -300i spectrometer looking the discharge glow with an optical fiber. Spectra are always recorded with the laser near threshold.

As needed, for further diagnosis, a multimode optical fiber looks the intensity of the glow. This can be monitored by a photomultiplier.

The capacitive discharge replaces the previous inductive discharge. In the capacitive discharge, the plasma contributes to the capacitance that determines the resonant frequency of the circuit and thus there is always impedance matching. This is not the case of the inductive discharge, where every little variation can change the power supplied to the gas. The capacitive discharge is, then, more stable that the inductive one and, moreover, irradiates less.

3.4 Active medium: gas mixture

Let us take a closer look to the active medium, on which many of the properties of our laser depend. The total pressure of the gas mixture ranges from 5.0 hPa to 6.0 hPa, much higher than usual for He-Ne lasers, in or-
der to facilitate single mode operations as discussed before in section 2.3.1. G-Pisa laser is typically filled with a 1:25 (roughly) mixture of neon and helium. This is an unusual ratio for He-Ne lasers (that uses much less helium), because we are not interested in high gain. Exact ratio can vary slightly between different fills of the gyro tank.

Neon supply is 50:50 mixture of two isotopes: $^{20}\text{Ne}$ and $^{22}\text{Ne}$. (The presence of both isotopes grants the two counter-rotating beams to be practically uncoupled, see section 2.3.1.) We currently use a specially produced isotopic neon sample, whose composition is well known. For the future we plan to use two separate supply of $^{20}\text{Ne}$ and $^{22}\text{Ne}$, mixing them in the desired ratio at hand during the filling of the cavity.

### 3.5 Optical setup

Several optical elements are used to analyze the gyro laser output. The main one is, of course, the readout of the Sagnac signal at corner 3. There are many others. The windows CW 1, CW 2 and CCW 2 are prepared with optical fiber couplers, mounted on shifters (see, for example, Fig. 3.6). The fibers can be aligned with the laser and then used with whatever instruments is necessary. In particular we shall describe here the setup used to investigate the mode structure of the gyro laser: one Fabry-Pérot analyzer (FP) and the beat of one of the beam with a reference laser (thus checking the absolute frequency of the gyro).
All optical fibers used are single mode, with angled physical contact (APC) to avoid backreflections.

As photodetectors we use mostly some Hamamatsu H5783 photomultiplier tube (the gyro beams are very weak, around 1 nW when there is only a mode in each direction). They can be directly connected to the fibers, for example for checking the intensity of one beam. Their gain can be controlled with a potentiometer.

3.5.1 Readout of the Sagnac signal

As already noted, the Sagnac signal can be simply obtained by heterodyne techniques, letting the counter-rotating beams beat.

The most reliable way to obtain Sagnac signal in G-Pisa is superimposing the two beams in air, using a polarizing beam splitter cube (Figs. 3.7 and 3.8). This is done with the CW and CCW beams exiting from corner 3. The cube is carefully aligned to combine the beams approximately with the same intensity. Beat note is recorded by a photodiode (PD) loaded on a transimpedance amplifier. A box surround all the optical components preventing leakage of room light on the PD.

The transimpedance amplifier is a Keithley 427 current amplifier. It can provide a $10^9$ V/A gain with a rise time of 1 ms. It grants both the amplification and the fast response needed to detect the Sagnac signal of the weak output of gyro laser (around 1 nW).

We usually employ this setup, most of the Sagnac signal data analyzed for this thesis use it. However in the attempt to obtain a faster response and improve the detection of the Sagnac signal we tried a variant using two photomultiplier tubes (PMT). In this alternative setup the beat note is obtained through an optical fiber coupler (X-shaped) at the exit of corner 2 (that is already prepared with optical fibers couplers). At both ends of the coupler we use the photomultiplier tubes to detect the signals. This approach let us perform a balanced signal detection: since the two beam are superimposed in counter-phase in the two arms of the coupler, subtracting the two signals of the PMT’s we can reject the common mode and increase the signal-to-noise ratio. Photomultipliers are current sources and need to be loaded on resistors, in our case we load them on 100 kΩ resistors. This value is enough to obtain a signal around 1 V. This setup performs better than the first one, especially at high frequency (PMT’s are faster detectors than the photodiode), but fibers are very sensible to acoustic noise.

3.5.2 Fabry-Pérot setup

The modes of the gyro laser are recorded from a confocal scanning Fabry-Pérot spectrometer as schematically represented in Fig. 3.9. The Fabry-Pérot (a Coherent 216-c) has a free spectral range $\Delta \nu_{\text{FSR}} = 300 \text{ MHz}$. The
Figure 3.7: Schematics of the readout of the Sagnac signal: CW, clockwise beam; CC, counter-clockwise beam; M corner supermirror; RM, mirrors used for beam-steering; BS, 50:50 beamsplitter; PD, photodiode.

Figure 3.8: Readout of the Sagnac signal in air using a beam-splitter cube and a photodiode around the corner 3. Usually this setup is closed in a box to prevent leakage of light on the photodiode.
Fabry-Pérot is slightly misaligned to avoid feedback. Both counter-rotating beams are coupled with single-mode optical fibers. For this purpose we use the CW 1 window and CCW 2 window; neither fibers looks directly the discharge glow. The Fabry-Pérot analyzer collects the light of both fibers by mean of an optical fiber coupler at one of its ends. A photomultiplier tube is located at the other end. The length of the Fabry-Pérot analyzer is slowly scanned (with a swept time of roughly 100 ms), making easier the detection of the low power gyro laser. The signal of the photomultiplier is further amplified (using a Tektronix AM S02 differential amplifier) before being sent to the oscilloscope. The amplifier has a low-pass filter rejecting noise with 3 dB point at 300 Hz. Other noise is eliminated letting the oscilloscope average several scans. Spectra are usually recorded in dark room, since light can leak in the interferometer.

This setup let us investigate both counter-propagating beams at the same time. Then, the source of each peak in the spectra can be discriminated shutting one or the other of the beams at the entrance of the fiber. The Fabry-Pérot finesse is too low to detect the Sagnac effect between the two beams.

3.5.3 About Bob

We can compare the optical frequency of the gyro with that of a reference laser, or lambda-meter. Before we turn to the description of the setup used to investigate the optical frequency of the gyro, we briefly describe here the behaviour of the frequency-stabilized laser at our disposal (called Bob). This will play a role in the stabilization of the perimeter of the gyro we are going
The lambda-meter is a He-Ne laser with internal-mirrors. It exploits a simple stabilization method originally proposed by Balhorn et al. \[7\].

The Doppler width of the neon emission line (474 THz) is about 1.5 GHz. Longitudinal modes in the laser cavity are separated approximately by a free spectral range (730 MHz). Thus, near central tuning, Bob supports only two axial modes within the Doppler width. Usually, these two modes are orthogonally polarized and their directions of polarization remain fixed.

Frequency stability is obtained by locking the cavity length to keep the difference of the intensity of the two modes constant. The two orthogonally modes can be separated by a polarization divider and their intensities are detected by two distinct photocells. The difference of the output of the photocells depends on the frequency of the two modes. It crosses zero when the modes are symmetrical respect to the gain curve (assuming the emission profile symmetrical, see Fig. 3.10). The cavity length is controlled by a heater driven by the difference signal. Basically, the difference is used as the error signal for frequency stabilization and no modulation is needed.

The laser output is single-mode, since a polarizer selects only one of the two modes. The two polarization modes can interchange position on the gain profile. So Bob can operate at two lock point, corresponding to the frequency of the output beam on the red side or blue side of the Doppler profile. That is, the frequency of the laser output change by 730 MHz when the locking position is switched. Lock point can be chosen via a switch, since only a change of sign in the error signal is involved.

It is worth noting that Bob does not contain any intra-cavity elements to define polarization. The polarization eigen-mode of the laser is permanently
fixed by the strain in the coatings of mirrors [11]. (Modern improved tech-
niques in mirror coating avoid this strain, and new, improved, mirror can
not be used in this type of lasers since the polarization direction would be no
longer stable.) The mirrors are individually assembled to the tube without
considering the strain direction, that is not easily identifiable. Nevertheless,
when the laser is started, the first mode to start will do so according to
some minimum loss resulting from the strain in the mirrors. Perpendicular
polarization of the second mode is then observed since it minimizes the gain
competition.

Short-term and long-term stability of polarization-stabilized lasers has
been reported by Niebauer et al. [22]. The short-term stability is very good,
better than 1 part in $10^{10}$ over about 1 h (a typical plot of the Allan deviation
of this kind of laser is shown in Fig. 3.11). The average of the frequency
of the red and blue lock point is expected to be more stable than either side
lock (the blue frequency should increase while the red one should decrease).
A typical center frequency drift of 4 MHz/a has been reported, correspond-
ing to a long-term stability better than 1 part in $10^8$ over one year. The
stabilization of the G-Pisa laser developed later in chapter 5 exploits only
one of the side lock, whose drift is approximately 10 MHz/a.

3.5.4 Absolute frequency investigation

As a further tool, we investigate the absolute optical frequency of the gyro
laser as shown in Fig. 3.12. The beat note between the clockwise beam
and Bob (see, later, section 3.5.3) is obtained using another optical fiber
Figure 3.12: Diagram of the setup used to check the absolute optical frequency of the gyro.

coupler and observed with a photomultiplier tube and a digital microwave signal analyzer (IFR Spectrum Analyzer 2399; it works between 9 kHz to 2.9 GHz). The fiber looks the clockwise beam through CW 2 window. The photomultiplier is loaded on a 100 kΩ resistor (this value of resistance is useful to check the gain of the PMT with an oscilloscope; however notice that the input impedance of the spectrum analyzer is 50 Ω, so, if the PMT is directly wired to the analyzer it is loaded on 50 Ω instead). A similar setup will be used in the frequency stabilization of the gyro, as we shall describe in section 5.4.

A neutral filter is used to scale down the intensity of the stabilized laser (whose power is 1 mW) to balance the gyro laser beam (whose power is about 1 nW). Usually, since the intensity of the beat note scales as the squared root of the product of the two intensity, the more intense the stabilized laser, the better. However we should not overload the PMT with too much light nor lose the beat signal under too much background light (that is, the intensity of the more powerful laser). In the best setup we have found the intensity of the stabilized laser (as read by the PMT) is about ten times the intensity of the gyro.

There are other reasons to scale down the intensity of the stabilized laser. Without neutral filter, a lot of power is scattered back at the entrance of the optical fiber in the laser itself: this effect can be so strong to cause the reference laser to lose the frequency stabilization. Moreover the laser light is scattered around in the coupler. Even with the filter a relevant portion of light is injected back to the gyro. Here it does not affect the gyro lasing mechanism (it has a slightly different frequency) but it is reflected by the rear of the supermirror, from CW 2 window to CCW 2 window, in the adjacent fiber and in other instruments. This cross-talk prevent the usage of instruments on CCW 2 window, when the stabilized laser is used to check the frequency of the beam exiting CW 2 window.

Using a optical fiber coupler is not the best setup for recording a beat
note. With the coupler we have not control over the polarization of both lasers, since any information about polarization is lost inside the optical fiber, whose length is of several centimetres. On the other hand a setup for recording the beat “in air” would be much more difficult (again, because the gyro laser is very weak). The optical fiber setup can be operated with the lights on, maintain alignment for prolonged time (the optical coupler moves with the mirror of the gyro), etc. However, this setup is certainly not the ultimate one and we can expect improvements in the future. For example, we plan to use an optical isolator at the entrance of the gyro. It is important that the isolator be very good, with minimal losses, because the gyro laser is very weak.

3.6 Data acquisition

The Sagnac signal can be acquired by an ADC board (16 bit) with a LabVIEW program running on a personal computer. The Sagnac frequency (or the phase) need then to be reconstructed from the signal, in order to get rotational information. This can be done in real time or, more often, off-line.

The Sagnac frequency is reconstructed in real time using an autoregressive second order algorithm. It assumes the signal to be a pseudo-sinusoidal signal with lorentzian lineshape so it need a narrow-band (20 Hz) filter around the expected frequency. Further, a frequency counter can also be used to read the Sagnac frequency.

For off-line reconstruction of the phase of the signal we use an elegant algorithm based on the Hilbert transform [33]. This method has a larger bandwidth than the autoregressive algorithm and it is used usually with an acquisition rate of 3 kHz. Velikoseltsev [35] gave a comparison of various method to obtain the Sagnac frequency.
Preliminary measures

In this chapter we present some measures characterizing the operation of the gyro laser: measurement of ring-down time, from which we can deduce the quality factor of our cavity; study of the contamination of the He-Ne mixture by hydrogen, that limits the working time of the gas; investigation of the mode taxonomy exhibited by the laser, both single and multimode.

4.1 Ring-down time measurement

Monitoring the quality factor of the cavity is an interesting test of the performance of the gyro. The mirrors are an important factor, for instance because better mirrors cause less backscattering. Some information can be obtained measuring the ring-down time $\tau$ of the laser. This is not an easy task: we can not simply turn off the power supply (the Hewlett-Packard 6543A), since it is specifically designed to turn off slowly. Moreover the capacitive coupling of the discharge has a slow and complex response and the plasma intervenes in the capacitance of the system. We adopt this solution: the discharge capacitor is short-circuited and the glow of the plasma fluorescence is monitored as well. The short circuit is simply performed touching with a screwdriver two bare contacts on the wires connecting the two armatures with the rest of the discharge circuit. Both the laser and the fluorescence are monitored by optical fiber coupling with two photomultiplier tubes (PMT). The photomultiplier tubes are loaded on $10\,\text{k}\Omega$: their time response is much less than the ring-down time we are going to mea-
Nevertheless we care that both instruments work with the same gain, since this can influence time response. Besides, we occasionally swap the two photomultipliers checking for differences. The signal of both PMT’s are recorded with an oscilloscope.

We performed this test with the gyro in its best shape (new gas, cleaned and carefully aligned mirrors, etc.). An example of the data obtained is shown in Fig. 4.1. The fluorescence decays first, driven by the residual RC circuit of the discharge. Then the laser follows. In the first microseconds the decay of the laser is driven both by the optical cavity and by the discharge. Nevertheless a rough estimate of the ring-down lifetime $\tau$ can be obtained by the tail of the curve, after the fluorescence had reached zero and the decay is, in good approximation, exponential. In fact a simply exponential decay will not fit all the data. To obtain a more accurate result, fitting all the data, we adopt the following model.

The laser dynamics, in particular the electric field amplitude $E$ can be described, at first order, neglecting saturation, by the equation (see, for example, Lamb [20]):

$$\frac{dE}{dt} = \frac{1}{2} \left( -\frac{\omega}{Q} + G \right) E,$$

where $\omega$ is the optical frequency, $Q$ the quality factor of the cavity and $G$ describes the gain of the active medium. We are not interested here in the complicated dependency of the gain $G$ on the frequency of the laser and other parameters. The term $1/\tau = \omega/Q$ characterizes the losses of the cavity, and it is the exponential decay time of the intensity $I = E^2$ when the gain is abruptly set to zero ($I \sim \exp(-t/\tau)$). The stationary solution $\dot{E} = 0$, instead, gives $G = 1/\tau$.

We assume that Eq. (4.1) stands also for gain variable with time. We assume the stationary condition is reached when the gain starts to decrease, at time $t = 0$, with an exponential-decay time constant $\sigma$, $G(t) = (1/\tau) \exp(-t/\sigma)$. Then we obtain:

$$\frac{dE}{dt} = \frac{1}{2\tau} \left( \exp(-t/\sigma) - 1 \right) E.$$

Solving the last equation gives the time dependency of the intensity of the laser:

$$I(t) = I_0 \exp \left\{ \left[ 1 - \exp \left( -\frac{t}{\sigma} \right) \right] \frac{\sigma}{\tau} - \frac{t}{\tau} \right\},$$

where $I_0$ is the initial intensity. We recover the usual exponential decay for $\sigma \ll \tau$ or for $t \gg \sigma$. This expression is used to fit the data of the laser decay, with the value of $\sigma$ as parameter deduced by the decay of the

\footnote{Time response of the PMT (Hamamatsu H5783) only is 0.78 ns, as declared by the manufacturer. In our experiment the limit is instead given by the load resistor and the residual capacitance of coaxial cables. With a similar setup, with a load of 100 k\Omega we are able to detect signals of hundred of megahertz, section 5.4.3.}
Figure 4.1: Example of the ring-down time measurement obtained with an oscilloscope. Both the laser decay and the discharge glow decay are reported. Dotted line fits the laser data as explained in the text.

discharge glow, given by the electronic circuit (neglecting the time constant of the fluorescence decay). It should be noted that Eq. (4.3) is no more invariant under time translations (as a standard exponential decay). Hence, we should fit also the origin of time, because of the delay introduced by the trigger of the oscilloscope.

An example of the fit obtained this way is also reported in Fig. 4.1. The agreement with the data is good enough. To obtain a reduced $\chi^2$ of 1 we should assign to each data point an uncertainty of 0.005 V, a value certainly compatible with the noise of the data.

We have repeated this measurement four times, obtaining a measure for the ring-down time

$$\tau = (36.6 \pm 0.5) \mu s,$$

where uncertainty is given by semidispersion. This value is an improvement over the roughly measure obtained before cleaning the mirrors (and considering only the tail of the decay), that was about 20 $\mu$s.

This is, of course, worse than the expected 100 $\mu$s computed with the nominal value of reflectivity for mirrors. This is expected because the mirrors were old ones used before in the Wettzell apparatus. This ring-down time corresponds to a cavity quality factor

$$Q = \frac{2\pi c}{\lambda} \tau = 1.1 \times 10^{11}$$
and thus to a finesse (all these formulas can be found in [32])

\[ \mathcal{F} = \frac{\lambda Q}{L} = 1.2 \times 10^4, \]

or a reflectivity of each mirror (assumed equal for all four mirrors)

\[ R = \left(1 - \frac{\pi}{2\mathcal{F}}\right)^{1/4} = 0.999968. \]

If the mirrors have not the same reflectivity, this value is the geometric mean of the four, and thus is a roughly estimate of the reflectivity of the worst mirror.

### 4.2 Gas contamination

Long term operation of the gyro laser is limited by the contamination of the He-Ne gas mixture. As the gas mixture is getting older, laser threshold increases until the laser action can no longer be maintained, even with maximum RF power supplied. At this point the laser should be refilled. Usually, in the last days of laser operation the gyro performance deteriorates.

Hydrogen is the first cause of contamination of the gas mixture (see, also, [14]). Hydrogen arises from outgassing from stainless steel and from the Pyrex tube. Other possible contaminants are oxygen (for example arising from the Pyrex capillary), nitrogen (air leakage) or water (from the Pyrex tube, it is broken in hydrogen and oxygen in the discharge).

Presence of hydrogen is revealed by the line at 656 nm as seen by the spectrometer (see, for example, Fig. 4.2). This line increases steadily and appreciably with time. Further studies indicates that the first source of hydrogen is the Pyrex tube of the discharge: growth of the hydrogen line is found to be correlated to how long the laser is kept turned on.

Some counter-measures was taken for increasing the working time of the laser: baking the chamber and using passive getter pumps. The gyro was able operate for about three weeks before these tricks. Baking the chamber increases the working period of the gas mixture to over a month. Figure 4.3 shows the growth of the hydrogen line before and after the baking (note, however, that the maximum of the intensity of the line is not correlated to the working time of the laser). The installation of the getter (SAES getter 5K0621) solved the problem radically: their activation reset the hydrogen line to zero in few minutes. This drop is reported in Fig. 4.4. The lifetime of the gas mixture using the getter is still under study and is expected to be of several months.

Why hydrogen influences so badly He-Ne lasing is still not well known. Graham [14] reported an exponential reduction in laser gain with increasing hydrogen pressure. As the partial pressure of hydrogen is increased further
Figure 4.2: Example of the spectrum of the discharge fluorescence around the 632.8 nm line. Intensity is normalized to the neon 650 nm line. The laser emission line and the 656 nm hydrogen line under study are indicated. This spectrum is obtained with the gas 8 days old. In the spectrum taken immediately after the filling the hydrogen line is absent.

Figure 4.3: Grow of the hydrogen 656 nm line intensity in function of time: ◦, before the baking, the gyro worked for 22 d; □, after the baking, the gyro worked 35 d. The first measure, of both series, was taken just after the refill. The last measures were taken just before the end of the gas mixture lifetime.
Figure 4.4: Effect of the getter on the hydrogen line. The first measure was taken with the gas used for 20 d.

(to much higher pressures than we would ever expect from hydrogen contamination as a result of outgassing) a small recovery in gain is reported. This behaviour is probably due to some complex, not well known, mechanism; a possible explanation is quenching of the active Ne level by molecular hydrogen in the discharge.

4.3 Mode taxonomy

G-Pisa laser can operate at different power levels and exhibits peculiar mode behaviours. The investigation of the mode structure has proved an invaluable tool for understanding the gyro laser.

Both the Fabry-Pérot analyzer and the spectrum of the beat note with the stabilized laser gives insight about the mode structure of the gyro laser. They provides complementary information. The Fabry-Pérot setup is best suited for recording the number and structure of modes and can investigate both beams at the same time (a very helpful feature, see next section). The spectrum of the beat note can be used to determine the number of modes but the picture is less clear (in the same frequency range there are the beat notes of each mode with the stabilized laser and each beat note between the modes of the gyro). On the other hand this spectrum let us check the time evolution of the optical frequency.

When everything is in place, we can observe Sagnac signal, Fabry-Pérot spectra and beat note with the stabilized laser at the same time, obtaining an excellent insight on the gyro operation.
4.3.1 Single mode operation

At low power the gyro laser works with just one mode in each direction. We refer to this behaviour as single mode, emphasizing the role of the single beam, even if the light modes in the cavity are indeed two. On the other hand, light circulating in only one direction is not usually expected. This single mode region is where ring laser gyro are usually designed to operate and the theory developed in chapter 2 refers to single mode operation.

Usually both beams lase in the same longitudinal mode, i.e., with the same mode number. In this case the spectra obtained with the Fabry-Pérot analyzer look like Fig. 4.5a. The Fabry-Pérot finesse is way too low to detect the Sagnac effect between the two beams and there is a single peak, albeit with repetition separated by a free spectral range of the FP analyzer. These repetitions are used for calibration. Both beams contribute to this peak. This behaviour is the standard operation for a ring laser gyro, and the beat note between the two counter-rotating waves is the usual Sagnac signal with frequency around 110 Hz.

Unfortunately, counter-propagating beams can lase on different longitudinal modes. As Hurst et al. [16], we call these occurrences split modes. In this case, FP spectrum looks like Fig. 4.5b. This spectrum is recorded in the same condition of Fig. 4.5a, only few minutes after. Instead of the single peak, there are two smaller peaks (and their repetition due to the FP spectrometer). Modes in the counter-clockwise and clockwise directions can be distinguished shutting the laser at one or the other entrance of the fibers. Their separation is 54 MHz, the free spectral range of the gyro laser.

Sagnac signal is lost with split modes. The beat note between the two split counter-propagating mode still exists, of course, but is about 54 MHz and the photodiode is not fast enough to detect it (see section 3.5.1). The first cause of split in G-Pisa laser is mode hopping due to thermal expansion. The counter-rotating beams are practically uncoupled, allowing them to mode jump independently of each other and, thus, to split. In the not-controlled condition of our laboratory, the typical time between one mode jump and the other could range between 5 min to 15 min. Split modes persist usually for some minutes, until both beams return on the same longitudinal mode. Then standard operation and Sagnac signal are recovered. These occurrences strongly limit the performance of the gyro, especially at low frequency, and prevent it to operate continuously. Active frequency locking of the gyro laser should solve the problem once and for all, as we shall demonstrate in section 5.5.

Split modes separated more than one free spectral range, are not ob-

\footnote{Sagnac effect is expected to contribute to this frequency, but only with about 100 Hz. Sorting this effect out with high accuracy would be a laborious task. However, this could be an interesting trick to avoid backscattering; this has already been done with the ultralarge ring laser UG-2 (21 m × 40 m) with FSR of 2.47 MHz [17].}
Figure 4.5: Fabry-Pérot spectrogram of the output of the gyro laser during single mode operations. Signals are repeated every 300 MHz. (a) Standard operation; both beams lase on the same mode; the peaks due to the two beams are superimposed, since the finesse of the FP is too low to detect the Sagnac effect. (b) Split modes; both counter-clockwise and clockwise direction are in single mode, but split by 54 MHz. Sagnac signal is lost.
served in our laser, but they were observed with larger ring lasers, thus with smaller free spectral range, as, for example, reported by Hurst et al. [16].

This picture has been confirmed checking the frequency of the beat note with the stabilized laser. For example let both beams lase on the same mode, say the $m$ one. Sagnac signal is present. The optical frequency slowly drifts across the gain profile, say it decreases, to fix ideas. It crosses the maximum of the gain profile when the beat note is approximately 60 MHz (see section 5.4.1). When it is far enough away from the maximum, the current lasing mode $m$ is no more the one with the highest gain (on both directions). However the $m$ mode and the $(m+1)$ mode are in competition, because of homogeneous broadening, and the $(m+1)$ mode will not start to lase immediately. The $(m+1)$ mode will overcome the $m$ one, giving a mode jump, almost randomly (usually when the beat is between 20 MHz to 30 MHz) and independently for the counter-rotating beams. This way the counter-rotating modes are split and the Sagnac signal is lost until also the second mode has (randomly) jumped. This is a noteworthy example of the hysteresis due to saturation and holes burning of the gain (section 2.3.1).

4.3.2 Stable multi-mode operation

Increasing power, the gyro begins to work multi-mode. Accordingly to power level, there can be 3 or 4 modes in each direction (see Fig. 4.6), accompanied with stable Sagnac signal. Two modes are practically never observed.

The modes in each direction are separated by two free spectral ranges (108 MHz). The lack of modes at one FSR (54 MHz) is due to mode competition. In fact, the homogeneous broadening in our plasma is large enough (around 200 MHz) to prevent growth of two near modes (in the same direction; see section 2.3.1).

Lasing multi-mode does not prevent the gyro going split modes, with behaviour similar to the single mode case.

Ring lasers are not usually operated multi-mode because in this condition the Sagnac signal is often lost or degraded. For example, each couple of counter-rotating mode with the same mode number is expected to beat at the Sagnac frequency. However, if the contribution of each couple does not add constructively in phase, the contrast of the Sagnac signal will fall, that is our signal will degrade. Instead, G-Pisa gyro shows often a stable good Sagnac signal, even with three or four modes in each direction. This evidence strongly suggests that modes are apparently mode-locked, or, there is a fixed phase relation between different modes, and this fixed phase is the

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3Contrast is intended as half of the ratio between the peak-peak amplitude and the DC level. DC level is measured respect the dark signal of the photodiode. It is an interesting parameter of the quality of the Sagnac signal, ideally saturating to 1.

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Figure 4.6: One order of the FP spectrogram during multi-mode operations. (a) Three modes in each beam; modes 1–2, 3–1’ are separate by two FSR. (b) Four modes in each beam; modes 1–2, 2–3, 4–1’ are separate by two FSR; modes 3 and 4 are near because of the folding effect of the Fabry-Pérot.
Figure 4.7: Contrast of the Sagnac signal obtained varying laser power and waiting for stable output. Number of modes ranges from one to the left to four to the right, as indicated by point style: ◦, single mode; △, three modes; ⋄, four modes. (Two modes are not usually observed.) Aside of some fluctuations, probably due to some hysteresis, contrast seems uncorrelated to number of modes and power.

same in both directions.\textsuperscript{4} That is, the contributions at the Sagnac signal of the different modes add constructively in phase (the counter-rotating \(m\) modes gives rise to Sagnac oscillation, as the \(m + 2\) modes, etc.).

Contrast of the Sagnac signal seems uncorrelated to intensity of the laser beam or number of modes. Figure 4.7 shows an example, with number of modes ranging between one to four (fluctuations are probably due to hysteresis effects and are not strictly reproducible). This is slightly different to the behavior reported for other ring lasers (see, for example, \cite{15, 16}), that show mode locking only at specific power levels. Dunn \cite{12} reported behaviour with a triangle ring laser (perimeter length 2.56 m) similar to what we have observed: mode-locked multimode operations with good Sagnac signal. However note that the reduction in the lock-in threshold he observed with multimode operation is probably due to the interference of the backscattered waves (section 2.2.2).

Indeed, the gyro sometimes does not mode lock, especially after power change, showing poor Sagnac signal (low contrast or modulation of amplitude were observed). We suspect these occurrences are surely connected

\textsuperscript{4}The term “mode locking” is usually associated with pulsed lasers. However pulsing does not occur in the G-Pisa laser. A more generic expression is “mode coupling” (see, for example, the discussion of Siegman \cite{31}). Even better we should speak of “phase locking”. Nevertheless, we adopt mode locking as already done in literature of ring laser gyros.
to the position of the various modes under the gain curve. However such conditions are temporary, and good Sagnac signal is recovered, usually in one or two minutes. Figure 4.7, for example, was recorded waiting for stable and high contrasted Sagnac signal at each power level.

In light of the discussion of section 2.2.2, the Sagnac frequency of each couple of modes should be differently affected by backscattering. Indeed different Sagnac frequencies for each couple may explain some of the bad signal recorded. However the good multimode Sagnac is sign of mode-locking, despite the different pulled Sagnac frequencies.

This good multi-mode behaviour suggests our gyro can work properly both single mode and multi mode, with different advantages. Single mode operation is simpler and better understood, but limited to work near laser threshold. On the other hand, working with four mode can achieve better signal-to-noise ratio.

4.3.3 Unstable multi-mode operation

If we further increase the power supplied, more modes kick in. At this point the situation begins to be less defined. The power of each mode starts to oscillate. If the Sagnac signal is still present, it suffers heavily of amplitude modulation. In these conditions, mode competition plays a big role, and decoupling effect of the two neon isotopes start to be less pronounced.
A ctive stabilization of the G-Pisa gyro laser is the main topic of this chapter. The intensity of one of the two beams is stabilized adjusting the radio frequency power supply of the discharge. The optical frequency of the laser, measured respect to a reference laser, is stabilized moving one mirror of the cavity with a piezo. With both stabilization the basic operation of the laser can be described as in Fig. 5.1

First we shall describe the implementation and the result of the amplitude stabilization. Then we shall discuss the predicted effects of the optical frequency stabilization on the Sagnac signal. Design, implementation and results of the frequency stabilization will conclude the chapter.

5.1 Motivation:

An example of the Sagnac signal recorder for half an hour, with the laser free-running, is reported in Fig. 5.2. Two striking features need an explanation and, possibly, a correction: the slow decrease in amplitude and the presence of gaps with no signal at all.

The decreasing of amplitude of the Sagnac signal (that is, of the laser power) is easily explained. Figure 5.2 is recorded during single mode operation, thus with laser near threshold. As the cavity is perturbed, for example by thermal expansion, the alignment of the mirrors gets lost. Since laser threshold is initially optimized moving the mirrors manually, any misalignment causes threshold to increase and laser power to decrease. Eventually lasing can stop. We can hope sometimes to work with negligible decrease in power for short periods, but in the long run (some hours) we expect a strong degradation as in Fig. 5.2.
Figure 5.1: Basic diagram of the gyro laser fully stabilized: the amplitude is stabilized adjusting the RF power supply; the optical frequency is stabilized moving a mirror with a piezoelectric transducer (PZT).

Figure 5.2: Example of a long acquisition of the Sagnac signal, before the stabilization were active. Notice the gaps in the signal, due to split modes. In these gaps no rotation information or Sagnac frequency can be read.
Working multimode can help, since the laser will not be constrained near threshold anymore. However, as previously discussed, single mode operation is simpler and preferable. An easier solution exists: amplitude stabilization acting on the RF power supplied to the discharge.

Gaps in the signal as in Fig. 5.2 are a trickier problem. The gyro works usually for some minutes, then the Sagnac signal is lost for a couple of minutes and eventually restored. These gaps are due to split modes, as already discussed, caused by thermal expansion (section 4.3). This behaviour, with the gyro often blind because of split modes arises from a particular unfortunate combination of thermal stability of the room, cavity length and homogeneous broadening in the active medium. With slower variation of temperature and stronger competition between modes, the laser will jump less frequently, working for longer periods. On the other hand with less competition both beams will more easily jump on the mode with maximum gain, shortening the blind intervals of split modes.

Unable to precisely control the temperature of the laboratory or to mount all the apparatus with over-the-top low-expansion materials, we are left with only a solution: active perimeter lock of the gyro laser. This is accomplished moving with a piezo one mirror as we shall discuss starting from section 5.3.

5.2 Amplitude stabilization

Amplitude stabilization of lasers is a standard example of negative feedback. Stabilizing a ring laser, instead of a linear laser does not add particular problems. Simply, we stabilize only one of the counter-rotating beams (usually the counter-clockwise one), hoping to stabilize the other as well.

We can monitor the signal from a single output beam with a photomultiplier tube (PMT) and stabilize its DC (direct current) component controlling the amplitude of the radio frequency that drive the plasma (see section 3.3).

When the amplitude is locked, the gyro laser can operate for long time. Increasing the RF power supplied by the discharge compensates any loss (due to misalignment of the cavity by thermal expansion, or by the slow contamination of the gas mixture, etc.). The stabilization should also guarantee a more stable operation of the laser.

5.2.1 Implementation

A bit of care is needed in the design of the loop bandwidth. Indeed, the amplitude of the single beam is modulated itself at the Sagnac frequency (110 Hz), because of backscattering. This modulation should not be corrected. Thus, the servo can be optimized using not only an integrator, that grants the required high gain at low frequency, but also a low-pass filter as in Figs. 5.3 and 5.4. The integrator have unity gain at 0.5 Hz. The filter
is a simple first order low-pass filter with cut frequency 1.5 Hz. The combined frequency response of the integrator and the filter is a $-10$ dB/decade slope for frequency below 10 Hz, but a $-20$ dB/decade slope above. This way we can increase the low-frequency gain without correcting the 110 Hz monobeam modulation.

The intensity is stabilized respect to a (variable) reference voltage. It is designed to take as input the signal of a Hamamatsu H5783 photomultiplier tube closed on a 100 kΩ resistor (notice, however, than its gain can be controlled) and control the discharge voltage through the prepared circuit that send RF power to the discharge (see section 3.3). It is possible to use the modulation input to control externally the reference voltage, thus the laser intensity, when the lock is engaged. (When the lock is not engaged the laser power can be controlled directly.) The monitor output can be used to access the signal before the integrator, that is, basically, the low-passed and shifted beam intensity.

A switch can be used to reset the output of the circuit to 0 V (discharging the capacitor of the integrator). Another switch can be used to temporary disengage the servo without varying the output.

It was implemented with OP77 operational amplifiers. The reference voltage is obtained by a REF03 integrated circuit. It gives a stable voltage of 2.5 V (between pin number 6 and pin 4; pin 2 is for power supply). Everything is supplied with ±15 V.

### 5.2.2 Results

Figure 5.5 shows an example of the performance of the stabilization loop. It shows the amplitude of one beam of the laser before and after the lock. This data are taken in a noisy contest (we were working around the laser,
Figure 5.4: Schematics of the servo circuit for amplitude stabilization.
Figure 5.5: Example of the amplitude stabilization. The intensity of the counterclockwise beam was recorded through a photomultiplier tube. At the point indicated by the arrow a prototype of the stabilization circuit is engaged.

etc.) with a prototype of the control loop. Here and in the following, the measurements of power are average made every 1 s, ironing out the modulation at the Sagnac frequency.

With the prototype circuit we have successfully operate the laser continuously for some days. This way we were able to record the Sagnac signal at night, avoiding a lot of noise due to the environment. Unfortunately the reference voltage used was not very good, and show some bad drift and noise. This has been corrected in the definitive setup.

We have monitored the intensity of both beams for prolonged time. Figure 5.6 shows the data collected with the amplitude stabilized for over two days. Topmost plot reports the intensity of both beams. Second plot is an enlargement of the variation of the stabilized beam (the CCW one, in this case). Last plot shows the voltage applied at the capacitor of the discharge, that is, the correction to the laser power. The beam used for the stabilization behaves as expected, showing little drift (probably arising from the temperature dependence of the reference level) and little noise.

Of course the other beam is not so well stabilized: it shows both a relevant drift and some random jumps. In particular it can mode hop independently by the other beam, and mode hopping is always accompanied by intensity jumps. Since we are going to lock the perimeter of the gyro to avoid mode jumps, this cause of intensity variation is not a concern. Drift may arise, for example, because the variation of intensity with RF power supplied is not the same for the two beams. Nevertheless, this beam is
enough stable to allow prolonged operation of the gyro.

In another occasion we were able to record also the temperature. Figure 5.7 shows the correction applied to the capacitor of the discharge along with the temperature of the room. Turning on and off of the air conditioner can be easily recognized in the temperature trend. In particular, the amplitude stabilization heavily responds when the conditioner is turned on. Here and there correction of mode jumps can be recognized in the discharge voltage.

5.3 Stability of the Sagnac signal under perimeter stabilization

Other considerations should be made about the stabilization of the perimeter of the gyro. In particular, what effect has the frequency stabilization on the Sagnac signal? Again all the problems arises because only one thing: the backscattering.

We describe backscattering using the equations used in section 2.2, that we summarize here. The total backscattering rate coefficient is given by

\[ R_\pm = r_\pm e^{ik\pm} = \sum_n A_n e^{2ikz_n} \tag{5.1} \]

where \( k = \frac{\omega}{c}, \) \( A_n \) are the coefficient of the backscatterer at position \( z_n \). These position \( z_n \) are basically, in our system, the distances between the mirrors—the only source of backscattering. Letting \( \rho_\pm = r_\pm E_\pm / E_\mp \) and \( \zeta = (\varepsilon_+ + \varepsilon_-) / 2 \) the net amplitude and phase of backscattering, and with the approximation \( \rho = \rho_+ = \rho_- \) the lock-in threshold is given by

\[ l = \frac{\rho}{\pi} \cos \zeta, \tag{5.2} \]

and the pulled Sagnac frequency is

\[ p = \sqrt{f^2 - l^2}. \tag{5.3} \]

The backscattering rate \( R_\pm \) and then the lock-in frequency \( l \) and the pulled frequency \( p \) depends critically on the separations between the mirrors \( z_n \), through both the amplitude \( r_\pm \) and phase \( \varepsilon_\pm \) of the backscattering.

Under thermal expansion, distances between mirrors changes uniformly at first order. Under this assumption the backscatter coefficient does not change, since the variation of \( z \) is compensated by the variation of the wavelength with the perimeter. That is, the backscattering coefficient and the Sagnac signal are insensible to change in the distance between mirrors proportional to change in the perimeter. We should note that this is true only without mode jumps. With each change in longitudinal mode, the phase in
Figure 5.6: Example of the amplitude stabilization. We report intensity of both beams, zoom of the variation of the stabilized intensity and discharge voltage.
Figure 5.7: Example of the correction to the discharge voltage (left axis) with temperature (right axis). Notice the huge effect of the air conditioner.

Figure 5.8: Example of a long acquisition of the Sagnac frequency. It is obtained with the amplitude stabilization engaged (but not the perimeter one). Data out of scale and vertical traces are failures during split modes (section 4.3.1) of the autoregressive algorithm that compute the Sagnac frequency. The plot shows clearly the change in pulling due to backscattering with each mode jump.
Eq. (5.1) is out of control. The effects of backscattering with the laser free running are shown in Fig. 5.8.

This phenomenon can be understood considering the variation of terms like \( \phi_n = 2kz_n \) in Eq. (5.1). That is \( \delta \phi_n = 2k \delta z_n + 2z_n \delta k \). If we assume no mode jumps (the laser wavelength is proportional to the perimeter) then

\[
\delta \phi_n = 2k \left( \delta z_n - z_n \frac{\delta L}{L} \right). \tag{5.4}
\]

That is, if the mirror separation changes \( \delta z_n \) are proportional to those in the perimeter, for any \( n \), the phases \( \delta \phi_n \) vanish and there is no change in backscattering, lock-in threshold and pulled frequency. A simple example of such a change is an (ideal) uniform expansion, in which the ring does not change shape.

Things are different when we consider the displacement induced by the piezoelectric transducer (PZT), or thermal expansion of the cavity under perimeter stabilization. Mostly because the mirrors curvature, under the movement of one mirror along the diagonal of the cavity, as happens in G-Pisa, the variation of the phases \( \delta \phi_n \) of the previous equation can be expected not to vanish. This is true whether the movement is induced from the outside or is induced automatically to keep the perimeter constant. In this last case the second term of Eq. (5.4) is zero, and only the first contributes. Then the lock-in threshold and the pulled Sagnac frequency change as the piezo is moved. Rodloff [25] and Schreiber et al. [28] reported this very effects of the interference of backscattering with their perimeter-stabilized ring lasers.

5.3.1 Numerical simulations

To understand better the problem we have done some numerical simulations of the effect of backscattering. Simulations are simple programs implemented with GNU Octave [13]. An analytic treatment is not impossible but will probably be much more obscure.

Our programs exploit the beam steering equations of Bilger and Stedman [9] to calculate the positions of laser spots on mirrors given tilts and positions of all mirrors. We consider the change in the position of the mirrors only in the plane of the cavity (in-plane and out-of plane movement are separable). Our program translates these displacement in the distances from the centers of the mirrors of each laser spot. Once we have the spot positions, a straightforward calculation gives the perimeter of the cavity and separations between mirrors. We consider the mirrors to be the only source of backscattering, so we can use Eqs. (5.1) to (5.3), or the slightly more general Eq. (2.23), to calculate the pulled Sagnac frequency. The wavelength is constrained to be an integer submultiple of the perimeter length.
Figure 5.9: Numerical simulation of the pulling of the Sagnac frequency moving one mirror along the diagonal, as under the effect of the PZT. The parameter of the simulation are: square cavity with side length of 1.40 m, perfectly aligned; all mirror with curvature $R = 6$ m; all the backscattering coefficients $A_{n \pm} = 17$ Hz (this value is chosen following the estimate of section 3.2.2).

With these simulations we can only obtain a qualitative description of these phenomena, since there are a lot of uncontrolled parameters (e.g., amplitudes and phase of backscattering at each mirrors, exact positions and tilts of mirrors, etc.). In all the following examples, to keep things simple and under control, we have assumed $E_+ = E_-$ (so that $\rho_\pm = r_\pm$). With the same motivation, the backscattering coefficient $A_{n \pm}$ is taken equal and real for all mirrors and for both directions. Nothing prevents to relax these conditions for testing more general cases.

In the simplest simulation we have varied the perimeter of the cavity moving one mirror along the diagonal, as under the action of a (virtual) piezoelectric transducer (Fig. 5.9). The Sagnac frequency shows variable pulling, periodic with the perimeter change. We assumed no mode jumps. The period of the pulling is mostly due to the geometry of the problem. Interesting this period is more than a wavelength ($2.8 \lambda$). The result is similar to the experimental data reported by Schreiber et al. [28]. Unfortunately we are not able to repeat this kind of experiment with our laser. In fact, we can not induce variations so wide (as the period of this variation) without incurring in mode jumps. Moreover we may have troubles with the variation of temperature in the room during the experiment, etc.

To obtain data more easily confronted with the real counterpart we have simulated the effect of backscattering with variation of temperature compen-
Figure 5.10: Numerical simulation of the variation of the Sagnac frequency with temperature when the perimeter stabilization is engaged. Perfectly aligned cavity and parameter as in Fig. 5.9. A thermal expansion for stainless steel $\kappa = 1.73 \times 10^{-5}$ K$^{-1}$ is assumed.

sated by the stabilization of the perimeter (Fig. 5.10). Thermal expansion is consider isotropic. The perimeter stabilization is virtually achieved by a simple bisection algorithm.

Clearly, these plots show the huge effect of the interference between the scattered waves. The picture becomes more complicated if we consider the more realistic case of a cavity not well aligned. Figures 5.11 and 5.12 show the result for both piezo-induced movement and dependency with temperature when, simply, two mirrors are tilted toward each other by 1.0 mrad. This (quite large) deformation is enough to move the laser spots some millimetres from the center of each mirror. This is entirely possible with our laser. The most evident effect of the deformed geometry is the appearance of variations with twice the period.

Concluding, these simulations provide interesting findings:

- the backscattering-induced pulling of the Sagnac frequency is periodic with perimeter variation;
- the period of this pulling is greater than a wavelength and it is fixed by the geometry of the cavity;
- the same periodic response is obtained with temperature variations under frequency stabilization;
- detail of these effects can be hard to predict because of the large amount of uncontrolled parameters.
Figure 5.11: Numerical simulation as in Fig. 5.9, but with the cavity not well aligned: the mirror moved by the piezo and an adjacent one are tilted by $-1.0\text{ mrad}$ and $1.0\text{ mrad}$ respectively.

Figure 5.12: Numerical simulation with the cavity not well aligned as in Fig. 5.11, showing variation of the Sagnac frequency with temperature under perimeter stabilization.
Seeing this, we should not be surprised that ring laser gyros are usually passively stabilized (with the notable example of the monolithic Zerodur block of G). The predicted periodic dependency of the Sagnac frequency with temperature is an unwanted side effect of the stabilization achieved with a piezoelectric transducer. So, why are we going to?

First, a passive stabilization is not always achievable or feasible: Zerodur monolith are not only (very) expensive but also hard to handle. Second, we are going to try it in order to avoid mode jumps and split modes, whose effect on the performance of the gyro are even less desirable than the backscattering one. Moreover, as clear in Fig. 5.8 on page 61, the laser free running is not insensitive to backscattering anyway. Third it is simply interesting to test the system, in order to get insight of the physics of the gyro laser.

5.4 Frequency stabilization

The stabilization of the perimeter of a ring laser gyro is a task that need some care. The beams are very weak (around 1 nW), and the measure of the optical frequency may take some patience. Moreover the laser is large, heavily influenced by thermal expansion, requiring generous corrections.

We talk interchangeably of frequency stabilization or perimeter stabilization, since

\[ m\lambda = L \quad \text{and} \quad \nu = m\frac{c}{L}, \]

or better

\[ d\nu = -\nu \frac{dL}{L}. \]

However there is a subtle observation. We are going to fix the frequency \( \nu \), stabilizing an absolute value (respect to a reference laser), but we can not fix the absolute value of the perimeter. We can only assure that the perimeter is not changing. This fact has a practical consequence in the laser gyro case: every time we are going to lock the laser we are getting a different backscattering interference. Thus some measures can not be reproducible. For example we could have, with the frequency stabilized and at a certain temperature, once the maximum of the pulled Sagnac frequency, other times the minimum.

The complete loop of the frequency stabilization is schematically shown in Fig. 5.13. The beat between the gyro laser and the stabilized laser (Bob) is recorded through a photomultiplier tube and an optical fiber coupler as already described in section 3.5.4. The key element of the loop is a Analog Devices ADF4108 evaluation board that can be used to discriminate two input frequencies. The photomultiplier signal is first amplified and then its frequency is compared by the ADF4108 chip with a reference frequency, given by a voltage controlled quartz crystal oscillators (VCO) or even a function generator. The output of the ADF4108 is a positive voltage if the
Figure 5.13: Block diagram of the frequency stabilization of the gyro laser. Detail of each stage can be found in the text. The ADF4108 chip is the key element of the loop: it discriminates if the frequency of the beat between the stabilized laser and the gyro is more or less than a reference frequency given by a voltage controlled oscillator (VCO).

signal frequency is higher than the reference, otherwise zero. This output is integrated and used to control the piezoelectric transducer, through a specifically designed servo circuit. For control purpose we monitored the beat with a microwave signal analyzer. Moreover the ADF4108 can give as output a signal which frequency is a fraction (usually 1/300) of the beat frequency. This frequency can be measured by a counter.

In the following we shall describe in detail each stage of the loop.

5.4.1 Choice of working point

Before proceeding further in the description of the loop, we should plan about the working point of the stabilization. That is, at which frequency the gyro laser should be locked? Or, about the same, at which frequency we should lock the beat between Bob and the gyro?

A good choice could be the maximum of the gain profile, where mode jumps are unlikely and the counter-rotating beams weakly compete. We do not need to hit the maximum precisely, we can settle for an estimate. Possibly we can finely tune the lock point later. The maximum should be found respect to the frequency of the reference laser.

The gain profile of the gyro laser and of Bob are sketched in Fig. 5.14, see
Figure 5.14: Estimate for the best working point of the frequency stabilization.
The gain of the gyro (higher only for clarity) is compared to the red (R) and blue (B) locking point of the stabilized laser. The origin of the frequency scale is the center of the $^{20}\text{Ne}$ transition. The gain of the $^{22}\text{Ne}$ is dashed.

also section 2.3. Working with both isotopes of neon, the gain of the gyro is proportional to the sum of the gain of the $^{20}\text{Ne}$ and of $^{22}\text{Ne}$. The resonant frequencies of the two species are separated by 850 MHz. The maximum of the gain is approximately halfway, neglecting possible differences in the concentration of the two isotopes and the slightly different mass of $^{20}\text{Ne}$ and $^{22}\text{Ne}$ (this influences the Doppler width of the two curves). The gain of Bob are approximately the gain of the sole $^{20}\text{Ne}$, neglecting possible traces of the other one in its gas mixture. The two lock points of the stabilized laser are symmetrically respect to the gain and separated by 730 MHz. Looking at Fig. 5.14 we can conclude that the desired target frequencies are 60 MHz and 790 MHz for the blue and red lock points of the reference respectively.

Detecting tens of megahertz is easier than detecting hundreds of megahertz. So, we choice to work with only the blue lock point of Bob and lock the gyro at 60 MHz.

The same locking point can be used to stabilize the laser working with three modes in each direction (our system should be able to lock the central mode). The target frequency 60 MHz should not be used to lock the laser with four modes (the modes distribution will be asymmetrical).
5.4.2 Piezoelectric transducer

The piezoelectric transducer is mounted under one of the corner turret holding mirrors. It can move the mirror along the diagonal of the square cavity, effectively controlling the perimeter length. Indeed, it does not move only the mirror, but also the mirror-holder and the corner turret (this way the piezo is not housed in the vacuum chamber). Movement between the corner and the tube along the side of the cavity is allowed by bellows. The amplification is set to the highest possible value that does not cause drowsiness.

The piezoelectric transducer is driven by a Piezosystem Jena 12V40 amplifier. It can be controlled with an applied voltage ranging between $-10\,\text{V}$ to $150\,\text{V}$. The piezo driver can be controlled in two ways. When the front input is enabled, the applied voltage can be controlled by a knob, letting us move the PZT by hand. A modulation signal of $0\,\text{V}$ to $10\,\text{V}$ can be added. Alternatively the rear input can be used. This way the knob is disabled but an input voltage of $0\,\text{V}$ to $10\,\text{V}$ can control all the PZT excursion (controlling the applied voltage on all the range $-10\,\text{V}$ to $150\,\text{V}$).

When using the rear input we can induce a frequency shift of approximately $2\,\text{GHz}/\text{V}$. That is, applying $1\,\text{mV}$ we shift the laser frequency of $2\,\text{MHz}$. Exploiting all the $10\,\text{V}$ excursion at the input we can move the piezo for 370 free spectral ranges of the gyro, or $240\,\mu\text{m}$. That is, we can hope to correct $370/2 = 185$ mode jumps, starting halfway.

Alternatively we can consider the linear thermal expansion of stainless steel (around $1 \times 10^{-5} \,\text{K}^{-1}$), and estimate a range of about $2\,\text{K}$ that can be corrected by the full range of the piezo.

This ranges are fairly wide, should let us to read the Sagnac signal continuously for hours or days, instead that for minutes.\(^1\) Technically it is possible to go further: every time the voltage applied to the piezo reaches upper or lower limit we can execute a “reset jump”; the voltage can be reported halfway, and then the frequency will re-lock, but with a different mode number. Unfortunately this is not so simple if we consider the stability of the Sagnac signal, as discussed in [25]. With the reset jump the backscatter coefficient, and the pulled Sagnac frequency with it, will change unpredictably (see section 2.2.2).

The response of the piezoelectric transducer at different frequencies need also to be investigated before designing the control loop. We expect the piezo to be the limiting factor in the stabilization of the perimeter. In fact, the piezo does not move only the mirror, but it moves all the corner turret, possibly against the bellow tubes. Thus we expect a very low frequency response of the transducer.

\(^1\)Unfortunately, currently, the air conditioning system of our laboratory can not be turned on at night. The resulting thermal excursion is more than our piezo can correct. See section 5.5.
We control the response of the piezo giving a signal of different frequencies to the modulation input of the driver and checking the effects on the optical frequency of the gyro with the spectrum analyzer.

The response of the piezo is flat until approximately 50 Hz. Then it increases slowly till 80 Hz. Between 80 Hz and 110 Hz it shows at least two resonances, approximately seven fold higher than the low frequency response. This is an unfortunate occurrence: resonances come with relevant phase shifts, that can change the sign of the feedback. We should quench the gain of the servo control at this frequencies to avoid oscillation. The piezo is unresponsive above 200 Hz.

### 5.4.3 RF circuit

We read the beat between the stabilized laser and the gyro using an optical fiber coupler and a photomultiplier tube (as we have already described in section 3.5.4). The exit of the PMT is followed by a chain of radio-frequency (RF) components amplifying the expected 60 MHz signal.

Setting down the circuit we encountered certain difficulties. We have done several tries before an acceptable result. We soon reached a good signal-to-noise ratio, however the signal was showing some resonances (some wide humps in the spectrum). Away from resonances the signal was damped, but recovered on top of them. We attributed this phenomenon to reflections in the coaxial cables. We tried to adapt in impedance our circuit; where we were not able to, we tried to use cables as short as possible, often linking directly the various component (for example at the exit of the PMT). We were not able to get rid of these resonances until we realized that the connector of the signal analyzer was faulty. Once repaired, the signal improves greatly and resonances are (mostly) gone.

The current design involves, in order (Fig. 5.15):

- a DC-block to protect the other following components;
- a 27 dB, low-noise amplifier (Advanced Control Components AC AM 7704) with bandwidth of 1 GHz;
- a “double-pi” low-pass filter, made with capacitors and hand-made inductors, with knee at 70 MHz: this filter quench the 130 MHz signal heavily irradiated by the discharge and picked by cables and other electrical circuitry;
- another amplifier (Tron tech W1G2H) with 27 dB gain and 2 GHz bandwidth.

An example of the signal obtained with this chain, as seen at the spectrum analyzer is shown in Fig. 5.16. The effect of the low-pass filter is evident. The width of the peaks is mostly due to the resolution bandwidth.
of the instrument. We have achieved a 20 dB signal-to-noise ratio, sometimes reach 25 dB with a resolution bandwidth of the spectrum analyzer of 300 kHz.

We plan to improve the amplification circuitry in the future. A definitive setup should be as compact as possible, and metal-shielded against irradiated noise (we can not shield the current setup, because it takes a lot of space).

5.4.4 The ADF4108

Once we obtained the beat signal between Bob and the gyro laser, we have to count it. This is done using an Analog Devices ADF4108 evaluation board. This chip is designed to work at much higher frequencies than our
The ADF4108 wants two inputs: a RF input (i.e., the beat note) and a reference frequency (given by a VCO). The expected value for both can be programmed via a personal computer. The device uses two frequency dividers to scale down both signal to about 200 kHz. This rescaled frequencies are compared by the phase frequency detector (PFD) of the device: when the RF signal is higher than the expected value the ADF4108 gives a high output (+5 V), else it gives 0 V. The transition is pretty sharp (we have check it to be less than 10 kHz wide at 60 MHz). The ADF4108 let us lock the frequency of the laser. The simplified behaviour of the chip is reported in Fig. 5.17.

For example we can set the RF input to be expected 60 MHz, while the reference frequency to be 10 MHz. Then the first frequency will be divided by 300 and the second by 50 to 200 kHz. Testing the ADF4108, its output will increase from zero to high when the frequency is changed from less to more than 60 MHz. However, the device does not contain any reference frequency and can be fooled: giving a reference frequency lower or higher than the expected, the frequency of transition change accordingly. Since the frequency of transition will be the locked frequency, this behaviour can be used to fine tune the locking point. This trick works only for small changes, since neither the reference frequency nor the beat one should be high enough for the device to work properly.

The ADF4108 can give the rescaled input frequency as output (if programmed specifically). This output is a squared (impulsed) waveform with the rescaled frequency of the input. That is, with a 60 MHz input, we obtain a 200 kHz output. This output can be easily read by a counter.
Unfortunately the device is not always accurate (probably there is a problem with the slew rate of the signal). When the beat signal is fed in the ADF4108, the transition can occur slightly higher or lower than the programmed one. Varying the gain of the photomultiplier tube (and thus the intensity of the signal), we can ensure that the transition is where we expect it. Surprisingly, at low gain the transition is lower than expected (for example the ADF4108 reads a real frequency of 50 MHz as 60 MHz). More surprisingly, increasing the intensity of the signal, the transition overshoots, and can be higher than expected. This behaviour is annoying, but it does not jeopardize the lock mechanism. Simply the locking point will be different than the expected and can be reported back on track varying the gain of the PMT.

This bad accuracy can be found elsewhere: the squared rescaled frequency is affected. If the reference frequency is correct, the output frequency at transition will be 200 kHz, no matter the real frequency of transition.

5.4.5 Servo loop

At this point it is clear that the servo loop circuit should be designed keeping in mind few points:

- it should stabilize the output of the ADF4108 board (0 V to 5 V), halfway;
- it should gain less than unity at the resonance frequencies of the piezo (around 100 Hz) to avoid self oscillation;
- it should drive the piezoelectric transducer with a voltage between 0 V to 10 V.

Figure 5.18 shows the simplified schematics of the resulting servo loop circuit (as we obtained it, after a few tries). A detailed schematics can be found in Fig. 5.19. The circuit takes as input the “vtune” exit of the Analog Device ADF4108 evaluation board and it is designed to control the Piezosystem Jena 12V40 piezo amplifier.

The circuit take as input the signal of the ADF4108. First, a offset of $-2.5 \text{ V}$ is applied: this way we have a zero-average error signal that can be integrated. The gain of the entire loop can be controlled using a variable amplifier. This amplification is set to the highest possible value that does not cause self-oscillations. A good value should be around unity gain. The integrator and a simple, first order, low-pass filter follow. The filter cuts signals whose frequencies are higher than 1.5 Hz. It is used in addition to the integrator to optimize the circuit: we can increase further the gain at low frequency without inducing to oscillation the resonances of the piezo. A similar combination was described for the amplitude stabilization (see section 5.2).
In the end the signal should be prepared to be sent to the control of the PZT: since at this point the signal can range between $-15\,\text{V}$ to $+15\,\text{V}$ (the supply voltage of the integrated circuits used) we need to cut it by one third and add another offset. It has prepared a modulation signal input that can be used when the locking is not engaged. Otherwise it can be moved manually on a small range.

The servo can be disengaged through a (double) switch, resetting the capacitor of the integrator and excluding the input. Unlocking and re-locking report the output voltage back to halfway (that is, could be used as a manual reset jump). The offset can be slightly varied to control the piezoelectric transducer when the system is unlocked (in this case the output of the integrator is set to zero).

It was implemented with OP77 operational amplifiers or their double version OP2277. The REF03 integrated circuit is used to obtain a negative stable voltage of $-2.5\,\text{V}$.

### 5.5 Results of frequency stabilization

On the night of 15 July 2009 we have been able to get the laser fully stabilized for the first time. Figure 5.20 on page 77 reports all the data collected: the Sagnac frequency as calculated by autoregressive algorithm, the intensity of the beam used for amplitude stabilization, the voltage at the capacitor of the discharge, the beat frequency between Bob and the gyro, the voltage applied at the PZT and the temperature. The temperature is measured with a Pt100 sensor in contact with the table, then the temperature data are smoothed to eliminate noise.

An unprecedented record of four hours of continuous Sagnac data is reported. After this time we reached the end of the piezo range, losing frequency stabilization. Unfortunately, with the current setup, we can not hope to obtain much longer time of locked operation. This limitation arises mostly because the air conditioning system of the laboratory automatically turns off in the evening and turn on in the morning. The resulting temper-
Figure 5.19: Schematics of the servo circuit for frequency stabilization (cf. Fig. 5.18 on page 74).
ature excursion (around 2°C) in the hot summer of Pisa is more than the piezo can correct. Currently we have no control over the operation of the air conditioning system, otherwise everything will be much simpler.

The switch on and off the air conditioner are particularly unforgiving against both the amplitude and frequency stabilization. See, for example, Fig. 5.7 that shows the excursion of the temperature (with the clear intervention of the air conditioner) and the correction applied to the voltage at the capacitor of the discharge to keep the intensity of the laser constant.

Despite the not long duration, the data collected are very interesting. The most noticeable feature is the oscillating Sagnac frequency as the piezo deforms the cavity to keep the perimeter constant. No mode jumps or split modes are recorded. Also the correction to the discharge power shows a variation with the same trend as the pulled Sagnac frequency: of course, the backscattering is influencing the laser intensity too. Translating the data as Sagnac frequency in function of temperature gives a result not dissimilar to the numerical simulation given before (cf. Fig. 5.21 and Fig. 5.10). Our simulation predict quite well the period of the pulling (also considering that the linear thermal coefficient of our table is not well known). Excursion of the pulled frequency is a bit higher than expected: in simulation we have underestimate the backscattering coefficient. Figure 5.22 shows another example. Some of the additional structures with twice the period (cf. Fig. 5.12) are recognizable. Of course, since the model used is quite simple, the simulations do not predict everything. For instance, Fig. 5.22 shows pushing of the Sagnac frequency that is unpredictable with our model (section 2.2.1). Modulation of the excursion of the effect is also greater than in any simulation tested. This can be a shortcoming of the model or maybe due to some variation in the scattering as the spots move across the mirrors or due to some anisotropy in the thermal expansion of our table. Nevertheless we can conclude that the interference in the backscattered waves is the dominant effect on the pulling of the Sagnac frequency in our laser.

As a last example we show in Fig. 5.23 a similar pulling effect recorded with the laser stabilized with three mode in each direction. Lasing multimode does not change qualitatively the backscattering effect.

Before the lock we were able to record the Sagnac only in windows of some minutes. Now we are able to record the Sagnac signal continuously, without incurring in split modes. The other side of the coin is the strong dependency of the Sagnac frequency with temperature. This is not really a downgrade. While in each window of good Sagnac signal, its frequency could be considered, usually, approximately constant, at each mode jump the frequency changed unpredictably because, changing the wavelength, the interference in the backscattering changed (see Fig. 5.8 on page 61).
Figure 5.20: Example of a record of data with the gyro fully stabilized.
**Figure 5.21:** Example of the dependency of the Sagnac frequency on the temperature under perimeter stabilization.

**Figure 5.22:** Other example of the dependency of Sagnac frequency on the temperature under perimeter stabilization. We can recognize also pushing (frequency higher than the expected one) and not only pulling.
Figure 5.23: Pulling of the Sagnac frequency versus temperature under perimeter stabilization with the laser running three modes in each directions.

5.5.1 Allan deviation of the beat frequency

To quantify the achieved frequency stability we measured the Allan variation $\sigma_y^2$ [2] of the beat frequency between Bob and the gyro. We have measured the 200 kHz output of the ADF4108 (the rescaled beat frequency) using a Hewlett-Packard 53132A universal counter. We set a gate time of $\tau_g = 0.5 \text{ s}$, the faster that the computer used for acquisition can handle. As pointed out by Rubiola [26], this counter is a Λ-counter and can be used to calculate Allan variance only for $\tau \gg \tau_g$. We consider dead time negligible. The total measurement time was 25 339.75 s (approximately seven hours) or 47 301 data samples (unfortunately this duration was not dictate by the PZT range, as we have often, but because of the failure of one amplifier in the RF chain). During this measure a Hewlett-Packard 33120A function generator is used as reference for the ADF4108. The counter 53132A and the function generator are supposed ideal. This measure was taken at night, with a supposed quiet environment.

Uncertainty is given at 1σ level of confidence (68%). It is computed as $I(\tau) \approx \sigma_y(\tau)\kappa_0 M^{-1/2}$, where $M$ is the number of data point used in the estimate and $\kappa_0$ is a constant whose appropriate value for white frequency noise is 0.87 [18].

The resulting Allan deviation $\sigma_y(\tau)$ (see Fig. 5.24) is mostly compatible with $\sigma_y(\tau) \sim \tau^{-1/2}$ or white frequency noise. This suggest there was no frequency drift during this measurement. In fact, a least-square fit of the frequency data (at 200 kHz) gives a linear coefficient $(-10 \pm 9) \times 10^{-4} \text{ Hz/s}$. This is a good result, indicating a well done locking able to follow the ref-
Figure 5.24: Allan deviation of the 60 MHz beat frequency. A least-square power-law fit is shown. Here $y$ indicate the relative frequency fluctuation.

Reference frequency of Bob without introducing extra noise. For example integrating for $\tau = 100$ s gives a Allan deviation for the 60 MHz beat note of 30 kHz. This correspond to a stability of the gyro perimeter (5.6 m) of 0.0006$\lambda$ or 0.4 nm (neglecting the stability of Bob, that for this sample time should be 5 kHz, see Fig. 3.11).

For sample time $\tau > 1000$ s the trends as $\tau^{-1/2}$ is less clear, but uncertainty in the data do not let us conclude anything else. We should increase the total measurement time to obtain a better picture, say up to one day or maybe more. Unfortunately we are now limited by piezo range and variation of the temperature in the room.

5.6 Can we do better?

In this section we shall discuss the possibility to overcame the periodic pulling effect with the frequency stabilized. Of course achieving passive stability, especially thermal, can do the difference. However, as in our case, this is not always possible. If we had the possibility to control the air conditioner, especially at night, a lot of problems, with the laser locked or free running, should be solved or alleviate.

We are interested in solutions of the problems introduced by the frequency stabilization using one piezoelectric transducer. Moving with four piezoelectric transducers all four mirrors is an obvious correction: this way we correct the perimeter uniformly changing the distances between mirrors. Of course four piezos are too many.
A cheaper and interesting solution could be obtained with only two PZT’s, diagonally arranged. This is something similar to what already proposed by Rodloff [25]. Rodloff used a triangular ring laser with two mirrors moved by a PZT. Simply, the same correction signal can be send to both PZT’s. The correction of the perimeter thus obtained is symmetrical enough to let the backscattering unchanged, at least in first approximation. This is true if the cavity is perfectly aligned. Even with a not well aligned cavity, change of the Sagnac signal with perimeter should be much less than under the movement of only one piezo. This is shown by simulations in Figs. 5.25 and 5.26 (notice the much larger scales) and confronted by the previous simulation in Fig. 5.27 (simulations obtained moving only a PZT are shown in Figs. 5.10 to 5.12 on pages 64–65). Of course one could always try to obtain a better alignment. Even if the pulling is not cancelled, surely its effects should be a lot mitigated, allowing, for example, to remove them post-processing the data.

One of the advantages of this scheme is the possibility to search a minimum of the pulling moving only a piezo and then engage the stabilization mechanism with both PZT. Unfortunately there is no guarantee to find such a minimum [25]. Moreover we should consider hysteresis effects in both PZT and the possibility that the two transducers react (slightly) differently to the same correction signal (introducing a small asymmetry, changing the interference of backscattering, etc.).

We can also imagine to use one of the transducer to lock the perime-

**Figure 5.25:** Numerical simulation of the pulling of the Sagnac frequency with perimeter variation induced by two PZT’s diagonally arranged. The movements of the two piezos are identical. No mode jump assumed. Cavity not well aligned and parameters as in Fig. 5.12.
Figure 5.26: Numerical simulation of the pulled Sagnac frequency under perimeter stabilization induced by two PZT diagonally arranged. No mode jump is assumed, even if the temperature excursion is quite wide. Cavity not well aligned and parameters as in Fig. 5.12.

Figure 5.27: Comparison of the pulling of the sagnac frequency obtained with: blue, only one piezo; green dotted, two PZT diagonally arranged. Cavity not well aligned and parameters as in Fig. 5.12.
ter and the other one to keep the backscattering under control (for example avoiding the problem of the different response of the two PZT).

The backscattering can be monitored in several ways (see section 2.2): the monobeam amplitude modulation at the Sagnac frequency is a good parameter; the phase between the two single modulation can give some insight; or the shape of the Sagnac signal, distorted by backscattering, can be used (it can be shown that the Fourier spectrum of the signal presents a geometric progression of harmonics, related to backscattering [33]).

A more exotic approach would be the use of external mirrors, moved by piezoelectric transducers so that the returned waves were in counter phase with the scattered waves, as suggested in [23].

Of course, any other technical improvement (cleaning of the mirrors or new mirrors) can reduce the backscattering and mitigate the problem.
CHAPTER 6

Conclusions

This thesis presents the intensive study of the G-Pisa ring laser gyro. Few points can be established by our experiments:

- ring-down time measure gives a quality factor of the cavity $Q = 1.1 \times 10^{11}$, obtained after cleaning the supermirrors;

- long term operation of the gyro was limited by hydrogen contamination: baking and installation of getters solve the problem;

- a good multimode mode-locked operation can be easily achieved, followed with good Sagnac signal;

- backscattering plays a crucial role and affects systematically the observed Sagnac signal; in our system the expected 111 Hz Sagnac frequency can vary up to 5 Hz because of backscatter-induced pulling;

- split modes and random mode jumps due to thermal expansion cause the gyro laser to be intermittently blind to rotation.

To solve this last problem and allow continuous operation of the gyro an amplitude stabilization and optical frequency stabilization loops was designed and tested:

- our frequency lock system can achieve a 30 kHz stability integrating for 100 s;

- split modes and mode jumps are avoided with the frequency stabilized;

- the current operations are limited by piezo range and thermal excursion of the laboratory: now we can work for several hours;
is significant in many ways. First, the large peak at the gyro laser is the power spectrum of the Sagnac frequency. Figure 6.1 spectra are recorded with the stabilization loops described in the previous measures at night, that shows the best spectra. The power spectrum has about 2 Hz has been recognized as due to resonant rotational movement of this dependency on the geometry of the cavity can be explained by a simple theory of backscattering.

We conclude studying the sensitivity of the gyro.

6.1 Sensitivity of the gyro

The sensitivity of the gyro can be described in terms of two main features: resolution and stability. Our principal instrument to check the resolution of the gyro laser is the power spectrum of the Sagnac frequency. Figure 6.1 shows three examples of such spectra, recorded at different times. These spectra are recorded with the stabilization loops described in the previous chapter engaged.\footnote{Indeed, in the last measurement the piezo had reached end of range, but we luckily found a good Sagnac signal.} Without the stabilization we were not able to record measures at night, that shows the best spectra. The power spectrum has been translated to an angular velocity for convenience.

A plot as Fig. 6.1 is significant in many ways. First, the large peak at about 2 Hz has been recognized as due to resonant rotational movement of
the optical table. Turning the table dampeners on or off shifts the resonant frequency. Second, the measure done at night (2:00 in the morning) is a lot quieter than the others. We can conclude that we are now limited by environmental (seismic) noise, mostly caught by the floating structure of the floor of the laboratory. Thus, these spectra are not the fundamental limit of the instrument. Moreover, the top resolution obtained, as low as $5 \times 10^{-9} \, (\text{rad/s})/\sqrt{\text{Hz}}$ at 0.5 Hz is encouraging. The different very-low frequency response of the three traces is surely imputable to a unequal backscattering pulling during the three measures.

On the other hand, the stability, especially on the long term, is given by the Allan variance of the Sagnac frequency. Two problems can affect such a measure: split modes and backscattering-induced pulling. The systematic effects due to backscattering are too wide for considering the Allan variance measure, especially with the perimeter lock engaged. Moreover, with the laser free running, split modes do not let us integrate for much time. We can give a significant measure of the Allan variance of the Sagnac signal only in the windows of signal without mode hops and split modes. Figure 6.2 shows three examples. These data is obtained with the amplitude stabilization engaged, but not the perimeter one: the laser was left running during a week-end. The Sagnac frequency data obtained (using an autoregressive algorithm) is searched for long period without mode jumps: the data reported refers to total measurements time of more than 30 min. (Let’s note, however, that such windows are usually much shorter, 10 min or even less, and do not let us integrate for much time.) Only windows that shows little drift of the frequency are considered. The sampling frequency is 1 Hz.

The Allan deviation $\sigma(\tau)$ decreases as $\sigma(\tau) \sim \tau^{-0.8}$ till $\tau \approx 100$ s. This gradient correspond to a dominant flicker phase noise (usually associated with noisy electronics) and white phase noise. The minimum at 100 s reaches 10 mHz or, expressed relatively to the nominal Sagnac frequency $0.9 \times 10^{-4}$. (Notice however that the systematic pulling of the Sagnac frequency are much larger.) After the minimum it flattens or increases slightly. This is related to environmental causes; the increasing is probably due to a (residual) backscattering-induced drift.

### 6.2 Future plans

A lot of other works and tests are planned for the future, ranging from small improvement to full-scale change in the apparatus. The amplitude and perimeter stabilizations, engaged accordingly the needs, will surely help, granting continuous operations without the fear of split modes.

Brand new electrodes of the discharge capacitor, of different lengths, are ready for testing. We want to study if a stabler or more symmetrical discharge can be achieved. All the locking circuit described in chapter 5 are
Figure 6.2: Three examples of the Allan deviation of the Sagnac signal. These was obtained from (particularly long) windows of signal without split modes, with the amplitude stabilization engaged, but not the perimeter one. The sampling frequency for every trace is 1 Hz. Total data length is as follows: ◦ 2052 s; △ 2068 s; □ 5958 s.

waiting definitive housing, preferably protected from irradiated and environmental noise.

Obtaining a better thermal stability of the room seems fundamental. Simply providing a portable air conditioner to be used ad hoc should mitigate greatly the change in backscattering with the perimeter stabilization engaged. Alternatively, since the floating floor of the room is also a limiting factor, even with the table dampers, moving in a stabler laboratory is currently under discussion. Waiting this change, modifying the servo loop of the perimeter stabilization so to execute reset jumps when the PZT is out of range could be a good compromise.

We plan to test a new set of four supermirrors. In the same time the cavity will be shrunk slightly, from a side length of 1.40 m to 1.35 m (to account for radius of curvature of the new mirrors). The new mirrors will be of better quality—with less backscattering—than the current ones, improving the performance of the system.

In the end, two ideas are intriguing us: the first is the purchase of a second piezoelectric transducer, that could open new possibility. The second is more radical: the change to a passive ring cavity, with a laser externally injected. This should overcome any trouble with backscattering, since of course is a very different system.


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