Searching for high—z galaxies through lensing by massive galaxy clusters

Tesi in Astrofisica e Scienze dello Spazio

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matr.242/012

Sessione di Luglio
Anno Accademico 2009/10
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Introduction

The epoch of formation of the first stars and galaxies is probably located between redshift 30 and 10 (corresponding to about 150 and 500 million years after the Big Bang). This period of cosmic time is generally known as the epoch of reionization, as the diffuse intergalactic neutral hydrogen (HI) was absorbing and scattering the UV photons. The epoch of reionization is one of the present most intriguing areas of research in observational cosmology: the aim is to investigate the nature of the first cosmic structures and to understand how they built up and evolved.

Present instruments and analysis techniques allow us to investigate the Universe at very high redshift, near the end of reionization epoch. Ultra deep imaging allows us to observe the farthest objects seen so far, quasars at $z \sim 6$, that are extremely bright objects founded in the nuclei of galaxies. High redshift quasars give us information about the intergalactic medium, but are not useful to enlarge our knowledge on sources that populated the reionization epoch: the large amount of radiation that they emit, dominates with respect to the host galaxy emission, preventing us to get information on the high redshift host galaxy.

Constraints on the evolution of the ultraviolet (UV)-luminosity function at high-redshift suggest a low-luminosity population of galaxies as the main source of UV photons that ionized the neutral hydrogen at the end of the epoch of reionization, in broad agreement with the so-called hierarchical scenario for galaxy evolution. However, the direct detection of low-luminosity galaxies at $z > 7$ is still difficult, and no candidate source has yet a confirmed spectroscopic redshift.

Gravitational Telescopes offer an efficient technique to search for high redshift galaxies lensed by foreground massive clusters. At high redshift the lensing magnification provided by massive galaxy clusters allows to probe the faint end of the luminosity function $\sim 1$ mag fainter than blank field surveys, although on much smaller sky areas. So, at present day, Gravitational Telescopes offer a unique way to observe high redshift faint galaxies. This thesis project is focused on the application of such astrophysical technique: first, we will determine the lensing properties of two massive galaxy clusters (that is: "calibrate the instrument"), and, second, we will search (using deep multiwavelength photometric data) for amplified sources at high-redshift. We will also determine (in a preliminary study) their average properties. This thesis work is indeed just a first step in a long-term project: next mandatory step is to obtain deep spectroscopic data of our high-redshift candidates in order to confirm their nature.

This thesis work is organized as follows. In Chapter 1 we introduce the
theoretical scenario and the observable probes that support the Reionization Epoch. Moreover we argue the candidate sources of the Reionization and the main techniques to investigate these sources. In Chapter 2 we review basic concepts and equations of Gravitational Lensing and the lensing phenomena expected for different lens model of the lensing system. In Chapter 3 we present applications of Gravitational Telescopes technique to search for high redshift galaxies and we introduce Strong and Weak lensing mass modelling of galaxy clusters. Then we search for high redshift \((z > 6) \) Lyman-Break Galaxies through the dropout technique: we fix photometric criteria to select high redshift candidate magnified by massive clusters, and then we estimate the photometric redshift of these candidates. Finally, in Chapter 4 we check the final sample to delete spurious selections and compare our final number of high redshift candidates with the expected number counts computed by simulations (Maizy et al. 2010). Then we correct the magnitudes of our sample for the magnification factors, we compute the stellar mass and the star formation rate (SFR) for the final candidates and we compare our results with other works. In the Appendix we also report about a complementary project (started in spring 2010), aimed at detecting galaxies at the *epoch of reionization* by targeting the host galaxy of the most distant gamma-ray burst known to date, GRB 090423, at \( z \simeq 8.2 \).
Chapter 1

The Re-ionization Epoch

The observation of the first stages of galaxy evolution, less than one billion years after the Big Bang, is finally within the reach of modern instruments. Present-day telescopes are now detecting sources at $7 \lesssim z \lesssim 10$, allowing us to investigate the epoch of formation of the first galactic systems, called the Epoch of Re-ionization (EoR). In this epoch, the neutral hydrogen (HI) of the intergalactic medium (IGM) is ionized by the UV radiation emitted by the first galaxy structures: this process rendered the Universe transparent, terminating the so-called Dark Ages. Studying the nature and the abundance of the structures responsible of this ionization is crucial to study the formation and evolution of the first cosmic structures.

1.1 The cosmological model

Our current understanding of the evolution of the Universe is based on the Big Bang theory, that describes its evolution from the initial singularity to the present day observed Universe. This theoretical model is based on the assumption of the Cosmological Principle and the observation of the expanding, isotropic Universe. The Cosmological Principle assumes that our place in the Universe is not a privileged one: at any given cosmic time, any observer would observe (on average) the same properties of the Universe. This is a generalization of the Copernican Principle to cosmology. Together with the observation of the Hubble flow$^1$, and the isotropy of the Cosmic Microwave Background Radiation (CMBR), this leads to a model of the Universe that

\[ v_{1,2} = H_0 d_{1,2} \]

$^1$For each pair of galaxies, the relative velocity $v_{1,2}$ and distance $d_{1,2}$ satisfy the Hubble law: $v_{1,2} = H_0 d_{1,2}$
is homogeneous and isotropic on large scales\(^2\). Such an expanding, isotropic Universe is described by the Friedmann-Robertson-Walker metric (e.g., Weinberg 2008).

A cosmological model is completely defined once that the values of the following cosmological parameters are fixed: the adimensional density of the Universe \(\Omega_0 = \Omega_m + \Omega_\Lambda + \Omega_\gamma\) \(^3\), the Hubble constant \(H_0 = 100h \text{ km} \text{ s}^{-1} \text{ Mpc}^{-1}\), the rms mass fluctuations on \(8h^{-1} \text{ Mpc}\) scale \(\sigma_8\), and the spectral index of the primordial density fluctuation \(n\).

The cosmological parameters \(\Omega_m, \Omega_\Lambda, \Omega_\gamma\) define the relative contribution to the overall mass-energy budget, of the matter component, the cosmological constant and the radiation respectively, and hence define the space-time geometry\(^4\) of the Universe.

At the present time the so-called \(\Lambda\)-Cold Dark Matter (\(\Lambda\text{CDM}, \text{hereafter}\)) cosmological model is the best available description of the observed Universe, on the basis of a large set of astronomical observations. This model assumes the presence of a large cosmological constant \(\Lambda\), and a dark matter component, in addition to the baryonic matter (i.e., electrons, protons and neutrons). On the basis of numerical simulations of evolution of cosmic structures, dark matter is expected to be made of collisionless particles interacting only through gravitational interaction and to be initially cold, having very small velocity dispersion. In this thesis we assume a standard \(\Lambda\)-CDM cosmological model with flat spatial geometry, for which the cosmological parameters are: \(\Omega_0 = 1.2 \pm 0.02, \Omega_\Lambda = 0.73 \pm 0.04, \Omega_m = 0.27 \pm 0.04, \Omega_b = 0.044 \pm 0.004, h = 0.71^{+0.04}_{-0.03}, \sigma_8 = 0.84 \pm 0.04\) and \(n = 0.93 \pm 0.03\) (Bennett et al. 2003). These values are determinate by experiments from the CMBR, being the ones that better fit the power spectrum of the CMBR temperature fluctuations; moreover they are in very good agreement with values estimated independently from experiments based on high-redshift Type Ia Supernovae (Knop et al. 2003).

According to the \(\Lambda\)-CDM model, after the Big Bang the Universe began to expand adiabatically and this caused its progressive cooling. At \(z \sim 1000\) the radiation temperature reached the value \(T \approx 4000\text{K}\) and the atomic particles (i.e. protons, electrons and neutrons) in the primordial plasma could combine in atomic forms (this epoch is called \textit{Recombination epoch}): the Universe became optically thin, letting the radiation to escape away. We

\(^2\)From observations of galaxy distribution we find that this principle is always verified on scales greater than few tens of Mpc.

\(^3\)where \(\Omega_m = \Omega_b + \Omega_{DM}\) describes the baryonic and dark matter.

\(^4\)As a fact, cosmologists still lack of a theory able to derive such numbers from first principles.
1.1 The cosmological model

Figure 1.1: Overview of the events from the Recombination to the present epoch. The blue regions represent epochs in which the Universe is in atomic form, while the red ones represent epoch of ionized Universe (Miralda-Escude 2003)

refer at this event as the last scattering surface, and the CMBR represents its observable fossil.

Because of their gravitational instability, primordial matter inhomogeneities hierarchically grew into structures that will become bounded systems: when fluctuations $\delta M/M$ reached the order of unity, the growth of inhomogeneities became non-linear and collapsed into the first structures of the Universe. These were the nurseries of the first stars born in the Universe, light-elements-massive stars, sources of a big amount of UV radiation. The period between the Recombination epoch and the ignition of first stars is called the Dark Age: in this epoch the Universe, composed by the CMBR, neutral baryonic- and dark-matter, continued to expand and to cool, with the contemporaneous growth of the density inhomogeneities. The EoR is the epoch in which the neutral inter-galactic medium of the Universe was ionized by a big amount of UV-radiation: nowadays the main candidates as sources of this UV radiation are the first galactic structures, thought to be small and low luminosity galaxies. The start of the EoR is related with the ignition of the first light-elements-massive stars and it ended when all the inter-galactic-medium (IGM) was ionized. At the moment, the end of the EoR is generally set at $z \sim 6$.

With the ignition of first stars, the UV radiation that they emit begins to ionize the neutral IGM in the surrounding regions, creating bubbles of ionized hydrogen (HII) around each star. This ionization proceeds enlarging the HII bubbles as long as the stars emit UV radiation and neutral IGM
1.2 Probes of Re-ionization

Evidences sustaining the EoR come from three main observational probes:

- the Gunn-Peterson effect, first observed in the high redshift quasar spectra;
- the damping and polarization of the CMBR anisotropies;
- the 21-cm signature of the HI.

The Gunn-Peterson effect and 21-cm signature give us direct information on the fraction of the neutral hydrogen (HI) in the IGM, while the anisotropies of CMBR give us information about the fraction of the HII.

1.2.1 Gunn-Peterson effect

The Gunn-Peterson effect corresponds to the absorption observed at low $\lambda$ in the spectra of sources at high redshift, and it was first observed in the...
1.2 Probes of Re-ionization

spectra of quasars at \( z > 5 \). Quasars are found in the nuclei of galaxies and are powered by the fall of matter of the accretion disk on the central black hole. They are very luminous objects with intrinsic broad-band spectra spanning from Radio to X frequencies. The study of these spectra will give us information about the interposed matter between the observed quasar and the observer. For example, comparing quasar spectra at different redshift we find that, the higher the redshift of the quasar, the higher the flux absorbed by the interposed matter (see Fig.1.2).

The absorption falls in the rest-frame region of the spectrum with \( \lambda \) shorter then the Lyman-\( \alpha \) wavelength \( \lambda_\alpha = 1216\,\text{Å} \), i.e. the photons emitted with energies higher than the Ly-\( \alpha \) energy are absorbed from interposed matter. This absorption is due to the neutral hydrogen of the IGM, that at each \( z \) between the \( z_{\text{source}} \) and \( z = 0 \), absorbs photons having \( \lambda(z) = \lambda_{\text{emitt}}(1 + z) = \lambda_\alpha = 1216\,\text{Å} \). The growth of absorption at redshift higher than 5 suggests a fraction of neutral hydrogen growing with \( z \). For quasars at redshift \( z \geq 6 \) the absorption due to the HI in the IGM seems to be almost complete. At lower redshift instead this absorption is not complete, and we find absorption lines come one after the other at \( \lambda \leq \lambda_\alpha(1 + z_{\text{source}}) \): in this case the radiation is only partially absorbed and this is related to a lower fraction of HI, that is the IGM must be in ionized form. This effect is called Lyman-\( \alpha \) forest (see Fig.1.3).

The photo-excitation cross-section of the Ly\( \alpha \) transition is

\[
\sigma(\nu) = \frac{e^2 f}{4\epsilon_0 m_e c } g(\nu - \nu_{\text{Ly}}) \tag{1.1}
\]

where \( \nu_{\text{Ly}} \) is the frequency of Lyman-\( \alpha \) transition, \( \nu \in [\nu_{\text{Ly}}, \nu_{\text{Ly}} + \Delta \nu] \), \( f = 0.416 \) is its oscillator strength, \( g(\nu - \nu_{\text{Ly}}) \) describes the absorption line profile and it is normalized so that \( \int g(\nu)d\nu = 1 \) (since this profile is highly peaked at \( \nu_{\text{Ly}} \) we can approximate \( g(\nu) \) with a delta function).

The optical depth due to the Lyman-\( \alpha \) scattering for radiation emitted by a source at redshift \( z_s \), with frequency \( \nu_1 \), that in absence of scattering would be observed at \( z = 0 \) with frequency \( \nu_0 \), is:

\[
\tau_{\text{GP}} = \int \sigma(\nu)N_{\text{HI}}(z)dl = \int \sigma(\nu)N_{\text{HI}}(z) \frac{dr}{1+z} = \int_0^{z_s} \frac{\sigma(\nu)N_{\text{HI}}(z)}{(1+z)H(z)}dz
\]

\[
= \frac{c}{H_0} \int_0^{z_s} \frac{\sigma[\nu_0(1+z)]N_{\text{HI}}(z)}{(1+z)[(\Omega_0 z + 1)(1+z)^2 - \Omega_\Lambda z(z+2)]^{1/2}}dz
\]

\[
= \frac{e^2 f}{4\epsilon_0 m_e H_0} \int_0^{z_s} \frac{N_{\text{HI}}(z)g[\nu_0(1+z) - \nu_{\text{Ly}}]}{[\Omega_0 z + 1)(1+z)^2 - \Omega_\Lambda z(z+2)]^{1/2}}dz
\]

\[
= \frac{e^2 f}{4\epsilon_0 m_e H_0 \nu_{\text{Ly}}} \frac{N_{\text{HI}}(z_{\text{obs}})}{[\Omega_0 z_{\text{obs}} + 1)(1+z_{\text{obs}})^2 - \Omega_\Lambda z_{\text{obs}}(z_{\text{obs}}+2)]^{1/2}}, \tag{1.2}
\]
1.2 Probes of Re-ionization

Figure 1.3: Spectra of the quasar Q1422 + 2309 at $z = 3.62$: we can observe the Lyα forest effect due to the clouds of HI interposed between source and the observer (Becker et al. 2004).

where we assumed a delta function for $g(\nu - \nu_{\text{Ly}})$. Here $N_{HI}(z)$ is the column density of HI (measured in atoms m$^{-3}$), while $z_{\text{abs}} = \nu_1 / \nu_{\text{Ly}} - 1$ corresponds to the redshift at which the radiation (redshifted to the Lyman frequency) can be scattered. Inserting the values of the constants we obtain:

$$\tau_{GP}(z) = 4 \times 10^4 h^{-1} \frac{N_{HI}(z)}{H(z)}.$$  \hspace{1cm} (1.3)

At low redshift, the optical depth assumes values lower than unity, while at high redshift, a tiny amount of HI (for example a fraction $x_{HI} \sim 10^{-4}$) causes a complete absorption of photons with $\lambda \leq \lambda_{\text{Ly}}$; in Fig.1.4 the Gunn-Peterson optical depth is plotted, as a function of the redshift, for nineteen quasars from the Sloan Digital Sky Survey (SDSS) with $z > 5.7$. For $z < 5.5$ the best fit is given by the relation $\tau_{GP} \propto (1 + z)^{4.3}$, while at higher redshift the evolution of optical depth accelerates as $\tau_{GP} \propto (1 + z)^a$ where $a > 10$. This growth of the optical depth with redshift is related to the increasing fraction of the HI, while the low values of $\tau_{GP}$ at lower redshift are related to the growth of the fraction of the ionized hydrogen. Because the Gunn-Peterson effect dominates from $z \sim 6$ to upper redshift, it means that around $z \sim 6$ something happened that made the IGM ionized. The gradual change from the Lyman-forest to the Gunn-Peterson effect suggests that the ionization of the IGM was not an instantaneous process, but that it was a process that spans a particular interval of time, and so can be observed on the correspondent interval of redshift. Gunn-Peterson effect allows us to fix a lower limit to the Re-ionization epoch that is at $z \sim 6$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{Q1422+2309_z=3.62.png}
\caption{Spectra of the quasar Q1422 + 2309 at $z = 3.62$: we can observe the Lyα forest effect due to the clouds of HI interposed between source and the observer (Becker et al. 2004).}
\end{figure}
1.2 Probes of Re-ionization

Figure 1.4: Evolution of the $\tau_{GP}$ for nineteen quasars from SDSS with combined Ly$\alpha$ and Ly$\beta$ results (Fan et al. 2006).

1.2.2 CMBR anisotropies

Anisotropies of the CMBR give us another evidence sustaining the Re-ionization. The CMBR is the fossil remnant of the Universe as it was when recombination occurred, that is the radiation that escaped away when the Universe became optically thin: it brings us information about the early Universe. As far as Recombination occurred, photons were highly scattered from free electrons in the primordial plasma, and only when the recombination in atoms of baryonic particles occurred, radiation started to expand freely.

The anisotropies observed in the CMBR give us information about initial condition of the Universe (primary anisotropies due to fluctuations of velocity, density and gravitational potential) and about the structures and dynamics of the Universe from the Recombination epoch to the present epoch (secondary anisotropies due to Integrated Sachs-Wolfe (ISW)$^5$ effect, gravitational lensing and global and local ionization effects); moreover fitting the power spectrum of CMBR anisotropies leads to constrain the cosmological parameters.

The Re-ionization can alter the power spectrum of the anisotropies of the CMBR due to the scattering of a fraction of this radiation by free electrons: due to the Thomson scattering primary anisotropies can be damped (small

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$^5$The Integrated Sachs-Wolfe effect corresponds to the redshift of CMBR-photons induced by gravitational potential (see Sachs and Wolfe 1967).
1.2 Probes of Re-ionization

Figure 1.5: Re-ionization damping of the power spectrum of temperature anisotropies for a model fully ionized $x_e = 1.0$ out to the Re-ionization redshift $z_i$.

scale anisotropies can be erased), while new secondary anisotropies can be generated, as the polarization anisotropies on large scales (the cosmic microwave background is polarized at the level of a few microkelvins). If the Re-ionization occurred instantaneously at the redshift $z$, then the total scattering optical depth $\tau_T = \int_0^z \sigma_T n_e dl$ due to Thomson scattering is (Hu W. 1995):

$$\tau = 0.041 \frac{\Omega_b h}{\Omega_m} \left\{ \left[ 1 - \Omega_m + \Omega_m (1 + z)^3 \right]^\frac{1}{2} - 1 \right\} = \begin{cases} 0.037 & \text{if } z = 5.8 \\ 0.10 & \text{if } z = 11.6 \\ 0.15 & \text{if } z = 15.3 \end{cases}$$

where $\sigma_T = 6.65 \times 10^{-25}\text{cm}^{-2}$ is the Thomson cross-section and we used the cosmological parameters value $\Omega_b = 0.045, \Omega_m = 0.3, h = 0.7$.

In Fig.1.5 they are shown simulations of the damping in the power spectrum of temperature anisotropies for different values of $\tau$. The damping of anisotropies is expected to grow with $\tau$: for $\tau \leq 1$ primary anisotropies are expected to be observed, but at higher value of the optical depth they are expected to be erased.

For angular scales lower than the horizon at the Re-ionization, in a direction $\mathbf{n}$ the damp in the CMBR temperature is given by $T'(\mathbf{n}) = e^{-\tau} T(\mathbf{n})$, which corresponds to the damp $C_l' = e^{-2\tau} C_l$ in the coefficient of the spherical harmonics decomposition of the CMBR anisotropies. Observations of small-scale anisotropies showed a peak in the power spectrum on the scale of $\sim 1^\circ$, as expected from primary anisotropies, which indicates that the Re-ionization damping, if present, is not very large, and it sets the upper limit for the Thomson scattering optical depth $\tau \approx 0.33$, that is Re-ionization
must occur at redshift \( z \leq 30 \).

Studying polarization anisotropies can help to determine better \( \tau_{TH} \), and so the redshift range in which Re-ionization could occur. When unpolarized radiation is scattered by free electrons, the outgoing radiation is polarized: the unpolarized radiation incident the free electron causes the electron to oscillate in the plane orthogonal to the incident radiation direction; the oscillating electron generates a dipole emission field with the electric field vector \( \mathbf{E} \) parallel to the oscillation plane of the electron, that is the emitted radiation is linearly polarized. For an isotropic radiation field, there will be balancement from incident photons having directions separated by 90° so that the outgoing radiation remains unpolarized. Polarization occurs only if the incident radiation field has a non zero quadrupole term in intensity or temperature (which has intensity peaks at 90° separations): then the result is a linear polarization of the scattered radiation. This is the CMBR case: the CMBR-field scattered by the free electrons resulting from the ionization process, will present polarization anisotropies that give us information about the Re-ionization epoch. The CMBR polarization presents two distinct components, the symmetric (\( E\)-mode) and antisymmetric (\( B\)-mode) components, being based on the symmetry and antisymmetry properties under parity transformations. The \( E\)-mode polarization anisotropies are due to the Thomson scattering by the free electrons in the Re-ionization epoch and cause peaks in the power spectrum at multipole \( l \simeq 1000 \) (or \( \simeq 10' \) angular scales). The \( B\)-mode arises in inflationary models that predict a primordial gravitational wave background.

Polarization anisotropies has been detected by WMAP at scale \( > 10' \): if Re-ionization was instantaneous, first-year data (Kogut et al. 2003) conduct to the value of \( \tau = 0.17 \pm 0.04 \), suggesting that the IGM was largely ionized by \( z \sim 17 \pm 4 \) (from equation 1.4), while WMAP three-year data (Spergel et al. 2006) show a smaller optical depth \( \tau = 0.09 \pm 0.03 \) based on the E-mode polarization measurements, indicating a largely ionized IGM by \( z \sim 10 \pm 3 \). In Fig.1.6 the temperature-polarization cross-correlation power spectrum obtained from WMAP three-years data is shown: the best fit of this anisotropies gives \( \tau = 0.09 \), that corresponds to \( z = 10.7 \), in the hypothesis of instantaneous Re-ionization. In the more realistic case of a gradual ionization of the IGM, this value of \( \tau \) fixes the redshift upper limit of the Re-ionization epoch at \( z \approx 30 \) (Page et al. 2007).

### 1.2.3 21 cm signature

We have seen that Gunn-Peterson effect and CMBR anisotropies allow us to fix lower \( (z \sim 6) \) and upper \( (z \leq 30) \) limit for the Re-ionization epoch.
1.2 Probes of Re-ionization

Figure 1.6: Temperature-polarization cross-correlation power spectrum $C^T_E$ obtained from WMAP three-years data: the solid line is the best fit for the WMAP cosmological model, with $\tau = 0.09$ (Fan et al. 2006).

During the Re-ionization epoch the IGM can be investigated studying the 21 cm signature of the HI.

Neutral hydrogen emits radiation at 21-cm because of the spin-flip transition from the triplet to the singlet hyperfine levels of the atomic ground-state ($n = 1$). In pre-Re-ionization epoch as long as the HI of the IGM and the CMBR are in thermal equilibrium, the IGM results unobservable. When some processes occur moving the hyperfine population levels away from thermal equilibrium, than the HI becomes observable in emission or absorption against the CMBR. When HI and radiation are in thermal equilibrium ($T_{HI} = T_{CMBR}$), the distribution of HI population in singlet ($n_0$) and triplet ($n_1$) state is described by

$$\frac{n_1}{n_0} = 3\exp \left\{ \frac{-T_s}{T_S} \right\}$$  \hspace{1cm} (1.5)

where $T_s = 0.7$K is defined by the 21 cm energy transition $k_B T_s = E_{21} = 5.9 \times 10^{-6}$eV, and $T_S$ is the hydrogen spin temperature, defined as the temperature that the gas would have if it was in thermal equilibrium with the background radiation, given the $n_1/n_0$ ratio. So if there is only the CMBR background, the HI reaches thermal equilibrium with $T_S = T_{CMBR} = 2.73(1+z)$K on the time scale $T_s/(T_{CMBR}A_{10}) \approx 3 \times 10^6(1+z)^{-1}$yr, where $A_{10} = 2.9 \times 10^{-15}$s$^{-1}$ is the spontaneous decay rate of the hyperfine transition. If gas and ra-
diation are not in thermal equilibrium, implying that $T_{HI} \neq T_{CMBR}$, then it is required an effective mechanism that decouples this two temperatures, and couples $T_S$ and $T_{HI}$ of the gas itself. There are two mechanisms available: collisions between hydrogen atoms and scattering by Ly$\alpha$ photons. The collision mechanism induces coupling between $T_S$ and $T_{HI}$ by the spin-exchange process between the colliding hydrogen atoms, but the estimate rate of this mechanism for realistic IGM densities at the redshifts of interest is too small. The coupling mechanism due to Ly$\alpha$ scattering mixes the hyperfine levels of neutral hydrogen in its ground-state: an atom in the n=1 state can be excited by a Ly$\alpha$ photon that puts it in the n=2 state; through a spontaneous decay it will come back to the ground-state but the final spin-state can be different from the initial one (Wouthuysen-Field effect). So due to the Ly$\alpha$ photons the initial thermalized HI is no more in thermal equilibrium, that is $T_{S} \neq T_{CMBR}$ with the consequent emission or absorption of the 21-cm line. The optical depth for a 21(1+z)-cm photon travelling through a region of neutral hydrogen with a uniform $T_S$ at high redshift $z$ is

$$\tau(z) = \frac{3c^3h^3m_{HI}(0)A_{10}}{32\pi H_0^2k_0^2 T_s^2 T_S} (1 + z)^{3/2} \approx 10^3 h_{50}^{-1} \left( \frac{T_{CMBR}}{T_S} \right) \left( \frac{\Omega_{IGM} h_{50}^2}{0.05} \right) (1 + z)^{1/2}. \quad (1.6)$$

The brightness temperature through the IGM is $T_b = T_{CMBR} e^{-\tau} + T_S (1 - e^{-\tau})$, so the observed differential antenna temperature of this region with respect to the CMBR will be

$$\delta T = (1 + z)^{-1} (T_S - T_{CMBR}) (1 - e^{-\tau}) \approx (0.011 K) h_{50}^{-1} \left( \frac{\Omega_{IGM} h_{50}^2}{0.05} \right) \left( \frac{1 + z}{9} \right)^{1/2} \eta, \quad (1.7)$$

where the 21-cm radiation efficiency is defined as

$$\eta \equiv x_{HI} \left( \frac{T_S - T_{CMBR}}{T_S} \right) \quad (1.8)$$

and $x_{HI}$ is the neutral fraction of HI in the region with $T_S \neq T_{CMBR}$.

The brightness temperature is related to the density of the neutral hydrogen: in ionized regions $T_j$ is proportional to the fraction of HI. For $T_S \sim T_{CMBR}$ no 21-cm signal is expected. If $T_S \gg T_{CMBR}$ it means that there are many Ly$\alpha$-photons heating the HI: $\eta \rightarrow x_{HI}$, and the IGM can be observed in emission (in this case the emission result independent of $T_S$). By contrast, when $T_S \ll T_{CMBR}$ the IGM is observed in absorption with $\eta \propto T_{CMBR}/T_S$.

Investigating the 21 cm signature result to be a useful method to get information about the evolution of neutral IGM in the Re-ionization epoch: indeed we can trace the 21 cm signature as a function of the redshift in the Re-ionization epoch (unlike the Gunn-Peterson effect that saturates at $z > 6$);
moreover from the 21 cm signature we can obtain 3D information about the evolution of cosmic structures (unlike the CMBR anisotropies that give us only projected information).

1.3 Sources of Re-ionization

As discussed above, there are different of probes of an epoch in the cosmic evolution in which the neutral intergalactic medium (IGM) was ionized by an intense UV field, in agreement with predictions by the Λ-CDM model. We now discuss the possible sources responsible of the ionization of the IGM, during the so-called Re-ionization epoch, that are quasars and AGN, star-forming galaxies, X-ray sources and decaying sterile neutrinos (Fan et al. 2006).

Quasars and AGN are high UV-photon emitters and their photon escape fraction is assumed to be of the order of unity, but the luminosity density of these objects (well determinate from surveys at low-redshift) shows a net fall at high-redshift: it decreases exponentially at redshift higher than $z \sim 6$, implying that their contribution to the Re-ionization radiation is not dominant in the ionization process (see, e.g., Fan et al. 2006).

X-ray sources at high-redshift (as mini-quasars or supernova remnants) could give an important contribution to the IGM ionization: X-ray photons could ionize the IGM significantly producing a rapid Re-ionization, consistent with the lower limit at $z \sim 6$, and providing the low optical depth obtained by the WMAP measurements. But, if these were the primary sources of the ionizing UV photons, then at present time we would observe a soft X-ray background higher than the one actually observed.

Decaying sterile neutrinos have been suggested as candidates for the Re-ionization epoch (Boyarsky al. 2009), but in this case too it is proved that, from the observed background radiation at present cosmic time, this contribute must be low.

Primeval star-forming galaxies result to be the optimal candidates as sources of Re-ionization: these galaxies must be composed by the very first population of stars (Population III stars) that must be metal-free, massive stars.

---

6The escape fraction is defined as the fraction of emitted UV-photons that escape a galaxy without being absorbed by its interstellar material.
1.3 Sources of Re-ionization

1.3.1 Population III stars

The first stars must be formed from the primordial metal-free IGM and, compared to present time stellar population, they are thought to be very massive structures ($M \sim 100 M_\odot$) characterized by a very short life-time.

To form stars, clouds of IGM must collapse following the collapse of dark matter halos. The dynamic of collapse is governed by unbalancement between gravitational and radiation pressure forces. For a cloud of gas, the cooling time $\tau_{\text{cool}}$ and the free-fall times $\tau_{\text{ff}}$ are

$$\tau_{\text{cool}} = \frac{3kT}{2n\Lambda(T)}, \quad \tau_{\text{ff}} = \left(\frac{3\pi}{32G\rho}\right)^{-1/2},$$

(1.9)

where $n$ is the total gas number density, $\Lambda(T)$ is the cooling function, $T$ and $\rho$ are the temperature and the density of the gas. Of course both these times must be lower than the Hubble time $\tau_H = H(z)^{-1}$ to allow the cloud to collapse.

If $\tau_{\text{cool}} \ll \tau_{\text{ff}}$ the thermal energy of the gas is efficiently lost through radiation and as long as the pressure is negligible the cloud evolves in an almost free-fall collapse. On the contrary, if $\tau_{\text{cool}} \gg \tau_{\text{ff}}$ the cloud collapses adiabatically reaching a quasi-equilibrium virialized state, from which evolves within the cooling time.

The cooling function is determined by:

1. Bremmstrahlung emission if $T_{\text{vir}} > 10^{5.5}\text{K}$ ,

2. collisional excitation of H and He if $10^4\text{K} < T_{\text{vir}} < 10^{5.5}\text{K}$ ,

3. molecular radiative emission if $100\text{K} \leq T_{\text{vir}} \leq 10^4\text{K}$ .

$\Lambda(T)$ decreases rapidly if $T_{\text{vir}} < 100\text{K}$, and the gas pressure is sufficient to prevent the collapse of the cloud. For usual values of IGM, we find that the virialized temperature falls in the $(100\text{K}, 10^4\text{K})$ range. The atomic H radiative emission can occur only if there are collisions able to excite and ionize the atoms, and this is not the case because of $T_{\text{vir}} < 10^4$. So the only coolant available in the first clouds is the molecular hydrogen, even if only a small fraction of the gas is in the H$_2$ form: this molecule, in our range of $T_{\text{vir}}$, can not be formed by collisions of atomic H, but only via reactions involving the species H$^-$ and H$^+_2$ formed by residual free electrons and protons from the early Universe. Simulations shows that first stars form in halos with $M \sim 10^6 M_\odot$ and $T_{\text{vir}} \approx 2000\text{K}$ (at lower temperature the cooling rate of H$_2$ is not sufficient to cool the gas): in these conditions the cooling rate
1.3 Sources of Re-ionization

results to be low, but leads to formation of a central core of H$_2$ cooled at $\sim 200$K with a mass of 100 to 1000 M$_{\odot}$, from which the first massive stars can be formed (Abel T. et al.(1998), Yoshida N. et al.(2003)). There are many uncertainties about the formation of Population III stars, but it is largely accepted that they are metal-free, massive stars formed by clouds of unmagnetized light-elements IGM. The lack of metal implies that the nuclear reactions sustaining the evolution of these stars must be the $p-p$ chain, and this implies that they must have very high temperature. The high masses and temperatures of these stars suggest that they are very luminous objects ($L \sim L_{\text{Eddington}}$) dominated by radiation pressure. Simulations shows that they are well described by polytropic models with index $n = 3$ (Ferrara 2007): from the Lane-Emden equation, once fixed the central density and temperature, the effective temperature can be estimate, finding the lower limit $T_{\text{eff}} = 3 \times 10^4 K$: the optimal candidates for these stars are the $O$-type stars. These stars emit a large amount of UV photons that ionize the HI of the IGM, changing the surrounding environment of the stars: bubbles of ionized IGM (HII) grows around each star as long as the UV photons are emitted and ionize the HI boundaries of the bubbles.

Because of the high values of mass and temperature of the Population III stars, these must produce a larger amount of photons than the Population II $O$-type stars: in the He II continuum they emit up to $10^5$ times more photons than Population II. In Fig.1.7 the estimated spectra of Population

Figure 1.7: Spectra of massive Pop III stars. Solid lines correspond to the flux at the surface of the star, while dashed lines correspond to the blackbody spectrum evaluated at the respective effective temperature $T_{\text{eff}}$; the lines are due to the transition of H and He II (Bromm et al. 2001).
1.3 Sources of Re-ionization

III stars are showed with mass $M = 100M\odot$ and $M = 300M\odot$ (Bromm et al. 2001).

1.3.2 Galaxies as Re-ionization sources

At present time, low-luminosity star-forming galaxies are thought to be the dominant source of ionizing photons in the EoR. As a matter of fact, according to the hierarchical model of galaxy evolution, low-mass and low-luminosity galaxies are the natural candidates as sources of the Re-ionization. This prediction has received a first support by observational studies of star formation rates (SFRs) and luminosity functions (LFs) of $z \geq 4$ galaxies (see, e.g., Bouwens et al. 2009).

In Fig. 1.8 it is shown the rest-frame UV LF at different redshift as resulted from high-redshift galaxies surveys. An evident evolution with redshift appears in the range $4 \leq z \leq 7$; assuming that this evolution continues up to $z \sim 8$, the UV LF has been extrapolated at $z \sim 8$ and it matches the few $z \sim 8$ candidate galaxies detected so far. We note that, as the redshift increases, the characteristic absolute UV magnitude $M_{UV}^*$ becomes fainter ($M_{UV}^* \sim -21$ AB mag at $z \sim 4$, $M_{UV}^* \sim -20$ AB mag at $z \sim 6$, $M_{UV}^* \sim -19.3$ AB mag at $z \sim 8$). Moreover, the higher the redshift, the steeper the faint-end slope of the luminosity function: this trend reflects a low-luminosity dominated population of star forming galaxies at early epoch, yet to be directly detected.
1.3 Sources of Re-ionization

Figure 1.9: SFR density, integrated to $0.06L^*_{z=3}$ (-18 AB mag): the blue region shows the SFR density as obtained directly from UV light; the orange region shows the SFR density as results after dust correction inferred from the UV-continuum slope measurements at $z \sim 5 - 6$ (from Bouwens et al. 2010).

From the UV LF it is possible to estimate the star formation rate, as a function of the redshift. In Fig. 1.9 it is shown the SFR density, as obtained directly from UV (blue region) and the one obtained accounting for dust-corrected estimates from UV-continuum slopes (orange region). In both the cases the SFR density presents a maximum at $z \sim 2$. At high redshift ($z > 5$) both the dust-corrected and uncorrected SFR density histories follow a decreasing trend. However, such estimates of the SFR history can be plagued by possible systematic errors. For instance, they strongly depend on the assumed extinction models, yet to be directly tested at redshift higher than $z \sim 6$.

In order to search high-redshift, low-luminosity galaxies, two techniques are used, based on the identification of their main spectral features in broad-band photometry: the Lyman-$\alpha$ Break and the Lyman-$\alpha$ (Ly-$\alpha$) emission at $\lambda_{rest} = 1216$ Å.

Lyman Break Galaxies (LBGs) are young galaxies with stellar population dominated by short-lived O and B stars, so they are very luminous in the UV rest-frame. They present a typical break in their rest frame spectrum at wavelength $\lambda$ bluer than $\lambda_{rest} = 912$ Å. This break is due to the fact that energetic photons with $\lambda \approx 912$ Å (produced by the OB stars) ionize neutral hydrogen and have therefore a short mean-free-path. Moreover, at high redshift ($z > 5$), these photons may also be absorbed by clouds of HI along the line of sight from the source-galaxy to the Earth (in correspondence of the
1.4 Searching for high-\( z \) galaxies: lensing or blank-field surveys?

Observations of high-\( z \) galaxies are difficult, due to their small sizes and their faintness. Two main observation strategies are used to detect them: blank field surveys and the so-called gravitational telescopes technique. These two strategies are complementary in order to constrain the UV-LF at \( z > 6 \). Indeed, blank field surveys\(^7\) are efficient to investigate the bright end of the UV-LF, while robust studies of the faint-end of the LF, with present instruments, are only possible through the lensing magnification by massive galaxy clusters (see, e.g., Maizy et al. 2010).

A gravitational telescopes is a massive galaxy cluster located along the line of sight between the observer and more distant sources: due to the gravitational lensing effects it acts as a gravitational lens, magnifying foreground sources. The magnification provided by the massive cluster increases the number of the observable background faint galaxies. Indeed, close to the so-called critical lines, images of the lensed background sources are highly magnified (by a factor as large as 10-30), enabling the observation of low-

\(^7\)A blank field survey is a deep imaging survey of a given sky region, where no lensing effect is expected due to foreground mass distributions.
luminosity galaxies at high redshift, otherwise not detectable with our present instruments. We can verify the gain in the number of detected sources using Gravitational Lensing, with respect to blank field surveys, by estimating the number of detectable high-$z$ galaxies in both blank and lensing fields, fixed the redshift, the field of view and the depth of the observations. Given the luminosity function

$$\phi(L)dL = \left(\frac{\phi^*}{L^*}\right)\left(\frac{L}{L^*}\right)^\alpha \exp\left(-\frac{L}{L^*}\right)dL, \quad (1.10)$$

the number of galaxies in the magnitude and redshift bins, $dm$ and $dz$, is:

$$N(m,z)dm\,dz = \phi(L,z)dL \,dV(z). \quad (1.11)$$

Integrating over the observable range of luminosities, we obtain the number of the detected sources in the redshift bin $dz$:

$$n(z)dz = dV(z)\int_{L_{\text{min}}}^{\infty} \phi(L,z)dL = dV(z)\Phi[L_{\text{min}}(z),z], \quad (1.12)$$

where $L_{\text{min}}$ is the faintest luminosity detectable by the detector.

Let us suppose that we are observing a region in which a massive galaxy cluster is along the line of sight. Due to the lensing effects, the detectable magnitude range for the background sources is shifted deeper by the mean magnification factor $\mu(z)$, while the effective field of view, and so the surface density, is reduced by the same factor. The final number of background galaxies becomes:

$$N'(m,z) = \frac{N[(m + 2.5\log_{10}\mu(z)),z]}{\mu(z)} \quad (1.13)$$

At faint luminosities, $\Phi(L)$ is approximated by a power law, so from Eq.1.12 we obtain that

$$n'(z)dz = \mu^{-1}\Phi[L_{\text{min}}(z)/\mu, z]dV(z) \simeq \mu(z)^{\beta(z)-1}n(z)dz, \quad (1.14)$$

where

$$\beta \equiv -\frac{d\ln\Phi[L(z), z]}{d\ln L(z)} \quad (1.15)$$

is defined as the effective index of the luminosity function. If $\beta > 1$, the number of faint sources observed having a galaxy cluster along the line of sight is greater with respect to the simple blank field case. On the other hand, if $\beta < 1$, many faint galaxies are expected to be detected through blank field surveys rather than through Gravitational Telescopes. Increasing
1.4 Searching for high-z galaxies: lensing or blank-field surveys?

Figure 1.10: Relative gain in number counts between lensing and blank fields as a function of the redshift. Three different type of LF and three fields of view (FOV) are considered, $6' \times 6'$ (black), $2.2' \times 2.2'$ (blue), $0.85' \times 0.85'$ (red) (Maizy et al. 2010).

the depth and the field of view (that determines the mean lensing magnification $\mu$) of the survey, $\beta$ decreases leading to a loss in efficiency of the gravitational lensing with respect to a blank field survey. In Fig.1.10 it is shown the relative gain in number of observed sources between lensing and blank field as a function of the redshift, for fixed depth, three different fields of view (FOV) and for three different shape of the LF at $z \geq 6$ (see Sec.4.4 for the parameters values of these LF):

- LF(a): we assume that LBGs have no-evolution from $z \sim 6$ (Beckwith et al. 2006);
- LF(b): a constant LF based on measurement in the Hubble UDF at $z \sim 6$ (Bouwens et al. 2008);
- LF(c): an evolutionary LF with $L^*$ decreasing with redshift (Bouwens et al. 2008).

In the three panels in Fig. 1.10 we see that the gain induced by gravitational telescopes grows as a function of the redshift of the sources in the range $6 \leq z_s \leq 12$. Moreover, this gain is greater for smaller FOV: this is due to geometric considerations, since the mean magnification decreases by increasing the FOV. Once we fix the FOV, the shape of the LF strongly in-
fluences the final gain. We deduce that comparison between blank and lensing field surveys of the same FOV will help in constraining the UV-LF. Furthermore, these two techniques are complementary, since, as we have seen from Eq. 1.14, deep blank field surveys are efficient to investigate the bright end of the UV-LF, while gravitational telescopes are more efficient in constraining its faint end.
Chapter 2

Basics of gravitational lensing

Gravitational Lensing theory studies deformation, amplification and multiplication of images of a source (S) due to the space-time distortion caused by a mass concentration, called gravitational lens (L), placed between the source and the observer (O). These phenomena are naturally and fully described by the General Theory of Relativity and they provided the first observational evidence ever of its validity (see, for instance, Weinberg 1972). In this Chapter, we present the physical and mathematical basis of gravitational lensing (GL), together with some of the parametric models used to describe the light deflection by galaxies and galaxy clusters.

2.1 Basic concepts and equations

In order to derive the equations of light deflection caused by an astrophysical object, a few simplifying assumptions are usually made. First of all, a weak gravitational field is assumed, that is we assume that the Newtonian potential of the lens is small (i.e., $\phi \ll c^2$) and that the relative velocities of source, lens and observer are small compared to the velocity of light ($v \ll c$).

We also note that the physical size of the lens is much smaller than the distances between lens and source, and lens and observer: hence, we can approximate the lens as a plane distribution mass (thin screen approximation), and so it is well described simply by a surface mass density. According to this assumption, all the deflection action due to the gravitational potential takes place at a given distance, that is at the lens distance (see Fig. 2.1).

Such assumptions hold for the gravitational field of most cosmic structures, including massive galaxy clusters. For example, let us consider a source at redshift $z \sim 1$ lensed by a galaxy cluster at redshift $z \sim 0.3$: the distances between source, lens and observer are of order of several Gpc, while the typ-
2.1 Basic concepts and equations

Figure 2.1: Diagram of a gravitational lensing system.

The typical size of galaxy clusters is of order of a few Mpc. Moreover, the typical potential of galaxy clusters is $|\phi| \ll 10^{-4}c^2 \ll c^2$, i.e. weak, in relativistic terms.

Such assumptions allow us to describe gravitational lensing phenomena using General Relativity at first order. Therefore, we can locally approximate the metric with the Minkowski metric perturbed by the gravitational potential of the lens:

$$ds^2 = \left( 1 + \frac{2\phi}{c^2} \right) c^2 dt^2 - \left( 1 - \frac{2\phi}{c^2} \right) (d\vec{x})^2 \quad (2.1)$$

Let us now consider a mass concentration placed at the angular-diameter distance $D_L$, deflecting the light rays coming from a source at the angular-diameter distance $D_S$. A typical diagram of such a lensing system is presented in Fig. 2.1: the optical axis of the system (the dashed line) is perpendicular to the lens and source planes and passes through the observer point;

---

1In an Euclidean space, given a source with physical size $x$ and angular size $\xi$, the angular-diameter distance $D$ is defined as $D = x/\xi$: this relation derives from the trigonometric relation $\tan(\xi) = x/D$, that for small angles ($\xi \ll 1$, condition always verified in astrophysics situations) can be approximate with $\tan(\xi) \approx \xi$. In a curved space it is not always verified that physical size $x$ is equal to the angular size $\xi$ times the distance $D$. It occurs if we define distances that holds this relation ($x = \xi D$) even in a non Euclidean space. Moreover, generally speaking, cosmological distances are not additive, so referring to Fig.2.1 we have $D_S + D_LS \neq D_S$ but $\theta D_S = \beta D_S + \alpha D_LS$. 

the angular positions on the source and lens planes are measured relative to this axis.

Due to the gravitational distortion the observer sees the source at the angular position $\vec{\theta}$ while actually it is at the angular position $\vec{\beta}$. If $\vec{\theta}$, $\vec{\beta}$ and $\vec{\alpha}$ are small ($\ll 1$), from geometric considerations we have that they are related by the following equation, called lens equation, that is, a mapping equation from the image plane to the source plane:

$$\vec{\beta} D_S = \vec{\theta} D_S - \vec{\alpha} D_{LS}. \quad (2.2)$$

The deflection angle $\hat{\alpha}$ is related to the gravitational potential $\Psi$ of the lens by the following equation\(^2\) (e.g., Schneider et al. 1992):

$$\hat{\alpha}(\theta) = \frac{2}{c^2} \int \vec{\nabla}_\perp \Psi \, dz, \quad (2.3)$$

where $\vec{\nabla}_\perp \Psi$ is the gradient of $\Psi$ perpendicular to the light path. If introduce the surface density of the lens $\Sigma$, given by the projection on the lens plane of the lens density $\rho(\vec{r})$

$$\Sigma(\vec{\xi}) = \int_0^{D_s} \rho(\vec{r}) \, dz, \quad (2.4)$$

where $\xi = D_L \theta$, then the deflection angle reads

$$\hat{\alpha}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}') \Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} \, d^2 \xi'. \quad (2.5)$$

For a symmetric circular lens with constant $\Sigma$ the deflection is:

$$\alpha(\xi) = \frac{D_{LS}}{D_S} \frac{4G \Sigma \pi \xi^2}{c^2} \Rightarrow \alpha(\theta) = \frac{4\pi G \Sigma}{c^2} \frac{D_{LS} D_L}{D_S} \theta. \quad (2.6)$$

Defining the critical surface density $\Sigma_{cr}$ as

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_S}{D_{LS} D_L} \quad (2.7)$$

the deflection angle for a symmetric circular lens with constant $\Sigma$ reads

$$\hat{\alpha}(\theta) = \frac{\Sigma}{\Sigma_{cr}} \theta. \quad (2.8)$$

\(^2\)For a light ray passing through a medium with non constant refraction index $n = n(l)$, the deflection angle of the ray is given by $\vec{\alpha} = - \int \vec{\nabla}_\perp n \, dl$; similarly, for a ray passing through a weak gravitational field we can define an effective refraction index of the field as $n = 1 - 2\phi/c^2$, and the deflection is given by $\vec{\alpha} = 2/c^2 \int \vec{\nabla}_\perp \phi \, dl$. 
If the lens has $\Sigma > \Sigma_{cr} \Rightarrow \alpha > \theta$, in this case there is strong deflection and we are in the so-called Strong Lensing domain: multiple images can be produced and, if the source is an extended object, Strong Lensing can generate highly deformed images. Indeed the light bundles coming from the source are deflected differentially, and it causes the shapes of the images to be distorted compared to the source shape.

If $\Sigma < \Sigma_{cr} \Rightarrow \alpha < \theta$, there is weak deflection and we are in the so-called Weak Lensing domain: in this case no multiple images are produced, nor high deformation in the lensed images can be detected. Weak Lensing causes only small distortions in the image shape.

Defining the reduced deflection angle as follows
\[ \bar{\alpha}(\bar{\theta}) = \frac{D_{LS}}{D_S} \alpha(\bar{\theta}) , \] (2.9)
the lens equation reads
\[ \bar{\beta}(\bar{\theta}) = \bar{\theta} - \bar{\alpha}(\bar{\theta}) . \] (2.10)

We can introduce the effective lensing potential $\psi(\bar{\theta})$, that is, the rescaled projection, on the lens plane, of the three-dimensional Newtonian potential:
\[ \psi(\bar{\theta}) = \frac{D_{LS}}{D_S D_L c^2} \int \Psi(D_L \bar{\theta}, z) dz . \] (2.11)

The gradient along $\bar{\theta}$ of $\psi$ gives the reduced deflection angle function:
\[ \nabla_{\bar{\theta}} \psi(\bar{\theta}) = \alpha(\bar{\theta}) . \] (2.12)
Indeed,
\[ \nabla_{\bar{\theta}} \psi(\bar{\theta}) = \nabla_{\bar{\theta}} \frac{D_{LS}}{D_S D_L c^2} \int \Psi(D_L \bar{\theta}, z)dz = \nabla_{\bar{\theta}} \frac{D_{LS}}{D_S c^2} \int \Psi(D_L \bar{\theta}, z)dz = \]
\[ \frac{D_{LS}}{D_S} \frac{2}{c^2} \int \nabla_{\bar{\theta}} \Psi(D_L \bar{\theta}, z)dz = \alpha(\bar{\theta}) \] (2.13)
The Laplacian of $\psi$ gives the convergence function $\kappa(\bar{\theta})$, defined as the normalization of the surface density $\Sigma(\bar{\theta})$ by the critical surface density $\Sigma_{cr}$
\[ \nabla_{\bar{\theta}}^2 \psi = 2 \frac{\Sigma(\bar{\theta})}{\Sigma_{cr}} \equiv 2\kappa(\bar{\theta}) . \] (2.14)

When the convergence $\kappa(\bar{\theta}) > 1$ the surface mass density $\Sigma(\bar{\theta})$ is larger than the critical surface density $\Sigma_{cr}$, and we are in the Strong Lensing domain; on the contrary, if $\kappa(\bar{\theta}) < 1$ we are in the weak lensing domain.
Through the lens equation, each surface element $\delta \vec{\theta}$ of the image plane is mapped into a surface element $\delta \vec{\beta}(\vec{\theta})$ on the source plane:

$$
\delta \vec{\beta} = \left( \delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) \delta \vec{\theta} = \left( \delta_{ij} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right) \delta \vec{\theta} = A \delta \vec{\theta}, \quad (2.15)
$$

where the indexes $i, j = 1, 2$ refer to the components of the angles on the lens and the source planes, while $A$ is the Jacobian matrix of this transformation. In the following, we will see that the matrix $A$ can be decomposed into an isotropic and an anisotropic terms.

Using the shorthand notation for the second derivatives of the potential

$$
\frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \equiv \Psi_{ij}
$$

we introduce the so-called shear matrix. This matrix is related to the projection of the gravitational tidal field and describes the shape distortion of the images generated by the lens:

$$
\begin{pmatrix}
A - \frac{1}{2} trA \cdot I
\end{pmatrix}_{ij} = \delta_{ij} - \Psi_{ij} - \frac{1}{2} (1 - \Psi_{11} + 1 - \Psi_{22}) \delta_{ij}
$$

$$
= -\Psi_{ij} + \frac{1}{2} (\Psi_{11} + \Psi_{22}) \delta_{ij} = \begin{pmatrix}
-\frac{1}{2} (\Psi_{11} - \Psi_{22}) & -\Psi_{12} \\
-\Psi_{12} & \frac{1}{2} (\Psi_{11} - \Psi_{22})
\end{pmatrix}
$$

$$
= - \begin{pmatrix}
\gamma_1 & \gamma_2 \\
\gamma_2 & -\gamma_1
\end{pmatrix}
$$

We have introduced the shear pseudo-vector $^2 \vec{\gamma} = (\gamma_1, \gamma_2)$ whose components are:

$$
\gamma_1(\vec{\theta}) = \frac{1}{2} (\Psi_{11} - \Psi_{22}) \quad (2.17)
$$

$$
\gamma_2(\vec{\theta}) = \Psi_{12} = \Psi_{21} \quad (2.18)
$$

The eigenvalues of the shear matrix are

$$
\pm \gamma = \pm \sqrt{\gamma_1^2 + \gamma_2^2}, \quad (2.19)
$$

thus there is a particular coordinate rotation (by an angle $\phi$) for which the shear matrix is written as:

$$
\begin{pmatrix}
\gamma_1 & \gamma_2 \\
\gamma_2 & -\gamma_1
\end{pmatrix} = - \gamma \begin{pmatrix}
\cos 2\phi & \sin 2\phi \\
\sin 2\phi & -\cos 2\phi
\end{pmatrix} \quad (2.20)
$$

$^2$The shear components are elements of a $2 \times 2$ tensor and not components of a vector.
Since
\[
\frac{1}{2} \text{tr} A = \left[ 1 - \frac{1}{2} (\Psi_{11} + \Psi_{22}) \right] \delta_{ij} = \left( 1 - \frac{1}{2} \Delta \Psi \right) \delta_{ij} = (1 - \kappa) \delta_{ij},
\]
we can rewrite the Jacobian matrix as:
\[
A = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ \gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} = (1 - \kappa) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos^2 \phi & \sin^2 \phi \\ \sin^2 \phi & -\cos^2 \phi \end{pmatrix}
\]
This expression shows clearly the meaning of both the convergence $\kappa$ and the shear: the convergence $\kappa$ represents the isotropic deformation, that is the images are simply rescaled relative to the source; on the contrary the shear represents the anisotropic deformation for which the shapes of the images are stretched along a privileged direction, given by the orientation of $\vec{\gamma}$.

The last effect we have to consider is the flux magnification of images respect to the source flux. Since there is no absorption nor emission related to gravitational lensing, the surface brightness of the source is preserved. Moreover, since the images are rescaled and stretched respect to the source, the solid angle under which they are seen is different from the one of the source. Thus, because of the Liouville theorem, we have that the fluxes of the images result to be magnified or demagnified.

The magnification matrix $M(\vec{\theta})$ is given by the inverse of the Jacobian matrix
\[
M(\vec{\theta}) = A(\vec{\theta})^{-1}.
\]
Where defined, the determinant of this matrix gives the total magnification
\[
\mu(\theta) \equiv \det M(\theta) = \frac{1}{\det A} = \frac{1}{(1 - \kappa(\theta))^2 - \gamma(\theta)^2}.
\]
The sign of $\mu$ gives us information about the parity of the images: for an image at the position $\vec{\theta}_i$, if $\mu(\vec{\theta}_i) > 0$, then the image has the same parity as the source, while if $\mu(\vec{\theta}_i) < 0$ it has a mirror symmetry compared to the source.

The family of points on the image plane where the magnification is infinite defines the critical lines: they are given by $\det A(\vec{\theta}) = 0$, i.e. by $\lambda_t = 1/\mu_t = 0$ and by $\lambda_r = 1/\mu_r = 0$, where
\[
\lambda_r = \frac{1}{\mu_r} = 1 - \kappa + \gamma,
\]
\[
\lambda_t = \frac{1}{\mu_t} = 1 - \kappa - \gamma.
\]
2.2 Time delay and occurrence of images

Due to gravitational lensing, when deflected light rays reach the observer, they will have a time delay with respect to unperturbed rays: this delay has a geometric and a gravitational component. The geometric component is caused by the different path followed by the lensed rays with respect to the path of unperturbed rays, and it is proportional to the square of the angular separation between the source and the image. The gravitational component is due to the slowing down of photons passing across the gravitational field of the lens and it is given by the effective lensing potential. The total time delay for a lens at redshift \( z_L \) is given by:

\[
 t(\vec{\theta}) = \frac{1 + z_L}{c} \frac{D_s D_L}{D_{LS}} \left[ \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right]. \tag{2.27}
\]

From Eq. 2.12 we have that

\[
 (\vec{\theta} - \vec{\beta}) - \nabla \psi(\vec{\theta}) = \nabla \left[ \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right] = 0 \tag{2.28}
\]

This relation implies that \( \nabla t(\vec{\theta}) = 0 \), that is the images satisfy the Fermat Principle. Thus images occur at stationary points of the time delay surface (given by the Eq. 2.27), i.e. at the maxima, minima and saddle points. The Hessian matrix of the time delay surface results to be proportional to the Jacobian matrix \( A \) (see Eq.2.15):

\[
 H(\vec{\theta}) = \frac{\partial^2 t(\vec{\theta})}{\partial \theta_i \partial \theta_j} \propto (\delta_{ij} - \Psi_{ij}) = A. \tag{2.29}
\]

Images that occur at minima of the time delay surface have both the eigenvalues of \( H(\vec{\theta}_{\text{min}}) \) positive, that is \( \text{tr}A > 0 \) and \( \det A > 0 \): therefore, they have positive magnification. Images that occur at maxima have both the eigenvalues of \( H(\vec{\theta}_{\text{max}}) \) negative, that is \( \text{tr}A < 0 \) and \( \det A > 0 \): they have positive magnification too. Finally for images that occur at saddle points the eigenvalues of \( H(\vec{\theta}_{\text{saddle}}) \) have opposite signs. In this case \( \det A < 0 \) and the images have negative magnification.
2.2 Time delay and occurrence of images

In Fig. 2.2 it is shown the time delay surface for an axially symmetric lens, for three different positions of the source. If the source and the lens are perfectly aligned along the optical axis the surface will have a Mexican hat shape, and images will occur at the central maximum and on the minima ring known as Einstein Ring (panel A). If the source moves away from the optical axis, the surface deforms from the Mexican hat shape, and there will be a central maximum, a minimum and a saddle point: the Einstein ring will break allowing the formation of two images, one in correspondence of the minimum point (with positive magnification) and the other in correspondence of the saddle point (with negative magnification).

Since $H(\vec{\theta})$ is related to the local curvature of the time delay surface, and $H(\vec{\theta}) \propto A = M^{-1}$, where images occur, smaller is the time delay surface curvature and larger is the total magnification. So in Fig. 2.2-B, at minimum and at saddle point, since the local curvature is small along the tangential direction, the images will be stretched in this direction leading to formation of tangential arcs. Moving the source away from the axis, the saddle point and the central maximum become closer and this causes the local curvature become smaller in the radial direction: in this case radial stretched images occur at the maximum and the saddle point. When the source is moved so far away from the optical axis that the maximum and the saddle point touch, the respective images will disappear, and only the minimum image will remain.
2.3 Lens models

In the previous section we derived the basic equations of the lensing theory. We now apply these equations to different lens models in order to describe realistic lensing phenomena due to galaxy-galaxy or galaxy clusters lensing. We will start our analysis with the simplest lens model, that is the point mass lens. Then, since in this work we are interested in the strong lensing by galaxy clusters, we will analyse the most used models of extended lens.

2.3.1 Point mass lens

Let us consider a mass concentration $M$ (i.e., the gravitational lens) at redshift $z_L$ and a source at redshift $z_S > z_L$. Treating the mass concentration as a point mass lens, the system results axisymmetric, that is no direction is preferred and we can reduce it to a one-dimension system. The gravitational potential is $\Psi = -\frac{GM}{r}$, so we obtain the deflection angle

\[ \hat{\alpha}(\xi) = \frac{2}{c^2} \int \nabla_{\perp} \Psi dz = -\frac{4GM}{c^2\xi} \hat{e}_r, \]  

(2.30)

\[ \Rightarrow \hat{\alpha} = \frac{4GM}{c^2\xi} = \frac{4GM}{c^2D_L\theta} \]  

(2.31)

where $\xi = D_L\theta$ is the impact parameter and $\hat{e}_r$ is the radial versor that, due to the symmetry of the system, can be identified with the coordinate axis that we prefer: in this case we identify it with the axis connecting the source and the point mass lens.

The lens equation reads

\[ \beta = \theta - \frac{4GM}{c^2D_L\theta} \frac{D_{LS}}{D_S} \]  

(2.32)

and defining the *Einstein radius* as follows

\[ \theta_E \equiv \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_LD_S}} , \]  

(2.33)

it can be written as

\[ \beta = \theta - \frac{\theta_E^2}{\theta} . \]  

(2.34)

Setting $y = \beta/\theta_E$ and $x = \theta/\theta_E$ we obtain

\[ y = x - \frac{1}{x} . \]  

(2.35)
2.3 Lens models

Multiplying by $x$ we get

$$x^2 - xy - 1 = 0 \quad (2.36)$$

which has two solutions:

$$x_\pm = \frac{1}{2} \left[ y \pm \sqrt{y + 4} \right] \quad \Rightarrow \quad \theta_\pm = \frac{1}{2} \left[ \beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right] \quad (2.37)$$

Thus, for a source at the angular distance $\beta$ from the point mass lens, we will have two images: we should have expected three images, corresponding to the maximum, the minimum and the saddle point of the time delay surface, but in this case there is a singularity on the time delay surface in correspondence of the point mass lens, so there is no maximum defined.

For $\beta = 0$, we have $\theta_\pm = \pm \theta_E$, that is if the source and the lens are perfectly aligned we will have a ring-shape images with radius $\theta_E$.

From the Jacobian matrix we get the magnifications of both the images:

$$\det A = \frac{y}{x} \frac{\partial y}{\partial x} = \left( 1 + \frac{1}{x^2} \right) \left( 1 - \frac{1}{x^2} \right) = \left( 1 - \frac{1}{x^4} \right) = \mu^{-1} \quad (2.38)$$

$$\Rightarrow \mu_\pm = \left( 1 - \frac{1}{x^4} \right) = \frac{x^4_\pm}{x^4_\pm - 1} = \frac{1}{2} \pm \frac{y^2 + 2}{2y\sqrt{y^2 + 4}} \quad (2.39)$$

From Eq.2.37 and Eq.2.39, we can see that for $\beta \to \infty$ we have

$$\begin{cases} \theta_- \to 0; & \mu_- \to 0 \\ \theta_+ \to \beta; & \mu_+ \to 1 \end{cases} \quad (2.40)$$

that is, the $\theta_-$ image disappears: enlarging the angular distance between the source and the lens, we move from strong lensing regime to weak lensing regime, where we have no multiple images but only weak distortion of the image shape.

2.3.2 Extended lens: circular symmetric lens

Let us consider now thin lenses with an extended mass distribution described by the surface density $\Sigma(\theta)$. In the case of axially symmetric mass distribution, the surface density reduces to $\Sigma(|\theta|)$, that is we can reduce the system to a one-dimension system.

The gravitational potential and the deflection angle are

$$\psi = -\frac{GM(\theta)}{D_L \theta}; \quad \hat{\alpha}(\theta) = \frac{4GM(\theta)}{c^2 D_L \theta} \quad (2.41)$$
In term of the critical surface density $\Sigma_{cr}$ and the reduced deflection angle $\alpha$ (see Eq. 2.9 and Eq. 2.7), the lens equation in this case reads:

$$\beta(\theta) = \theta - \frac{M(\theta)}{\pi D_L^2 \theta \Sigma_{cr}} ,$$

(2.42)

where $M(\theta)$ is the lens mass enclosed within the radius $D_L \theta$.

Defining the Einstein radius as

$$\theta_E = \frac{1}{D_L} \left( \frac{M(\theta)}{\pi \Sigma_{cr}} \right)^{\frac{1}{2}},$$

(2.43)

the lens equation reads

$$\beta = \theta - \frac{\theta_E}{\theta}$$

(2.44)

The Jacobian matrix $A$, in polar coordinates $(\theta, \phi)$, is

$$A = \begin{pmatrix}
1 - \frac{1}{\pi D_L^2 \Sigma_{cr}} \frac{\partial}{\partial \theta} \left( \frac{M(\theta)}{\theta} \right) & -\frac{1}{\pi D_L^2 \Sigma_{cr}} \frac{\partial}{\partial \phi} \left( \frac{M(\theta)}{\theta} \right) \\
-\frac{1}{\pi D_L^2 \Sigma_{cr}} \frac{\partial}{\partial \theta} \left( \frac{M(\theta)}{\theta} \right) & 1 - \frac{1}{\pi D_L^2 \Sigma_{cr}} \frac{\partial}{\partial \phi} \left( \frac{M(\theta)}{\theta} \right)
\end{pmatrix} = \begin{pmatrix}
\frac{d\beta}{d\theta} & 0 \\
0 & \frac{\beta}{\theta}
\end{pmatrix}
$$

(2.45)

The first element of the Jacobian matrix gives the radial distortion factor, that is $\left(\frac{d\beta}{d\theta}\right)^{-1}$, while the second diagonal element gives the tangential distortion factor, that is $\left(\frac{\beta}{\theta}\right)$. The family of points where the diagonal elements of $A$ are zero defines the radial and tangential critical lines.

If the surface density has a power law profile $\Sigma(\theta) = C_0 \theta^{-a}$ with $a < 2$, the Jacobian matrix reads

$$\begin{pmatrix}
1 - \frac{2C_0}{\Sigma_{cr}} \left( \frac{1}{2-a} \right) \frac{1}{\theta^a} & 0 \\
0 & 1 - \frac{2C_0}{\Sigma_{cr}} \left( \frac{1}{2-a} \right) \frac{1}{\theta^a}
\end{pmatrix}
$$

(2.46)

and we have that both the distortion factors define circular critical lines on the image plane.

If the lens is not singular, the radial critical line given by $d\beta/d\theta = 0$ is a circle with radius

$$\theta_R = \frac{1}{D_L} \left( \frac{M(\theta_R)}{\pi \Sigma_{cr}} \right)^{\frac{1}{2}} \left( \frac{1}{2 \Sigma(\theta_R)} - 1 \right)$$

(2.47)
while, if the lens is singular, it reduces to a point coincident with $\theta = 0$.

The tangential deformation at the radial critical line is

$$\left| \frac{\theta}{\beta} \right|_{\theta=\theta_R} = \left| 1 - \frac{1}{a} \right| . \quad (2.48)$$

Also the tangential critical line, given by $\beta/\theta = 0$, is circular, with radius given by the Einstein radius

$$\theta_T = \theta_E = \frac{1}{D_L} \left( \frac{M(\theta_E)}{\pi \Sigma_{cr}} \right)^{\frac{1}{2}} \quad (2.49)$$

and the radial distortion factor is

$$\left( \frac{d\beta}{d\theta} \right)^{-1}_{\theta=\theta_E} = \frac{1}{a} \quad (2.50)$$

From the lens equation Eq. 2.44, we can see that points of tangential critical lines are mapped in $\beta = 0$ on the source plane, that is the circular symmetric lens have point tangential caustic, while points of the radial critical lines are mapped on a circle caustic.

The number of the multiple images is odd (see the Section on the occurrence of images), except for the case of lens with singular distribution mass, when multiple images are in even number since the central maximum in the surface density is not defined. The position of the source with respect to the caustics determines the number of the multiple images generated: for a non-singular circular symmetric lens, if the source is within the radial caustic than three images will be produced (we are in the strong domain), while if the source is out of the radial caustic there will be only one image (weak domain). In this case of axially symmetric lens, since the tangential caustic correspond to the point $\beta = 0$, the image multiplicity depends only on the radial caustic. Of course, if the source is extended then both the radial and the tangential distortions of the images are more evident than if the source is point-like (see Fig.2.3 and Fig.2.4).

**The Singular Isothermal Sphere density profile**

In order to model the mass profile of gravitational lenses, different circular symmetric density profiles are used. Most of the time these models are not representative of the mass distribution of real astrophysical systems, like galaxy clusters, but they give the advantage of a simple analytic form.

For $\Sigma(\theta) = C_0 \theta^{-a}$ with $a = 1$, we have the case of the so-called singular spherical lens. If we assume that the lens has an axially symmetric mass
distribution and that its matter content behaves as an ideal gas in thermal and hydrostatic equilibrium, we reduce to the singular isothermal sphere model (SIS). Even if this model has a central singularity and its total mass diverge (so it does not represent a physical mass distribution) it is often useful because of its very simple analytic form and because its radial mass density profile matches the observed profiles in elliptical galaxies.
The mass density profile of such a model is given by

\[ \rho(r) = \rho_0 \left( \frac{r_0}{r} \right)^2 = \frac{\sigma^2}{2\pi G r^2} \]  

(2.51)

where \( r_0 \) is a scale factor, \( \rho_0 \) is the density at \( r_0 \) and \( \sigma \) is the velocity dispersion along the line of sight.

Integrating \( \rho \) along the line of sight, we obtain the surface density

\[ \Sigma(\theta) = \frac{2 \sigma^2}{2\pi G} \int_0^\infty \frac{dz}{(D_L \theta)^2 + z^2} = \frac{\sigma^2}{\pi G D_L \theta} \left[ \arctan \left( \frac{z}{D_L \theta} \right) \right]_0^\infty = \frac{\sigma^2}{2GD_L \theta}, \]  

(2.52)

while the mass enclose within a radius \( D_L \theta \) is given by

\[ M(\theta) = \frac{\pi \sigma^2 D_L \theta}{G}. \]  

(2.53)

To remove the singularity, we can introduce a finite core radius \( r_c \), so that the previous quantities become

\[ \rho(r) = \rho_0 \left( 1 + \frac{r^2}{r_c^2} \right)^{-1} \]  

(2.54)

\[ \Sigma(\theta) = \frac{\pi \rho_0 D_L \theta_c^2}{\sqrt{\theta^2 + \theta_c^2}} \]  

(2.55)

\[ M(\theta) = 2\pi^2 D_L^3 \rho_0 \theta_c^2 \left( \sqrt{\theta^2 + \theta_c^2} - \theta_c \right) \]  

(2.56)

where \( \rho_0 \) is the central density this time.

The Nawarro Frenk & White density profile

The Nawarro, Frenk & White (NFW) density profile is a general density profile obtained by fitting the output dark matter halos of N-body simulations. It has been shown (Navarro et al. 1997) that its shape is independent of the halo mass, of the initial density fluctuation spectrum and of the value of cosmological parameters used.

Within the mass range \( M_{\text{vir}}/(h^{-1}M_\odot) \in [3 \times 10^{11}, 3 \times 10^{15}] \) the general expression of this density profile is

\[ \rho(r) = \frac{\rho_s}{\left( \frac{r}{r_s} \right)^\alpha \left( 1 + \frac{r}{r_s} \right)^{1+\alpha}} \]  

(2.57)
where \( r_s \) and \( \rho_s \) are the scale radius and the characteristic density of the halo. For the \( \alpha \) exponent it is usually adopted the value 1, so the NFW density profile reads

\[
\rho(r) = \frac{\rho_s}{\left( \frac{r}{r_s} \right)^2 \left( 1 + \frac{r}{r_s} \right)^2}.
\]

(2.58)

For small radii, \( r \ll r_s \), we have \( \rho(r) \propto r^{-1} \), while for large radii, \( r \gg r_s \), we find that \( \rho(r) \propto r^{-3} \). So with respect to the SIS, which density profile follows \( r^{-2} \), the NFW profile results to be flatter in the inner region of the halos and steeper in the outer one.

The main parameters of a NFW halo are \( r_{200} \), \( M_{200} \) and \( c \), where \( r_{200} \) is the radius of the sphere, centered on the halo, having as average density a density that is 200 times the critical density of the Universe\(^1\); \( M_{200} \) is the mass enclosed in the sphere with radius \( r_{200} \); finally \( c \) is the concentration of the halo and it is defined as \( c \equiv r_{200}/r_s \). It is assumed that the concentration value reflects the value of the mean cosmic density at the time the halo formed: this is supported by numerical simulations showing that earlier the halo forms, greater is its concentration. By integrating the density \( \rho(r) \) along the line of sight, we get the surface mass density:

\[
\Sigma(x) = \frac{2\rho_s r_s}{x^2 - 1} f(x), \quad \text{where} \quad x = \frac{D_L \theta}{r_s},
\]

(2.59)

with

\[
f(x) = \begin{cases} 
1 - \frac{2}{\sqrt{x^2 - 1}} \arctan \sqrt{\frac{x - 1}{x + 1}} & (x > 1) \\
1 - \frac{2}{\sqrt{1 - x^2}} \arctanh \sqrt{\frac{1 - x}{1 + x}} & (x < 1) \\
0 & (x = 1)
\end{cases}
\]

(2.60)

In Fig. 2.5 they are compared some properties of the NFW profile and the SIS profile: the reduced deflection angle \( \alpha(\xi) \) and the logarithms of the lensing potential \( \Psi \), the convergence \( \kappa \) and the shear \( \gamma \). In particular we note that in the inner region \( r < r_s \) the deflection angle is smaller for the NFW than the SIS profile, that is the NFW is less efficient in splitting and deforming images; on the contrary, since the amplification is tightly related to the curvature of the potential, in the inner region the NFW profile results to be more efficient than the SIS profile.

\(^1\)In a flat Universe, the critical density gives the value of the average density at present time: it is given by \( \rho_c = 3H_0^2/8\pi G \) and correspond to \( \approx 10^{-29} g/cm^3 \).
2.3 Lens models

Figure 2.5: Main properties for the NFW and the SIS profiles as a function of the distance $\xi = D_L \theta / \xi_0$ from the center of the profiles (where $\xi_0$ is a scale factor: for NFW $\xi_0 = r_s$): reduced deflection angle (upper-left panel), effective lensing potential (upper-right), convergence (bottom-left) and shear (bottom-right).

2.3.3 Extended lens: Elliptical lens

More realistic models of the mass distribution of galaxy clusters are the ones characterized by an elliptical symmetry, rather than a spherical symmetry. Starting from the ellipsoidal mass distribution $\rho(\vec{r})$ of a lens, the surface mass distribution $\Sigma(\theta)$ can be obtained projecting $\rho(\vec{r})$ on the lens plane. Given $\Sigma(\theta)$, from Eq. 2.14 and 2.12, we can derive the lensing potential, the deflection angle and the convergence functions for the elliptical lens. Since it is analytically complicated to derive these functions, in order to have a first and simpler approach to elliptical lens modelling we can use the following variable substitution in the surface mass density $\Sigma(\vec{x}) = f(\vec{x})$ and in the lensing potential $\Psi(\vec{x}) = g(\vec{x})$:

$$\vec{x}(x_1, x_2) \rightarrow X = \sqrt{\frac{x_1^2}{1-e} + x_2^2(1-e)} \quad (2.61)$$

where $e = 1 - b/a$ is the ellipticity of the lens and $a$ and $b$ are the major and minor axes respectively. With this substitution we obtain an elliptical model characterized by elliptical iso-density contours with major axis along
2.3 Lens models

Figure 2.6: Upper panel: Deflection angle maps for a NFW lens model at redshift $z = 0.3$ for different value of the ellipticity of the lens potential. The source is assumed at $z = 1$. Bottom panel: Critical lines (blue curves) and caustics (red curves) for the same model (Meneghetti 2006).

Figure 2.7: Examples of multiple images produced by an elliptical lens. In the left panel we can see the images produced by a compact source as it is moved away from the optical axis crossing the caustic in a fold point. In the right panel we have the same but for a source crossing a cusp caustic (Meneghetti 2006)
the \(x_2\)-direction. The resulting Cartesian components of the deflection angle are:

\[
\begin{align*}
\alpha_1 &= \frac{\partial \Psi}{\partial x_1} = \frac{x}{(1 - e)X} \hat{\alpha}(X) \\
\alpha_2 &= \frac{\partial \Psi}{\partial x_2} = \frac{x_2(1 - e)}{X} \hat{\alpha}(X)
\end{align*}
\]

(2.62)

where \(\hat{\alpha}\) is the axially-symmetric unperturbed deflection angle at the distance \(X\). In Fig. 2.6 they are shown the deflection angle map, the critical lines and caustics for a source at redshift \(z = 1\), for a NFW lens model with mass \(10^{15} M_\odot h^{-1}\) at redshift \(z = 0.3\), with different value of ellipticity. In the axially symmetric model \((e = 0)\) the critical lines are circles and the caustics are a circle and a point. Increasing the ellipticity of the lensing potential, the shear field becomes stronger: the caustics enlarge, deform and develop cusps, while the critical lines deform toward a dumbbell shape.

In Fig. 2.7 there are examples of multiple images produced by elliptical lenses: in this case, since the tangential caustic is not point-like, as for the axially symmetric case, for sources that are within both the tangential and radial caustics five multiple images are produced.
Chapter 3

Galaxy clusters as Gravitational Telescopes

Gravitational Telescopes offer a unique tool to observe high-z galaxies: lensing magnification provided by massive galaxy clusters allows to investigate the faint end of luminosity function at high redshift \((z \geq 6) \sim 1\) magnitude fainter than blank field surveys, although on much smaller sky areas. In the present Chapter, we apply such a technique to two massive clusters of galaxies.

3.1 The strategy

The first step in the research of high redshift galaxies through Gravitational Telescopes (GTs) is to build a robust model of the mass distribution of the galaxy cluster that acts as the gravitational lens. Indeed, given the mass distribution of the GT, though the lensing theory, we can determine the maps of magnification induced by the cluster in the background sources and we can trace the critical and caustic curves for lensed sources at different redshifts. The knowledge of the critical and caustic curves allows us to identify regions on the image plane where high magnifications occur, that is regions where we have more chances to detect faint sources highly magnified. Moreover, detailed magnification maps allow to obtain the intrinsic luminosity of lensed sources giving the amplification factor for which we have to correct the observed magnified fluxes. Two approaches can be used to reconstruct mass distribution of galaxy clusters through gravitational lensing: the strong- and the weak-lensing mass modelling methods.

Strong lensing mass modelling uses a parametric technique to reconstruct the mass distribution of a cluster, using positions and shape of multiple im-
ages as constraints to optimize the model (see Sect. 3.3). On the contrary, weak lensing mass modelling uses a statistical approach that allows to reconstruct cluster mass distributions studying weak distortions imprinted on the shape of distant galaxies in the field of view (see section 3.4).

Both approaches should be used to have a complete reconstruction of cluster mass distribution. Indeed, the strong lensing approach allows a good mass reconstruction in the inner region of the cluster, where $\Sigma(\theta) > \Sigma_{cr}$ (i.e. in the strong lensing domain), while the weak lensing analysis is more efficient in the outer region of the cluster, where $\Sigma(\theta) < \Sigma_{cr}$ (i.e. in weak lensing domain).

In this chapter we study two massive clusters (CLG 0152-57 and RDCS 1252) at different redshift ($z = 0.83$ and $z = 1.23$, respectively) to search for magnified high-redshift galaxies. Using the software LenseTool (Kneib 1993, Jullo et al. 2009), we reconstruct CLG 0152-57 mass distribution through strong lensing mass modelling; while for RDCS 1252 we use the map of the mass distribution obtained by Lombardi et al. 2005 through weak lensing reconstruction.

Once we obtain the cluster mass distributions, we can determine the magnification and correct the photometric data of our clusters by the lensing effects: this step is crucial to start a study of the physical properties of the background sources lensed by the GTs.

For both the clusters we collected a dataset of optical, near- and mid-infrared imaging data from HST/ACS, VLT/ISAAC and Spitzer/IRAC respectively.

### 3.2 Observational data and catalogs

We studied two massive galaxy clusters at different redshift, CLG0152 ($z \sim 0.83$) and RDCS1252 ($z \sim 1.2$), for which we collected three subsets of archival images: optical from HST/ACS, near-infrared from VLT/ISAAC and mid-infrared from Spitzer/IRAC.

For CLG0152 we have three HST/ACS images in the filters f625w ($r$-band), F775w ($i$-band), F850lp ($z$-band): these are $\sim 1000\text{Å}$ wide with central $\lambda$ at $\sim 6250\text{Å}$, $\sim 7750\text{Å}$ and $\sim 8500\text{Å}$ respectively. The VLT/ISAAC images are in the filters J and H (centered at $\sim 12500\text{Å}$ and $\sim 16500\text{Å}$, and wide $\sim 3000\text{Å}$). The Spitzer images are in 4 filters: ch1 at $\sim 3.6\mu$m (channel1 wide $\sim 1\mu$m), ch2 at $\sim 4.5\mu$m (channel2 wide $\sim 1.5\mu$m), ch3 at $\sim 5.8\mu$m (channel3 wide $\sim 2\mu$m) and ch4 at $\sim 8.0\mu$m (channel4 wide $\sim 4\mu$m).

For RDCS 1252 we have two optical images from HST/ACS in the filters F775w ($i$-band) and F850lp ($z$-band), two images from VLT/ISAAC in the
3.2 Observational data and catalogs

Table 3.1: Dataset collected for the two galaxy cluster.

<table>
<thead>
<tr>
<th></th>
<th>CLG0152</th>
<th>RDCS1252</th>
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<tbody>
<tr>
<td><strong>HST/ACS</strong></td>
<td>F625w</td>
<td>F775w</td>
</tr>
<tr>
<td></td>
<td>F775w</td>
<td>F850lp</td>
</tr>
<tr>
<td><strong>VLT/ISAAC</strong></td>
<td>J</td>
<td>Js</td>
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<tr>
<td></td>
<td>H</td>
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<td></td>
<td>~</td>
<td>Ks</td>
</tr>
<tr>
<td><strong>Spitzer/IRAC</strong></td>
<td>ch1</td>
<td>ch1</td>
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<td></td>
<td>ch2</td>
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<td>ch3</td>
<td>ch3</td>
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<tr>
<td></td>
<td>ch4</td>
<td>ch4</td>
</tr>
</tbody>
</table>

filters Js (centered at $\sim 12400\,\AA$ and wide $\sim 1300\,\AA$) and Ks (centered at $\sim 21600\,\AA$ and wide $\sim 2700\,\AA$). In Table 3.1 the set of filters are summarized for each cluster.

We extracted sources from optical and near-infrared images, and through photometric analysis we selected high-redshift LBG candidates (see section ??), identified as optical and near-infrared dropouts: a dropout is an object that is not detected in all bands, but is detected in the redder ones and not in the bluer ones. This behaviour is due to a strong rest-frame break in the spectrum of the object that is redshifted in the wavelength range covered by our set of filters. If we assume that the spectral break is the Lyman-break at $\lambda_{\text{rest}} = 1216\,\AA$, than an $r$-dropout, that is an object not detected in $r$-band and detected in the other redder filters, is estimated to be at redshift $z > 5$, an $i$-dropout (i.e. not detected in the $i$-band) at redshift $z > 6$, a $z$-dropout at $z > 7$, and so on.

Since we are using broad-band filters, as a matter of fact for each dropout object we can only identify the approximative redshift range $[z_{\text{min}}, z_{\text{max}}]$ to which it could belong: our broad bands are $\sim 1000 – 3000\,\AA$ wide, so we can not tightly identify at which $\lambda$ the break occurs (such information needs a further narrow band (NB) analysis). We can only fix the ends of the redshift range at which the drop-outs could belong, given by $z_{\text{min}} = (\lambda_{\text{min}}/1216) - 1$ and $z_{\text{max}} = (\lambda_{\text{max}}/1216) - 1$, where $\lambda_{\text{min, max}}$ are the $\lambda$-ends of the first filter where the dropout is detected. Of course, these redshift ranges are approximative, since we should take into account of the transmission curve of the filters. In Table 3.2 we listed the redshift ranges related to each dropout for our broad bands and the respective mean redshift. The Spitzer images will be used to check that the selected optical and near-ir dropouts
Table 3.2: Redshift range and mean redshift for each dropout (see text for details).

<table>
<thead>
<tr>
<th>Dropout</th>
<th>z-range</th>
<th>mean $\bar{z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$-dropout</td>
<td>[4.1, 6.6]</td>
<td>5.4</td>
</tr>
<tr>
<td>(detected in i-band)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i$-dropout</td>
<td>[5.0, 7.0]</td>
<td>6.0</td>
</tr>
<tr>
<td>(detected in z-band)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z$-dropout</td>
<td>[7.0, 10.5]</td>
<td>8.7</td>
</tr>
<tr>
<td>(detected in Js-band)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(detected in J-band)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J$-dropout</td>
<td>[10.1, 15.0]</td>
<td>12.5</td>
</tr>
</tbody>
</table>

are detected in the mid-infrared too as they are expected to be.

We used the tool SExtractor (Bertin & Arnouts 1996) to identify sources in each image. There are some detection parameters to set so that, though SExtractor, we can distinguish when a number of nearby pixels have to be considered as an object and when as background (e.g. the threshold, the minimum value of the pixel above threshold to be considered as object). For the extracted objects SExtractor estimates both photometric and geometric parameters as magnitudes, fluxes, position angle, ellipticity.

The output file is a catalog where for each extracted object all the parameters (requested in an output.para file) are listed: identification number (ID), coordinates, photometric and geometric parameters.

We used SExtractor in double mode: having a set of images, we chose one of the images of our set as our reference image; then SExtractor extracts objects from the reference image, it computes an objects-mask of the extracted objects and it uses this mask to extract objects in the same positions and with the same shapes from all other images of our set. Since we are interested in optical dropouts, we chose as reference filter the reddest one for each subset, that is, $z$ filter for HST/ACS subset and filter H for VLT/ISAAC data subset.

Therefore, using SExtractor in double mode for each image of our data, we obtained final catalog for each subset (the optical and the nir), where identification number ID, coordinates, magnitudes and magnitude errors are listed. Magnitudes are measured within a diameter-aperture of $3''$ both for the optical and the near-infrared images. To match these two catalogs we used as matching-parameters the coordinates using as constraint that the central position of the matched objects can not differ more than $0.9''$. We
Table 3.3: Magnitude limits in each filter.

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>i</th>
<th>z</th>
<th>J</th>
<th>Js</th>
<th>H</th>
<th>Ks</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLG0152</td>
<td>25.32</td>
<td>25.08</td>
<td>24.14</td>
<td>23.60</td>
<td>~</td>
<td>23.66</td>
<td>~</td>
</tr>
<tr>
<td>RDCS1252</td>
<td>~</td>
<td>25.55</td>
<td>24.61</td>
<td>~</td>
<td>26.28</td>
<td>~</td>
<td>26.04</td>
</tr>
</tbody>
</table>

Table 3.4: Extinction corrections calculated with the NED extinction calculator. See text for details.

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>i</th>
<th>z</th>
<th>J</th>
<th>H</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLG0152</td>
<td>0.038</td>
<td>0.029</td>
<td>0.021</td>
<td>0.013</td>
<td>0.009</td>
<td>0.005</td>
</tr>
<tr>
<td>RDCS1252</td>
<td>0.199</td>
<td>0.145</td>
<td>0.127</td>
<td>0.067</td>
<td>0.043</td>
<td>0.027</td>
</tr>
</tbody>
</table>

used the tool TOPCAT\textsuperscript{1} to work on catalogs and to match them.

Since the sky area covered by the VLT/ISAAC mosaics (2.7′ × 2.7′ for CLG0152 and 4.4′ × 4.4′ for RDCS1252) is smaller than the FOV of HST/ACS images (6.2′ × 6.2′ for CLG0152 and 6.0′ × 6.0′ for RDCS1252) there are objects extracted from the optical images that are not in the near-infrared FOV: these objects are \textit{not observed} in the VLT/ISAAC FOV and we associate them the value −99 at the nir magnitudes and at their respective errors.

While for objects that are \textit{not detected} in all filters (that is they are in both the FOV of our dataset but are not detected in all bands), in that filters we used the magnitude limit of the filters where they are not detected. In each band we use as magnitude limit 3 times the magnitude of the average background $\sigma$ of the image: an object is \textit{not detected} if its magnitude is lower than 3$\sigma$. In Table 3.3 the magnitude limits are listed for each image of our subsets.

Since we are working on optical images from space-ground observations and NIR images from ground-based observations, we should \textit{seeing-match} the optical images to the NIR images; however, since we choose a large aperture (3") to measure magnitudes and, moreover, in this work we are interested in detection and non-detection of objects in each band, we can neglect this correction.

Finally we corrected the magnitude for each filter by the extinction using the corrections obtained by the NED extinction calculator: these corrections are listed in Table 3.4.

\textsuperscript{1}TOPCAT is an interactive graphical viewer and editor for tabular data. See: http://www.star.bris.ac.uk/mbt/topcat/.
3.3 Strong lensing mass modelling

In order to use the Gravitational Telescope strategy, an accurate model of the mass distribution of the massive cluster is of primary importance since it allows us to build the magnification map of the lens and to trace critical lines and caustics for sources at different redshifts.

Let us consider galaxy clusters as composed by a main mass component, corresponding to the dark halo and the intra-cluster gas, and a secondary mass component, corresponding to all the sub-structures of the cluster, as the galaxies. We can therefore write the surface mass density of the cluster as a multipole expansion:

$$\kappa(\vec{\theta}) = \kappa_0(\theta) + \sum_{n=1}^{\infty} \kappa_n(\theta) \exp(in\phi),$$  \hspace{1cm} (3.1)

where $\kappa_0$ correspond to the monopole term and represent the axially symmetric part of the lens; for $n = 1$ we have the dipole term and for $n = 2$ we have the quadrupole term that describes the degree of ellipticity of the iso-density contours; the higher order multipoles describe the influence of the sub-structures.

The surface density profile $\kappa(\vec{\theta})$ is a fundamental characteristic of the gravitational lens, since it determines its lensing properties: the multiplicity of images, the strength of the shear and magnification fields. Moreover the weight of the sub-structures depends on the shape of the density profile: substructures, adding convergence and shear, enhance strongly the lensing ability of a cluster with a density profile shallower in the inner regions: for example, including the sub-structure component will have a stronger impact on the features of the lensing model for a NFW profile rather than for a SIS profile. The shape of the density profile affects the sensitivity of the lens to external perturbation too: even in this case density profiles shallower in the inner region, are more sensitive to external perturbations. In Fig. 3.1 it is shown how the inclusion of external perturbations and sub-structures can modify the shape of critical lines and caustics, both for NFW and SIS profiles.

In order to model galaxy clusters through strong lensing, two approaches can be used: a parametric method or a non-parametric one. The non-parametric approach aims at solving the lens equation for the observed lensed images, obtaining local constraints on the surface density: it assumes that the effective lensing potential and the deflection are linear functions of the surface density. We have seen that the surface density can be decomposed in multipoles as in Eq. 3.1, but in general it can be decomposed in density functionals whose linear combination gives out the surface density.
3.3 Strong lensing mass modelling

Figure 3.1: Left panel: critical lines (red) and caustics (blue) of NFW (up panel) and SIS (bottom panel) axially symmetric lenses with mass $M = 10^{14} M_\odot$. The lens is at $z = 0.3$ and the source is at $z = 1$. Central panel: deformation of the critical lines and the caustics induced when an external perturbation, described by the external shear $\vec{\gamma}$ of amplitude 0.1, is applied to both the models. Right panel: deformation due to the add of the external shear and of 30 axially symmetric sub-structures with $10^{10} M_\odot < M < 10^{11} M_\odot$. The red lines are the critical lines after populating the halo with the sub-structures, while the blue lines are the critical lines before the population. (Meneghetti 2006)

In this case we can write the lens equation for the image $i$ as

$$\vec{\beta}_i = \vec{\theta}_i - A_i \vec{k}, \quad (3.2)$$

where $A_i$ is the matrix that gives the deflection at the position of the images $i$ as a function of the coefficients $\vec{k}$ of the multipoles decomposition of the surface density: they are the unknowns that we want to estimate. For example, we can choose to decompose $\Sigma$ in pixels, that is, we can divide the lens plane in $N_c$ cells, each of them containing a mass $m_j$ with $1 \leq j \leq N_c$. In this case we have $N_c$ coefficients of the $\Sigma$ decomposition corresponding to the masses $m_j$, while the matrix $A$, that transforms the mass vector into the deflection angle, is a $2N_{\text{img}} \times N_c$ matrix, where $N_{\text{img}}$ is the number of
observed lensed images.

If we know the sources position (that is $\beta_i$), there are $2N_{\text{img}}$ linear equations for $2N_{\text{img}} + N_c$ unknowns, and to solve this system of equations we need to introduce constraints. Usually sources positions are unknown, so we have more unknown quantities than $2N_{\text{img}} + N_c$.

If multiple images of a source are identified, we can reduce the number of unknowns removing the source position and if there are enough constraints the Eq. 3.2 can be solved. For example, if we have $N_{\text{img}} = 2$ multiples images of a source, subtracting the two lens equations of the images, we can delete the source position obtaining the equation:

$$\vec{\theta}_1 - \vec{\theta}_2 = (A_1 - A_2)k$$

which can be solved taking the inverse of the matrix $(A_1 - A_2)$:

$$k = (A_1 - A_2)^{-1}(\vec{\theta}_1 - \vec{\theta}_2).$$

Usually, studying galaxy clusters there are more degree of freedom than constraints and we can not solve the system of lensing equations through the non-parametric approach.

The parametric method uses analytic density profiles to describe the matter components of the lens cluster: these density profiles introduce a number of parameters that define the features of the matter components. The first parameters are introduced through the analytic density profile used to model the dark-matter-halos and intra-cluster gas components: for example, if we use a NFW profile, we introduce the scale radius $r_s$ and the characteristic density $\rho_s$ of the halo. Other than the intrinsic parameters of the analytic profiles, there are parameters related to position, orientation and ellipticity of the dark-matter-halos and intra-cluster gas component. Moreover, if we introduce sub-structures too, these will add other parameters related to the analytic density profiles used to model them and to their position, orientation and ellipticity. The number of free parameters grows rapidly adding components to the lens model; as a consequence a large number of constraints is needed too.

In this case, given the parameters $p$, the lens equation reads:

$$\vec{\beta}_i(\vec{\theta}_i) = \vec{\theta}_i - \vec{\alpha}(\vec{\theta}, p),$$

(3.3)

where $\vec{\theta}_i$ are the positions of the lensed images and they are the main constraints in the parametric method. Position, shape and flux of multiple images depend on the properties of the lens and on the redshift of the source and let us to fit the free parameters of the density profile and to determine...
the source redshift, if unknown. On the contrary, if the redshift of the source is known, it is a strong constraint on the lens model and in this case also single images can be used as constraints. Also merging multiple images\(^2\) can be used as constraints, since we can set a constraint on the critical line passing in between the two merging images.

Given the positions of multiple images, the faster and simpler way to proceed is to solve the lens equation in the source plane: we project the images positions \(\vec{\theta}_i\) in the source plane, and then we compute the best \(\beta_i(\theta_i)\) minimizing the \(\chi^2\) written as

\[
\chi^2 = \sum \left( \frac{\bar{\beta} - \beta_i(\bar{\theta}_i)}{\mu_i \sigma_i} \right)^2,
\]

(3.4)

where \(\beta\) is the unknown parameter, \(\beta_i(\theta_i)\) are given by Eq. 3.3, \(\sigma_i\) are the errors on the \(i\)-image position and \(\mu_i\) are their magnifications. Through the \(\chi^2\) minimization we search the minimal scatter between the \(\text{estimated}\) position \(\beta_i(\bar{\theta}_i)\) of the sources, that leads to the observed images at \(\theta_i\), and the free parameters \(\bar{\beta}\). This procedure helps to rapidly individuate the region in the parameters space of the best fit for the lens model. To determine the final lens model, it is better to run the \(\chi^2\) minimization in the image plane, that is to estimate the set of parameters for which we have the minimal scatter between the observed image position and the ones estimated by solving the lens equation in the image plane, even if this is more difficult than solve it in the source plane. In this case the \(\chi^2\) to minimize is

\[
\chi^2 = \sum \left( \frac{\bar{\theta}(\bar{\beta}) - \bar{\theta}_i}{\sigma_i} \right)^2.
\]

(3.5)

Modelling galaxy clusters the number of free-parameters highly grows when the model is complex, and this leads to an high degeneracy of the parameters: there are multiple sets of values of the free parameters for which the \(\chi^2\) is minimal. The Maximum Likelihood (ML) method is very sensitive to local maxima of the Likelihood distribution. When observables are not enough to strongly constrain the model, a Bayesian approach is better suited since it helps to reduce the parameters degeneracy.

### 3.3.1 Bayesian approach to model galaxy cluster

The Bayesian statistic introduces the so-called \textit{Prior} information about the probability distribution function (PDF) of free parameters: the prior infor-

\(^2\)Such merging multiple images are produced by sources that are in the nearby of caustics curves (see the green source in Fig. 2.7).
mation lets one to include *a priori* knowledge about the PDF of parameters.

Starting from the likelihood distribution of free parameters and the *Prior information* about them, in the Bayesian statistic we estimate the *Posterior PDF*, that is a final probability distribution function of the parameters (Jullo et al. 2007). These quantities are related by the Bayes theorem

\[ Pr(p|D,M)Pr(D|M) = Pr(D|p,M)Pr(p|M) \] (3.6)

where:

- \( Pr(D|p,M) \) is the Likelihood of getting the data \( D \) given the parameters \( p \) for the assumed model \( M \) of the studied system;

- \( Pr(p|M) \) is the prior PDF, that is the probability to have the parameters \( p \) given the model \( M \);

- \( Pr(D|M) \) is called *evidence*, it gives the probability of getting the data \( D \) given the assumed model \( M \);

- \( Pr(p|D,M) \) is the posterior PDF, that gives the *a posteriori* probability of getting the parameters \( p \) given the data \( D \) and the model \( M \).

To better figure the meaning of this probabilities, in Fig.3.2 it is shown as example the PDFs of galaxy redshifts: in this case the free parameter is the redshift \( p = z \), the data are the observed magnitudes and colors of the galaxies \( D = \{C,m_0\} \) and the models are the galaxies spectral types \( M = T \). In the \( a \) panel we have the likelihood function that gives the probability of getting the colors \( C \) given the redshift \( z \) and the spectral type \( T \) (E-S0: elliptical-lenticular galaxies; Sp: spiral galaxies; Irr: irregular galaxies). In panel \( b \) it is shown the prior information, obtained from surveys studies, that gives the probability of a galaxy with magnitude \( m_0 \) to have spectral type \( T \) and redshift \( z \). In panel \( c \) we have the posterior PDF, that is the probability to have a galaxy with spectral type \( T \) at redshift \( z \) given the observable data \( \{C,m_0\} \) : this is obtained multiplying the likelihood function for the prior information, \( p(z,T|C,m_0) \propto p(z,T|m_0)p(C|z,T) \). Finally in the last panel we have the posterior PDF integrated over all the model, i.e. over all the spectral type in this case.

It is evident the high difference to use the likelihood distribution and the posterior PDF: for example, the maximum likelihood (ML) method expects
Figure 3.2: Example of the probability distributions used in the Bayesian statistic for the PDF of galaxies redshift (see text). Panel a): Likelihood function for different model (spectral type T) of galaxies; panel b): Prior information for each spectral type; panel c): posterior PDF given by the multiplication of the likelihood function and the prior function; panel d): posterior PDF integrated over all the spectral type (Benítez N. 2000).

to have more spiral galaxies at high redshift, while after including the prior information, the probability to find spiral galaxy at high redshift drastically reduces. The main difference between this two approaches is that the ML identify the highest maximum in the likelihood distribution as the best parameter, without taking into account of the plausibility of this estimation; on the contrary Bayesian approach weighs the likelihood distribution by the prior probabilities: this reduce spurious solutions due both to degeneracy of
3.3 Strong lensing mass modelling

parameters and noise of the observations.

Finally we introduce the negative entropy defined as

$$I = \int Pr(p|D,M) \frac{\log[Pr(p|D,M)]}{Pr(p|M)} dp$$

(3.7)

where the integral is done over all the parameters space. This quantity measures the information that we obtained computing the posterior PDF $Pr(p|D,M)$ from the prior PDF $Pr(p|M)$ and it represents the distance between the posterior PDF and the prior PDF. The negative entropy should be of the order of unity, since this means that the posterior PDF is similar to the prior PDF, that is the final PDF that we estimate given our data, is similar to the PDF known a priori.

3.3.2 Modelling galaxy clusters with Lenstool

To model the mass distribution of galaxy clusters we used the parametric gravitational lensing package *Lenstool* (Kneib 1993, Jullo et al. 2007): chosen the density profile to describe each mass component of the cluster and given in input the positions of a set of multiple images, through the Bayesian statistic, this software explores the free parameters space sampling the posterior PDF and searching for the sample of parameters where the posterior PDF is higher and the negative entropy is $\sim 1$.

Initially the negative entropy is $\gg 1$ because the output posterior PDF is 'distant' from the input prior PDF: to reduce this 'distance', we need to run an optimization to converge progressively the posterior PDF to the prior PDF. In *Lenstool* it is used a Markov chain Monte Carlo (MCMC) method to converge the posterior PDF to the prior PDF (Jullo et al. 2007): in each optimization step, ten new PDF samples are drawn randomly from the posterior PDF of the current step, these PDF sample are weighted according to the their likelihood; the ones with the worst likelihood are deleted, while the one with the best likelihood are used for the next step to draw new PDF samples (for example, at the first step ten PDF are drawn from the prior PDF, the ones with the best likelihood are selected and in the second optimization step, from these selected PDFs other PDF samples are drawn, and so on). This procedure leads to the convergence of the posterior PDF toward the prior PDF; of course more samples are generated and more completed is the exploration of the parameters space, but this imply that more time is needed for the optimization.

The values of the free parameters with the higher probability in the PDF are estimated at each optimization step, and in correspondence of the lens model related to these values the scatter between positions of the observed
and the estimated images is computed, i.e. the $\chi^2$ in Eq.3.5 is estimated for each posterior PDF drawn.

### 3.3.3 Modelling ClG0152-1357

ClG0152-1357 is a rich cluster of galaxies at redshift 0.83 composed by different clumps, likely in a merging phase (see Fig.3.7). It shows a high variety of lensed images and gravitational arcs of sources at higher redshift lensed by the cluster: we can select samples of multiple images to use as constraints to build the lens model using Lenstool.

To build a good model of a cluster we must take into account at least of three mass components: the galaxies, the intra-cluster gas and the dark matter halo.

X-ray observations from *Chandra ACIS* (Huo et al. 2004) allowed the identification of two main interacting clumps in this cluster, whose projected distance is $\sim 1.5'$ (0.7 Mpc): there are two peaks in the X-ray emission where, in the optical images, two regions with galaxy overdensity are found (see Fig.3.3). Both clumps show a symmetric shape, elongated in the NE-SW direction. The NE clump is centered at $\alpha = 01:52:44.1, \delta = -13:57:22.3$ (with an offset of $\sim 6''.0$ respect to the optical center of the clump) and if modelled with a NFW profile has $r_s = 37''.5^{+2.6}_{-2.4}$; the SW clump is centered at $\alpha = 01:52:39.7, \delta = -13:58:30.1$ (with an offset of $\sim 4''.9$ respect to the optical center) and if modelled with a NFW profile has $r_s = 19''.34^{+2.17}_{-1.46}$ (Huo et al. 2004).

From photometric and spectroscopic analysis of the galaxies of the cluster, Jorgensen et al. (2005) found that the cluster is at redshift $z_{\text{cluster}} = 0.8350 \pm 0.0012$ and it has a line-of-sight velocity dispersion of $\sigma_{\text{cluster}} = 1110 \pm 163$ km/s, while for the two clumps they found $z_{\text{NE}} = 0.8372 \pm 0.0014$ and $z_{\text{SW}} = 0.8349 \pm 0.0020$, with line-of-sight velocity dispersions $\sigma_{\text{NE}} = 681 \pm 232$ km/s and $\sigma_{\text{SW}} = 866 \pm 266$ km/s.

For both the two clumps, to model the smooth halos of DM and gas, we adopted a NFW density profile, so we have 7 parameters for each clump: central position $(x_c, y_c)$, ellipticity $e$, position angle PA, scale radius $r_s$, concentration $c$ and the exponent $\alpha$ of the power law of the NFW profile.

As initial values in the optimization process we used for these parameters the ones found in the literature (Huo et al. 2004, Jorgensen et al. 2005) listed in Table 3.6: we optimized these initial values through the Bayesian optimizations described above, running 10 sampling iterations in the first optimizations (to identify the best model region in the space parameters),
Due to the offset between the center of the clumps as measured in the X-ray data and in the optical images (6″ for the NE clump and of 4.9″ for the SW clump, see Huo et al. 2004), we used the optical coordinate for the center of the clumps (centered on the central Giant Elliptical (cD) galaxies of the clumps) optimizing both δ and α within a range of 5″. For the ellipticity we assumed as initial value 0.5, using as optimization interval the whole range [0,1]. Because of the evident elongation of both the clumps in the NE and SW direction, we used as initial position angle the value θ = 120° varying it within Δθ = 60°.

The initial values used for the two scale radii are 280kpc for the NE clump and 140 kpc for the SW clump (Huo et al. 2004); for the concentration we used an initial value of 4 for both the clumps, and finally for the exponent of the power-law we assumed the constant value α = 1.

Figure 3.3: Left: R-band image of CLG0152 from LRIS (Keck Telescope) on which the X-ray emission is overplotted (black lines are iso-emission contours in the X-ray). Right: zoom of the two regions of galaxy overdensity coincident with the peaks in the X emission (Huo et al. 2004)
The total number of free-parameters related to the clumps is 12.

To build a sample of galaxy members of the cluster to include as sub-structures in the lens model, we selected from optical images galaxies within the red-sequence of the cluster, with $r \leq 26$ and $1.6 \leq r - z \leq 2.2$, and with distance from the center of the clump lower than 40" for the NE clump (being the initial $r_s = 37^\prime.5^{+26}_{-23.4}$) and lower than 20" for the SW clump (being its initial $r_s = 19^\prime.3^{+17.2}_{-1.46}$) (Huo et al. 2004). Then we checked that the selected sample of galaxies were cluster members matching them with the galaxies with known spectroscopic redshift $z_{sp}$ from catalogs of members and field galaxies (from Demarco et al. 2005, Blakeslee et al. 2006): we obtained a final catalog of 29 galaxies members of the cluster.

All the galaxies were modelled with the Pseudo-Isothermal Elliptical Mass Distribution (PIEMD, Kassiola and Konver 1993) model which surface mass density is given by

$$\Sigma(x, y) = \frac{\sigma_0^2}{2G} \frac{r_{cut} - r_{core}}{r_{core}^2 + \rho^2} \left[ \frac{1}{r_{core}^2 + \rho^2} - \frac{1}{(r_{cut}^2 + \rho^2)} \right], \quad (3.8)$$

where $\rho^2 = [(x-x_c)/(1+\epsilon)]^2 + [(y-y_c)/(1-\epsilon)]^2$, $(x_c, y_c)$ is the central position with respect to the cluster center, $\epsilon$ is the ellipticity, $\sigma_0$ is the central velocity dispersion, and $(r_{core}, r_{cut})$ are the core and cut radii. For each galaxy we fixed position and ellipticity using the ones computed through SExtractor, while $r_{core}, r_{cut}$ and $\sigma_0$ are free parameters.

From our catalog of galaxy members, we selected 3 cD galaxies (the two central cD galaxies of the NEclump and the one of the SWclump) that we optimized as individual sub-structures: each of them add 3 parameters to our list of free parameters. For the other galaxies we assume that their parameters follow relations scaling with the galaxy luminosity (Jullo E. et al. 2007):

$$\sigma_0 = \sigma_0^* \left( \frac{L}{L^*} \right)^{1/4} \quad (3.9)$$

$$r_{core} = r_{core}^* \left( \frac{L}{L^*} \right)^{1/2} \quad (3.10)$$

$$r_{cut} = r_{cut}^* \left( \frac{L}{L^*} \right)^{\alpha} \quad (3.11)$$

Using this scaling relations, we add only three free parameters for these galaxies altogether: $\sigma_0, r_{core}, r_{cut}$. In Fig.3.4 both the smoothed halos and the galaxies sub-structures profiles included in our model are shown.
Finally we need to select the multiple images to use as constraints. There is a set of three multiple images for which the spectroscopic redshift of the source ($z_{sp} = 3.928$) is known (Umetsu et al. 2005): this is the strongest constraint that we have for the lens model. We selected other families of arclets from the color image of the cluster computed with the optical images from the ACS/WFC3 (see Fig. 3.7). For the NE clump (Fig. 3.5) we identified five families of arclets other than the one with the known spectroscopic redshift. For the SW clump (Fig. 3.6), even if we can identify many lensed images, we are not able to identify families of multiple images: there is only an arc that shows a symmetric distribution in luminosity with two bright peaks at its ends and so, assuming that it is a fold image, merging of two multiple images, we split it into two multiple images. From an inspection of the HST/ACS color image (see Fig. 3.7), it seems plausible that the critical line for this arc passes in the middle of it.

Figure 3.4: The CIG0152-1357 NE and SW smooth clump-scale halos (in green) and the selected galaxies spectroscopically confirmed to belong to the clumps (in red).
Figure 3.5: NE clump: families of the arclets used as constraints for the NE clump; the green family (ID: A) is the one with the known redshift of the sources $z_{sp} = 3.928$.

We decomposed arcs that show substructures in different sub-families of multiple images instead of using them as extended lensed images, giving as constraint that this sub-families must have the same redshift: for example see the sub-families $D$ and $E$: they are three multiple images of substructures of the same source. All the selected families of multiple images are listed in Table 3.9.

Each multiple image introduces two constraints in the model (i.e. on its position), so for each family with $n$ multiple images we have $2 \times n$ constraints. If the redshift of a family of multiple images is unknown, there is another free parameter to determine for our model. Other than multiple image positions we introduce as constraints also some critical points, that is the middle points between two merging multiple images of the same family, where the critical line is expected to pass. We introduce four critical points between A1-A2, B1-B2, I1-I2, K1-K2 images.

### 3.3.4 Optimization of the lensing model

Given the starting values of the parameters for the two clumps, we run the first optimization of the model of the cluster giving in input only two families of multiple images, one for the NE clump (the one with the spectroscopic
redshift known) and the one for the SW clump. Then we added the other
groups of arcs in the NE clump one by one in the following optimizations,
using them to help us to restrict the ranges of optimization of the free-
parameters and to determine their best values. When we notice that a free-
parameter converges to a fixed value, after a set of optimization we assume
that value as the best value for our model and use it as a fixed parameters,
to reduce the number of free-parameters: this is what happened for example
for the PA and the ellipticity $e$ of the NE clump that converged to the values
$PA \sim 134^\circ$ and $e \sim 0.57$.

The best lens model that we obtained has negative entropy $= 10.15$ and
$\chi^2_{tot} = 24.34$, having 21 degrees of freedom (dof). The best values at which
the parameters converge to are listed in Table 3.7 while in the Table 3.8 the
output redshifts of the input families of the multiple images are listed.

To obtain this best model, we had to omit two arcs in the NE clumps: the
$D.2 - E.2$ arc and the $F.1 - G.1 - H.1$ arc. If we include them in the input
set of arcs, then the best optimization that we obtained correspond to $\chi^2_{tot} =
178.6$ with 27 degrees of freedom. Since we are modelling the cluster only
taking into account of the smoothed halos and the galaxies structures, we
are neglecting every other mass concentration components. So we decided to

Figure 3.6: SW clump in the galaxy cluster ClG0152: the gravitational arc is made
of two well-separated images, allowing a constrain on the matter distribution in
the SW clump (see text for details).
Figure 3.7: HST/ACS color image of the sky region centered on the cluster ClG 0152-1357.
exclude these two arcs, assuming that there should be some inhomogeneities in the matter distribution that we are not taking into account. In Fig. 3.8 the critical lines and caustics of the best model for a source at redshift \( z_s = 4 \) and a source at redshift \( z_s = 7 \) are shown. Given the best model of the mass distribution for CLG0152, through Lenstool we can obtain the convergence and the magnification maps for all the source redshift \( z_{source} \) that we want.

Figure 3.8: HST/ACS r-band image of cluster CLG0152. The overplotted lines are the critical lines (red) and the caustics for sources at redshift \( z_s = 4 \) (the inner lines) and at \( z_s = 7 \) (the outer lines).
3.4 Weak-lensing mass modelling

The weak lensing mass modelling technique uses a statistical approach to reconstruct the mass distributions of the clusters, studying weak distortions imprinted on the shape of background galaxies: the large number density of these galaxies, \( \sim 30 - 40 \) galaxies per arcmin\(^2\) of the sky (and it can reach \( \sim 100 \) galaxies per arcmin\(^2\) in deep observations), allows a statistical analysis of weak deformations of their shapes. The intrinsic shape of these sources is not known, but if we assume that they are randomly oriented, the average shape over all the sources should be circular, due to their large number. When weak lensing deforms the shape of a circular source with radius \( r \), it appears elliptical, and, assuming \( \kappa, \gamma \ll 1 \), the ellipse axes are given by (Meneghetti 2006):

\[
a = \frac{r}{1 - \kappa - \gamma} \quad (3.12) \\
b = \frac{r}{1 - \kappa + \gamma}. \quad (3.13)
\]

The ellipticity of the source results

\[
e = \frac{a - b}{a + b} = \frac{\gamma}{1 - \kappa} \approx \gamma, \quad (3.14)
\]

that is, the elliptical distortion of an intrinsic circular source directly measures the shear at the position of the source. If we have a large number of background galaxies, which average shape is expected to be circular, due to weak lensing distortion, the average ellipticity over all the sources around a position \( \vec{\theta} \) across the cluster gives the average value of the shear in the region around \( \vec{\theta} \):

\[
\langle e \rangle = \langle \gamma \rangle.
\]

Since \( \vec{\gamma} \) is related to the lensing potential \( \psi \) through the relations Eq. 2.17 and Eq. 2.18, if we know the shear at different positions \( \vec{\theta} \) across the cluster, we can deduce the lensing potential and then the convergence \( \kappa(\vec{\theta}) \) at that positions up to an arbitrary additive constant (mass-sheet degeneracy). A way to break this degeneracy is through the maximum likelihood method, by estimating the potential \( \psi \) that minimize the \( \chi^2 \) defined as

\[
\chi^2 = \sum_{\text{pixel}} \frac{[\gamma_1 - \gamma_1(\psi)]^2 + [\gamma_2 - \gamma_2(\psi)]^2}{2\sigma_\gamma^2} \quad (3.15)
\]

where \( (\gamma_1, \gamma_2) \) are the measured shear components, \( \sigma_\gamma \) is the shear uncertainty and \( (\gamma_1(\psi), \gamma_2(\psi)) \) are the estimated values related to the potential \( \psi \) by the relations 2.17 and 2.18. Minimizing the \( \chi^2 \) we can estimate the potential \( \psi \) at each position where the \( \gamma \) has been measured: so if we measure the average
shear in correspondence of a large sample of positions across a cluster, we can trace a detailed lensing potential map, from which a detailed convergence map can be deduced.

### 3.4.1 Galaxy cluster RDCS1252

RDCS1252 is a massive galaxy cluster at $z = 1.23$, first detected through its X-ray emission (Rosati et al. 2004). The optical center of the cluster is located at $\alpha=12:52:54.5$, $\delta=-29:27:17.5$, where two cD galaxies are observed. X-ray observations with the *Chandra* ACIS-I detector shows a strong emission peak shifted of $\sim 8''$ (corresponding to $\sim 65$ kpc at the cluster redshift) with respect to the optical center. This likely reflects an offset between the baryonic and dark matter components. Moreover, the X-ray emission shows an elongation in the gas mass distribution in the Est-West direction that, together with the central offset, suggests that the cluster is in a post-merging phase. From HST/ACS deep images we can identify different images candidate as gravitational arcs (see Fig. 3.9), but we can not identify families of multiple images (of the same sources), so we can not use such lensed images to build a model of the cluster through the strong lensing methods. Therefore for this cluster we used weak lensing data in order to constrain the magnification map of the cluster.

A weak lensing analysis of the cluster was carried out by Lombardi et al. (2005) using the same HST/ACS images data used in this thesis. They measured the ellipticity in both filters (F775w, i.e. $i$-band, and F850lp, i.e. $z$-band) for a selected sample of faint background galaxies, from which they obtain the shear $\gamma_{i\text{-}band}$ and $\gamma_{z\text{-}band}$ as a function of the sky position. Since $\gamma$ depends on the galaxy redshift, to reconstruct a shear map all the galaxies were assumed to be at the effective redshift $^{3}z_{\text{eff}} = 1.72$. The estimated shear map was first smoothed by a Gaussian kernel to obtain a continued shear field, then the convergence field has been obtained by inverting this shear field. Fig. 3.10 shows the final convergence map obtained by combining both the $i$- and $z$-band measurements.

To break the so-called mass-sheet degeneracy, the mass distribution was fitted with parametric mass model minimizing the $\chi^2$ defined as

$$
\chi^2 = \sum_{n=1}^{N} \frac{|g_n - g_n(\vec{\theta})|^2}{\sigma_n^2},
$$

where $g_n$ is the measured shear, $\sigma_s$ is the related uncertainties, $g_n(\vec{\theta})$ is the

---

This value is estimated by resampling the photometric redshift catalogs of the Hubble Deep Field (see Lombardi et al. 2005).
3.4 Weak-lensing mass modelling

Figure 3.9: HST/ACS color image of the sky region centered on the cluster RDCS 1252.
3.4 Weak-lensing mass modelling

Figure 3.10: Convergence map obtained from the weak lensing analysis of F775lp and F850w images from HST/ACS. Contours delimit overdensity (red) and under-density (green) regions with respect to the average convergence over all the field (blue contour). Contours are spaced by 0.015, that is the median error of the map estimated over all the field (Lombardi et al. 2005).

<table>
<thead>
<tr>
<th>Model</th>
<th>$\theta_0$ (arcsec)</th>
<th>$\sigma_v$ (km/s)</th>
<th>$r_c$ (arcsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIS</td>
<td>-9.61 0.10</td>
<td>1185</td>
<td>5.91</td>
</tr>
<tr>
<td>Model</td>
<td>$\theta_0$ (arcsec)</td>
<td>$\Sigma_0$(M$_\odot$/pc)</td>
<td>$r_s$(arcsec)</td>
</tr>
<tr>
<td>NFW</td>
<td>-10.17 1.78</td>
<td>523</td>
<td>52</td>
</tr>
</tbody>
</table>

Table 3.5: Best-fit parameters for RDCS1252 model (see text for details).

estimated shear and the sum is over all the galaxies of the background galaxies sample. Both the SIS and the NFW mass profiles were used to model the mass distribution of RDCS1252, both giving results in good agreements with the X-ray analysis of the cluster.

In Table 3.5 the parameters of the best model for both the profiles are listed: the center of the model $\theta_0$ (given in celestial coordinates $\Delta \alpha$ and $\Delta \delta$ with respect to the central peak of the weak lensing map at $\alpha=12:52:55.4; \delta=-29:27:19.6$), the velocity dispersion $\sigma_v$ and the core radius for the SIS model, the scale radius and the density $\Sigma_0 = \rho_s r_s$ for the NFW model. We used the best SIS model parameters as reference to estimate the mass distribution of RDCS1252 and to compute its amplification maps through the Lenstool package for sources at different redshift.
3.4 Weak-lensing mass modelling

Table 3.6: Initial values for the NFW profile parameters of the NE and SW clumps of CLG0152.

<table>
<thead>
<tr>
<th>parameter</th>
<th>NE clump</th>
<th>SW clump</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{\text{opt}}$</td>
<td>01:52:43.8 ± 5.0</td>
<td>01:52:39.7 ± 5.0</td>
</tr>
<tr>
<td>$\delta_{\text{opt}}$</td>
<td>-13:57:18.8 ± 5.0</td>
<td>-13:58:25.8 ± 5.0</td>
</tr>
<tr>
<td>$e$</td>
<td>0.5 ± 0.5</td>
<td>0.5 ± 0.5</td>
</tr>
<tr>
<td>$PA(\degree)$</td>
<td>120 ± 60</td>
<td>120 ± 60</td>
</tr>
<tr>
<td>$r_s(\text{kpc})$</td>
<td>37.5 ± 20</td>
<td>19.34 ± 20</td>
</tr>
<tr>
<td>$c$</td>
<td>4 ± 3</td>
<td>4 ± 3</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.7: Final values for the NFW profile parameters of the NE and SW clumps of CLG0152 obtained with the best cluster model.

<table>
<thead>
<tr>
<th>parameter</th>
<th>NE clump</th>
<th>SW clump</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{\text{fin}}$</td>
<td>01:52:43.0 ± 0.6</td>
<td>01:52:38.6 ± 0.9</td>
</tr>
<tr>
<td>$\delta_{\text{fin}}$</td>
<td>-13:57:18.6 ± 0.7</td>
<td>-13:58:21.2 ± 1.9</td>
</tr>
<tr>
<td>$e$</td>
<td>0.623 ± 0.023</td>
<td>0.595 ± 0.077</td>
</tr>
<tr>
<td>$PA(\degree)$</td>
<td>132.8 ± 0.4</td>
<td>110.1 ± 3.1</td>
</tr>
<tr>
<td>$r_s(\text{arcsec})$</td>
<td>35.9 ± 3.8</td>
<td>27.3 ± 2.1</td>
</tr>
<tr>
<td>$c$</td>
<td>4.60 ± 0.04</td>
<td>4.99 ± 0.04</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.8: Redshift estimated by Lenstool for the families of arcs used as constrains in modelling CLG0152.

<table>
<thead>
<tr>
<th>ID</th>
<th>B-C</th>
<th>D-E</th>
<th>F-G-H</th>
<th>I</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>2.39 ± 0.07</td>
<td>1.76 ± 0.04</td>
<td>3.44 ± 0.17</td>
<td>1.88 ± 0.15</td>
<td>2.85 ± 0.13</td>
</tr>
</tbody>
</table>
Table 3.9: Selected families of arclets in the NE and SW clumps of CLG0152; the ID color refers to the signature colors of the arcs in the images in Fig. 3.5-3.6

<table>
<thead>
<tr>
<th>ID</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$z_{spec}$</th>
<th>ID Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>28.1890</td>
<td>-13.9521</td>
<td>3.9277</td>
<td>Green</td>
</tr>
<tr>
<td>A.2</td>
<td>28.1883</td>
<td>-13.9511</td>
<td>3.9277</td>
<td>Green</td>
</tr>
<tr>
<td>A.3</td>
<td>28.1852</td>
<td>-13.9488</td>
<td>3.9277</td>
<td>Green</td>
</tr>
<tr>
<td>B.1</td>
<td>28.1872</td>
<td>-13.9544</td>
<td>unknown</td>
<td>Blue</td>
</tr>
<tr>
<td>B.2</td>
<td>28.1858</td>
<td>-13.9529</td>
<td>unknown</td>
<td>Blue</td>
</tr>
<tr>
<td>B.3</td>
<td>28.1832</td>
<td>-13.9508</td>
<td>unknown</td>
<td>Blue</td>
</tr>
<tr>
<td>C.1</td>
<td>28.1868</td>
<td>-13.9537</td>
<td>unknown</td>
<td>Yellow</td>
</tr>
<tr>
<td>C.2</td>
<td>28.1865</td>
<td>-13.9534</td>
<td>unknown</td>
<td>Yellow</td>
</tr>
<tr>
<td>D.1</td>
<td>28.1848</td>
<td>-13.9565</td>
<td>unknown</td>
<td>Magenta</td>
</tr>
<tr>
<td>D.2</td>
<td>28.1832</td>
<td>-13.9544</td>
<td>unknown</td>
<td>Magenta</td>
</tr>
<tr>
<td>D.3</td>
<td>28.1813</td>
<td>-13.9534</td>
<td>unknown</td>
<td>Magenta</td>
</tr>
<tr>
<td>E.1</td>
<td>28.1845</td>
<td>-13.9566</td>
<td>unknown</td>
<td>Dark Yellow</td>
</tr>
<tr>
<td>E.2</td>
<td>28.1832</td>
<td>-13.9541</td>
<td>unknown</td>
<td>Dark Yellow</td>
</tr>
<tr>
<td>E.3</td>
<td>28.1810</td>
<td>-13.9534</td>
<td>unknown</td>
<td>Dark Yellow</td>
</tr>
<tr>
<td>F.1</td>
<td>28.1825</td>
<td>-13.9600</td>
<td>unknown</td>
<td>Dark Blue</td>
</tr>
<tr>
<td>F.2</td>
<td>28.1792</td>
<td>-13.9584</td>
<td>unknown</td>
<td>Dark Blue</td>
</tr>
<tr>
<td>F.3</td>
<td>28.1774</td>
<td>-13.9562</td>
<td>unknown</td>
<td>Dark Blue</td>
</tr>
<tr>
<td>G.1</td>
<td>28.1824</td>
<td>-13.9599</td>
<td>unknown</td>
<td>Cyan</td>
</tr>
<tr>
<td>G.2</td>
<td>28.1794</td>
<td>-13.9585</td>
<td>unknown</td>
<td>Cyan</td>
</tr>
<tr>
<td>G.3</td>
<td>28.1774</td>
<td>-13.9559</td>
<td>unknown</td>
<td>Cyan</td>
</tr>
<tr>
<td>H.1</td>
<td>28.1822</td>
<td>-13.9598</td>
<td>unknown</td>
<td>Light Blue</td>
</tr>
<tr>
<td>H.2</td>
<td>28.1796</td>
<td>-13.9585</td>
<td>unknown</td>
<td>Light Blue</td>
</tr>
<tr>
<td>H.3</td>
<td>28.1773</td>
<td>-13.9558</td>
<td>unknown</td>
<td>Light Blue</td>
</tr>
<tr>
<td>I.1</td>
<td>28.1834</td>
<td>-13.9567</td>
<td>unknown</td>
<td>Red</td>
</tr>
<tr>
<td>I.2</td>
<td>28.1830</td>
<td>-13.9564</td>
<td>unknown</td>
<td>Red</td>
</tr>
<tr>
<td>K.1</td>
<td>28.1651</td>
<td>-13.9770</td>
<td>unknown</td>
<td>Dark Red</td>
</tr>
</tbody>
</table>
Chapter 4

Candidate galaxies at high-z: selection and first results

In this Chapter we present and discuss the candidate galaxies at redshift $z > 6$ that we selected on the basis of their photometry in the field of the two GTs that we analysed. We present the photometric criteria that we adopted for our high-z candidates selection and the estimation of their photometric redshift. Then we discuss the expected number of high-z sources and compare it with the observed number in our survey. In the last part of this work, we estimate the rest-frame ultraviolet luminosity function and star-formation rate (SFR) of the selected sample, and we compare our results with recent ones from literature, finally discussing the next steps in our long-term project.

4.1 Photometric selection criteria

To select a sample of high redshift candidates we fixed some photometric criteria that such sources must satisfy. We are searching for galaxies at $z \geq 6$: for such galaxies the Lyman-α break (see 1.3.2) is redshifted at $\lambda > 8500$Å, so we can identify this sources at least as $i$-dropouts. This means that $z \geq 6$ candidates galaxies must be undetected in the $r$ and $i$ optical bands: we consider a source undetected in a filter $f$ if its flux $F_f$ is lower than $3\sigma_f$, where $\sigma_f$ is the background flux of the image. In terms of magnitudes, this reads $m_f > \mu_f - 1.193$ (where $\mu_f$ is the background magnitude), so in each filter we can set the detection condition as:

$$\begin{cases} m_f > \mu_f - 1.193 & \text{the object is not-detected in the filter f} \\ m_f < \mu_f - 1.193 & \text{the object is detected in the filter f} \end{cases}$$

(4.1)
4.1 Photometric selection criteria

So, given the filters $f_{\lambda 1}$ and $f_{\lambda 2}$ with $\lambda 1 < \lambda 2$, an object is an $f_{\lambda 1}$-dropout if it is not-detected in the $f_{\lambda 1}$ filter while it is detected in the redder filter $f_{\lambda 2}$, i.e. it must satisfy the *dropout conditions*:

$$\begin{cases} m_{\lambda 1} > \mu_{\lambda 1} - 1.193 \\ m_{\lambda 2} < \mu_{\lambda 2} - 1.193 \end{cases}$$

(4.2)

By subtracting the previous conditions, we deduce a color condition that a $\lambda 1$-dropouts must satisfy:

$$m_{\lambda 1} - m_{\lambda 2} > \mu_{\lambda 1} - \mu_{\lambda 2}$$

(4.3)

In summary, the strong photometric conditions that our high-$z$ candidates must satisfy are:

$$\begin{cases} m_{\lambda 1} > \mu_{\lambda 1} - 1.193 \\ m_{\lambda 1} - m_{\lambda 2} > \mu_{\lambda 1} - \mu_{\lambda 2} \end{cases}$$

(4.4)

To have $z > 6$ candidates $f_{\lambda 1}$ must be the $r$ or a redder band. Moreover, to be a *strong* $f_{\lambda 1}$-dropouts we required that an object must be not-detected in all the filters of our dataset bluer than $f_{\lambda 1}$ and that it is detected in the redder filters.

In Table 3.3 we listed the magnitude limits (given by $3\sigma$) for our dataset for each cluster; the photometric conditions that we adopt to select $z > 6$ candidates are listed in Table 4.1: the sources that satisfy these conditions are included in the sample of $z > 6$ candidates for which we estimated the photometric redshifts using the LePhare package (Arnouts & Ilbert 2010).

Through these photometric criteria we selected:

- a sample of 36 $i$-dropouts and 41 $z$-dropouts for CLG0152;

- and a sample of 128 $z$-dropouts and 9 $Js$-dropouts for RDCS1252.

We can not directly classify these selected objects as real dropouts since we used the mean background values to fix the magnitude limits in each band: due to the dishomogeneity of the background in our images we need a further visual check of these objects to be sure they are dropouts. Moreover a further visual check is needed to remove spurious selection, as objects near the edges of the fields or close to bright sources.
4.2 Photometric redshifts

There are two main techniques used to estimate redshifts from photometric data: the spectral energy distribution (SED) fitting and the training-set (based on neural networks) methods. The SED-fitting method needs a reference library of template spectra: the measured magnitudes of an object in different bands are fitted with the template spectra, after that these are redshifted and corrected for intergalactic extinction. The best-fit result returns the photometric redshift, the extinction and the template spectrum which better approximates the input photometric data. Therefore, in the case a large number of photometric points is available, also the physical properties of the underlying stellar populations could in principle be estimates (e.g., age, metallicity, star-formation rate).

The training set methods instead needs a sample of galaxies with known spectroscopic redshift: through a multiparametric fit, a relationship $z(c, m_0)$ between the redshift and the apparent magnitude and colors is extrapolate from the sample of galaxies with known $z_{sp}$. This relationship is then used to estimate the photometric redshift for all objects for which we have the apparent magnitude and colors. We note that this method is based on the assumption that $z = z(c, m_0)$ is a function defined in the $(c, m_0)$ space. To have a strong $z(c, m_0)$ relationship we need a large sample of spectroscopic confirmed galaxies to exhaustively explore the $(z, c, m_0)$ space. This is not our case, since we have only a small sample of spectroscopic confirmed galaxies in the sky regions of our clusters; moreover the sample that we have spans a tight redshift interval around the cluster redshift, and so it does not allows to sample the $z(c, m_0)$ relationship in the high redshift range that we are want to investigate.

To estimate the photometric redshift of the $z > 6$ candidates that we selected with photometric criteria, we used the tool Le PHARE (Arnouts & Ilbert 2010)\(^1\): this package computes photometric redshift through a SED-

---

Table 4.1: Photometric limits for $z > 6$ candidates

<table>
<thead>
<tr>
<th></th>
<th>i-dropouts</th>
<th>z-dropouts</th>
<th>J-dropouts</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLG0152</td>
<td>$i &gt; 25.08$</td>
<td>$z &gt; 24.14$</td>
<td>$J &gt; 23.60$</td>
</tr>
<tr>
<td></td>
<td>$i-z &gt; 0.94$</td>
<td>$z-J &gt; 0.54$</td>
<td>$J-H &gt; -0.06$</td>
</tr>
<tr>
<td>RCDS1252</td>
<td>$i &gt; 25.55$</td>
<td>$z &gt; 24.61$</td>
<td>$J &gt; 26.28$</td>
</tr>
<tr>
<td></td>
<td>$i-z &gt; 0.93$</td>
<td>$z-J &gt; -1.67$</td>
<td>$J-H &gt; 0.24$</td>
</tr>
</tbody>
</table>

\(^1\)PHotometric Analysis for Redshift Estimations (LE PHARE) is available at the website http://www.oamp.fr/people/arnouts/LEPHARE.html.
4.2 Photometric redshifts

![Output spectrum determined by LePhare for the image A.1 (with $z_{sp} = 3.928$) at $z_{LP} = 3.6 \pm 0.4$. The bottom panel on the right is the associated probability distribution function as a function of redshift.]

fitting method between the observed magnitudes and template spectra, but it also gives the chance to optimize the fitting procedure through a training upon a sample of spectroscopic confirmed galaxies.

Through LePhare we use a standard $\chi^2$ fitting method to fit the observed magnitude with template spectra, with $\chi^2$ defined as (Ilbert et al. 2006)

$$\chi^2(z, T, A) = \sum_{f=1}^{N_f} \left( \frac{F_{\text{obs}}^f - A \times F_{\text{pred}}^f(z, T)}{\sigma_{\text{obs}}^f} \right)^2,$$

where the index $f$ refers to the filters of our observations, $N_f$ is the number of filters, $F_{\text{obs}}^f$ is the flux observed, $\sigma_{\text{obs}}^f$ is its error, $F_{\text{pred}}^f(z, T)$ is the estimated flux for the redshift $z$ and the spectral type $T$, and finally $A$ is a normalization factor. The photometric redshift is computed searching for the solution $(z, A, T)$ for which the $\chi^2$ is minimal.

Giving in input the catalog of magnitudes and magnitude uncertainties of the selected objects, LePhare returns in output a catalog where, for each object in the input catalog, there are listed the $\chi$ of the best fit, the estimated redshift, the extinction and other optional informations, e.g. star formation rate, age or absolute magnitudes. Moreover, for each object LePhare computes the redshift PDF and determines the best fitting spectrum from the input library of templates.
4.2 Photometric redshifts

Running Lephare on a sample of galaxies with known spectroscopic redshift allows us to estimate and correct systematic offsets in the flux measurements. The offset in each filter \( f \) is computed on the sample of the bright galaxies \( (i \leq 22.5) \) minimizing the sum (Ilbert et al. 2006)

\[
\psi^2 = \sum \left( \frac{A \times F_{\text{pred}} - F_{\text{obs}} + s_f}{\sigma_{\text{obs}}^f} \right)^2,
\]

where the sum is performed over all the bright galaxies in the spectroscopic sample and \( s_f \) is a free parameter. For random and normally distributed uncertainties, the average \( \langle s_f \rangle \) should be zero, but if there is an offset in the flux measurements than \( \langle s_f \rangle \neq 0 \) and its values gives the mean offset in the \( f \) filter.

We collected samples of all galaxies with spectroscopic redshift that are in the sky regions of our clusters (both members and field galaxies) and used these sample to compute the flux offsets for all our filters (see Table 4.2): then we corrected the magnitudes of our input catalogs by the computed offsets.

Using the lens model of CLG 0152, we estimate the redshifts of the families of multiple images for our final best model (see Sect. 3.3.3): we checked these redshift values with the ones computed by LePhare. We note that, even if for some families the best redshift computed by LePhare \( (z_{LP}) \) matches within the errors the one predicted by the lensing model \( (z_{\text{lens}}) \), but for other families we have very different redshift values. Therefore, we checked the PDF computed by LePhare, and found that some PDFs have more than one peak. For our families of multiple images we found that, when the \( z_{\text{lens}} \) does not match the best value computed by LePhare, corresponding to the higher peak in the PDF, it matches the value related to the secondary peak in the redshift PDF. In Table 4.3 we listed, for each family\(^2\), the redshifts predicted by the lensing model\(^3\), and the photometric ones computed by LePhare, in correspondence of the primary and secondary peaks of the PDF.

\(^2\)We omitted C and F families since their multiple images are too faint to be extracted by SExtractor.

\(^3\)For the A-family we used the spectroscopic redshift measured by Umetsu et al. (2005).

Table 4.2: Magnitudes offsets computed with LePhare

<table>
<thead>
<tr>
<th>Cluster</th>
<th>( r )</th>
<th>( i )</th>
<th>( z )</th>
<th>( J )</th>
<th>( J_s )</th>
<th>( H )</th>
<th>( K_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLG0152</td>
<td>0.272</td>
<td>0.178</td>
<td>0.042</td>
<td>-0.071</td>
<td>( \sim )</td>
<td>-0.013</td>
<td>( \sim )</td>
</tr>
<tr>
<td>RDCS1252</td>
<td>( \sim )</td>
<td>-0.005</td>
<td>-0.129</td>
<td>( \sim )</td>
<td>0.130</td>
<td>( \sim )</td>
<td>-0.079</td>
</tr>
</tbody>
</table>
Table 4.3: Redshifts computed by LePahre in correspondence of the primary ($z_{LP1}$) and secondary ($z_{LP2}$) peaks and by Lenstool ($z_{lens}$) for the families of multiple images that we used as constraints to model mass distribution in CLG0152.

<table>
<thead>
<tr>
<th>ID</th>
<th>A</th>
<th>B</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z_{L1}$</td>
<td>3.928±0.07</td>
<td>1.76±0.04</td>
<td>1.76±0.04</td>
</tr>
<tr>
<td></td>
<td>$z_{LP1}$</td>
<td>3.6±0.4</td>
<td>2.4±0.3</td>
<td>3.8±0.4</td>
</tr>
<tr>
<td></td>
<td>$z_{LP2}$</td>
<td>2.8±0.3</td>
<td>1.3±0.2</td>
<td>1.8±0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ID</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z_{L1}$</td>
<td>3.44±0.17</td>
<td>1.88±0.15</td>
<td>2.85±0.13</td>
</tr>
<tr>
<td></td>
<td>$z_{LP1}$</td>
<td>3.24±0.2</td>
<td>3.8±0.1</td>
<td>2.4±0.2</td>
</tr>
<tr>
<td></td>
<td>$z_{LP2}$</td>
<td>2.2±0.3</td>
<td>3.0±0.1</td>
<td>3.6±0.2</td>
</tr>
</tbody>
</table>

Except for the $I$ family, we find that for all other families the redshifts computed by Lenstool for our best model of CLG0152 match (within the estimated errors) the redshift computed by LePhare in correspondence of the primary, or at least the secondary, peak. As an example, in Fig. 4.1 we show the output spectrum for the best fit and the photometric redshift PDF for the image A.1

By performing the SED analysis on the sample of dropouts selected through the photometric criteria, we obtained photometric redshifts for our selected sources. We used as template library the CWW-KINNEY library: it contains four empirical galaxy spectra (including the SED of an Elliptical (E0), two spirals (Sbc, Scd), and an Irregular (Irr)) from Colemann, Wu and Weedman (1980), plus two empirical spectra of starburst galaxies (SB1, SB2) from Kinney et al. 1996. All templates were extrapolated into ultraviolet and infra-red wavelengths using the GISSEL synthetic models (Bruzual and Charlot, 2003). We corrected irregular (Irr) and starburts (SB1, SB2) template for extinction adopting the Calzetti et al. (2000) extinction law, with color excess E(B-V) ranging over (0, 0.3).

We selected all the source for which LePhare estimated the best photometric redshift $z_{ph} \geq 6$: for each of this object we checked the $\chi^2$, related to the spectrum that best fit the observed magnitudes, removing from our final sample the object with bad best fit ($\chi^2 > 25$).

Our final samples of high-$z$ candidates includes 13 objects from CLG01252 and 80 objects from RDCS1252.
4.3 Selection of high-$z$ galaxies

The final sample of high-redshift candidates that we collected includes objects that satisfy our photometric criteria and that have photometric redshifts $z \geq 6$, within the estimated error bars.

First of all, in order to delete spurious selections (due to noise fluctuations), a visual check is performed, introducing also the mid-infrared images from Spitzer/IRAC in which high-$z$ candidates are expected to be detected. Indeed, Spitzer/IRAC is a four-CCDs instrument covering the wavelength range from about $3.6 \mu m$ to $8 \mu m$ with four broad filters: in these bands we are observing the optical rest-frame region, from 4000 to 11000 Å approximately, of sources at redshift $z \sim 6$.

Through this visual check we remove objects that were included in our dropout-sample probably due to local fluctuations in the image background. Indeed for simplicity, our selection was performed by using as detection threshold a value three times the average background (3σ) in each image, although the background maps produced by SExtractor reveal that the background is not homogeneous over all the image field.

After the visual check, our final high-$z$ candidate sample counts 3 candidates in the field of the galaxy cluster CLG0152 and 11 in the field of RDCS1252: all the final candidates are presented in Fig. 4.2 for CLG0152.
4.3 Selection of high-$z$ galaxies

Figure 4.3: Strong high-redshift candidates in RDCS152, in the optical ($i,z$), near-infrared (Js,Ks) and mid-infrared ch1 and ch2) filters (each panel is $10'' \times 10''$). The identification numbers of the candidates are, from top to bottom: #9019, 9031, 9033, 9034, 9048, 9049.

and Fig. 4.3-4.4 for RDCS1252. In each row, we present a small sky region around the candidate source in the optical, near- and mid-infrared images.

We distinguish between faint and strong high-$z$ candidates, that is between sources that in their respective dropout-band are just below the local threshold (faint candidates) and sources that are not detected at all (strong candidates). We remind that we assigned the magnitude value -99 at objects not observed in a filter (e.g. object #2532 lies out of the FOV of VLT/ISAAC, so in the photometric catalog J=H=-99.0), while we assign the magnitude limits to object not detected in an image (e.g. all the high candidates in RDCS1252 were not recognized as objects by SExtractor in
Figure 4.4: Strong high-redshift candidates in RDCS152, in the optical (i,z), near-infrared (Js,Ks) and mid-infrared ch1 and ch2) filters (each panel is $10'' \times 10''$). The identification numbers of the candidates are, from top to bottom: 9053, 9054, 9057, 9064, 9078.

In the optical images, while they were extracted in the near-infrared images: at these objects we assign the higher magnitude limit 28.8 efficient in all the optical bands).

In CLG0125 two candidates (ID # 9001 and #9003) are weakly detected in the $r$-band, showing a signal lower then the local threshold, while in RDCS1252 there are 7 candidates showing a weak signal ($\lesssim 3\sigma$) in their dropout-band: in each case, such very low-signal detections can be local noise fluctuations, or possibly low optical-rest frame flux from galaxies actually at lower redshift ($z \sim 2$ contaminants). All the other candidates are completely not detected in their respective dropout-band.

In particular we note the candidate #9078 in the field of RDCS1252: from the visual inspection, we can clearly identify two close objects. One is visible also in the optical images but not in the infrared ones, while the other one is an optical-drop-out, and only appears in the near-infrared images. This
4.3 Selection of high-$z$ galaxies

system is probably a chance (and rare) superposition of two distinct sources at different cosmic epochs: VLT spectroscopy is necessary here to clarify its nature. Of course, on the basis of the available photometric data we refer at the second source as #9078 $z > 6$ candidate.

4.3.1 Redshift distribution

We separate the final candidates into three groups, according to their redshift probability distribution function (PDF, hereafter) computed by the tool Le-Phare. Such quantity is a function of the redshift and allows to estimate the robustness of the photometric redshift assigned by the SED fitting method. We have therefore the followings subgroups:

- (A): objects with one (or more) well defined maximum in their PDF($z$) at high redshift ($z \geq 6$);
- (B): objects with two comparable maxima at high ($z \geq 6$) and low ($z \leq 3$) redshift;
- (C): objects with an approximatively flat PDF, for which the best photometric redshift solution is not well defined.

In Tables 4.5 we listed magnitudes, dropout-band${}^4$, photometric redshift, $\chi^2$ of the spectral best fit and the PDZ-group for the final high-$z$ candidates in CLG0152 and RDCS1252. Finally in Fig. 4.5 -4.6 the PDF($z$) are presented for each of the candidate of both the clusters.

---

${}^4$We use here "$\leq 3\sigma$" to identify weak candidates
Table 4.4: Final high-z candidates

<table>
<thead>
<tr>
<th>id</th>
<th>r</th>
<th>i</th>
<th>z</th>
<th>J</th>
<th>H</th>
<th>Ks</th>
<th>dropout-band</th>
<th>$z_{ph}$</th>
<th>$\chi^2$</th>
<th>PDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2532</td>
<td>29.7±6.6</td>
<td>26.0±0.4</td>
<td>24.0±0.2</td>
<td>-99.0</td>
<td>-99.0</td>
<td>-</td>
<td>r-drop</td>
<td>5.8±0.6</td>
<td>0.04</td>
<td>B</td>
</tr>
<tr>
<td>9001</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>22.8±0.1</td>
<td>21.8±0.1</td>
<td>-</td>
<td>r-drop</td>
<td>8.2±0.1</td>
<td>20.2</td>
<td>A</td>
</tr>
<tr>
<td>9003</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>23.7±0.1</td>
<td>22.6±0.2</td>
<td>-</td>
<td>r-drop</td>
<td>9.2±0.1</td>
<td>0.8</td>
<td>A</td>
</tr>
<tr>
<td>9019</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>25.88±0.14</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>z-drop</td>
<td>9.38±0.19</td>
<td>0.309</td>
<td>B</td>
</tr>
<tr>
<td>9031</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>26.61±0.26</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>z-drop</td>
<td>9.65±0.23</td>
<td>0.267</td>
<td>B</td>
</tr>
<tr>
<td>9033</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>24.88±0.06</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>i-drop</td>
<td>9.53±0.04</td>
<td>2.020</td>
<td></td>
</tr>
<tr>
<td>9034</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>24.66±0.05</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>i-drop</td>
<td>9.500±0.001</td>
<td>8.292</td>
<td>A</td>
</tr>
<tr>
<td>9048</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>25.40±0.10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>z-drop</td>
<td>9.57±0.14</td>
<td>0.262</td>
<td>A</td>
</tr>
<tr>
<td>9049</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>24.37±0.04</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>i-drop</td>
<td>9.35±0.06</td>
<td>0.422</td>
<td>A</td>
</tr>
<tr>
<td>9053</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>24.99±0.07</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>z-drop</td>
<td>9.50±0.16</td>
<td>0.472</td>
<td>A</td>
</tr>
<tr>
<td>9054</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>25.04±0.07</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>i-drop</td>
<td>9.21±0.18</td>
<td>0.360</td>
<td>A</td>
</tr>
<tr>
<td>9057</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>24.99±0.04</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>i-drop</td>
<td>9.46±0.02</td>
<td>2.574</td>
<td>A</td>
</tr>
<tr>
<td>9064</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>24.37±0.04</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>z-drop</td>
<td>9.50±0.01</td>
<td>1.398</td>
<td>A</td>
</tr>
<tr>
<td>9078</td>
<td>28.8±2.0</td>
<td>28.8±2.0</td>
<td>25.46±0.10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>i-drop</td>
<td>9.69±0.01</td>
<td>0.297</td>
<td>A</td>
</tr>
</tbody>
</table>

Table 4.5: Final high redshift candidate in CLG0152 (id # 2532, # 9001 and # 9003) and RDCS1252 (the other ones id#).

In order from left to right we have: observed magnitudes, dropout band, photometric redshift, $\chi^2$ of the best fit template, the PDF type (following our classification, see text). The symbol - stay for no data; for objects lensed by RDCS1252, the J magnitude refers to the Js band.
4.4 Expected vs observed number of high-z galaxies

To estimate the expected number of high redshift source in a lensing survey we must take into account of different components:

1. **Sources properties**: in particular, the UV luminosity function that describes galaxy population at high redshift and the typical size of these sources;

2. **Gravitational Telescope properties**: the mass distribution and redshift of the massive cluster, in order to determine the effect of the gravitational lens;

Figure 4.6: Probability distribution functions of redshift for the candidates in RDCS1252.
3. **Survey properties**: depth and FOV of the different photometric data used in the survey.

Gravitational telescopes (GTs), as compared with blank field surveys, introduce two main effects: an increase in the survey effective depth by the amplification factor \( \mu \), and the reduction of the effectively explored sky background area by the same factor. Moreover, we have to take into account of the fraction of FOV occupied by the bright cluster galaxies, i.e., of the light contamination of these galaxies that obstruct to observe the Universe behind them. Unfortunately, this effect is quite important in the high-density regions like the galaxy cluster core.

The number counts \( N(z, m_0) \) of objects at redshift \( z \) with apparent magnitude \( m_0 \) is given by (e.g., Maizy et al. 2010):

\[
N(z, m_0) = \phi^* \int_{x,y} M(x, y) \int_{L(\mu, z, m_0)}^\infty \frac{C_v(x, y, z)}{\mu(x, y, z)} \phi \left( \frac{L(\mu, z, m_0)}{L^*} \right) d \left( \frac{L}{L^*} \right) dx dy ,
\]

(4.7)

where \( M(x, y) \) is a 2D function which takes into account the position of the galaxies of the cluster, \( \mu(x, y, z) \) is the lensing amplification, \( C_v(x, y, z) \) is the comoving volume and \( \phi(L) \) is the Schechter (1976) luminosity function depending on the parameters \( L^*, \phi^*, \alpha \).

When comparing observational program using GTs or targeting blank fields, the most important parameters are: sky areas and depth (i.e., limit magnitude). If the aim is to find a large sample of the most massive and rare galaxies, it is natural to choose a program in which surveyed areas is a more important parameter than depth. However, when exploring the faint-end of the luminosity function, the magnification factor provided by GTs offers an unique possibility, before the advent of 30m-class telescopes like the European Extremely Large Telescope. Therefore, the two techniques are complementary to study high-z sources, since the former is more efficient to investigate faint sources in shallow surveys while the latter is more efficient to study bright sources in moderately deep surveys (see Sec. 1.4).

By means of simulations, Maizy et al. (2010) estimated the expected number of high redshift sources in a lensing field as a function of the assumed luminosity function, the cluster mass distribution and redshift, the area and the depth of the survey. They assumed sources at \( z > 7 \) to be compact sources since from simulations their angular size is expected to be small (\( \lesssim 0.1'' \)) and such sources remain generally spatially unresolved in ground.

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5The E-ELT is an ESO project, expect to be completed at Cerro Armazones, Chile, in 2020. Web site: [http://www.eso.org/public/teles-instr/e-elt.html](http://www.eso.org/public/teles-instr/e-elt.html)
### 4.4 Expected vs observed number of high-z galaxies

#### Table 4.6: Parameters of the three luminosity functions used to predict the observed number of \( z > 6 \) galaxies.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \phi^* )</th>
<th>( M^* ) Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>LF(a)</td>
<td>1.6</td>
<td>( 0.4 \times 10^{-3} \text{Mpc}^{-3} )</td>
</tr>
<tr>
<td>LF(b)</td>
<td>1.74</td>
<td>( 1.1 \times 10^{-3} \text{Mpc}^{-3} )</td>
</tr>
<tr>
<td>LF(c)</td>
<td>1.74</td>
<td>( 0.4 \times 10^{-3} \text{Mpc}^{-3} )</td>
</tr>
</tbody>
</table>

Based observations. Moreover, Oesch et al. (2009) measured the average intrinsic size of \( z \approx 7 - 8 \) Lyman-break galaxies (LBG) to be \( 0.7 \pm 0.3 \) kpc, and smaller physical sizes are expected for higher redshift and intrinsically fainter galaxies.

Three different types of UV-LF were considered in the simulations:

- **LF(a)**: it assumes that LBGs have no-evolution since \( z \approx 6 \); the LF at \( z \approx 6 \) shows the same shape of the LF at \( z \approx 3 \) but it is normalized with a factor 3 times smaller (Beckwith et al. 2006);

- **LF(b)**: a constant LF based on measurement in the Hubble UDF at \( z \approx 6 \) (Bouwens et al. 2008); respect to the LF(a) this one presents a turnover at bright magnitude;

- **LF(c)**: an evolutionary LF with \( L^* \) decreasing with redshift (Bouwens et al. 2008).

In Table 4.6 the parameters for these three LFs are listed.

In the cited work, authors considered three different well-studied galaxy clusters: A1689, A1835, AC114 massive clusters, located at redshift \( z = 0.18, 0.23, 0.32 \), respectively. They have been chosen has representative of the different configurations of the mass distribution in massive clusters (relaxed or not relaxed, showing a single massive central galaxies or a merging pair). The mass distribution of the clusters determines the magnification maps in the image plane, but this also depends (see Chapter 2) on the cluster and sources redshifts: in Fig. 4.7 histograms of the fraction of magnified FOV is shown as a function of the magnification, for the three clusters shifted at identical redshifts; the source is fixed at \( z = 8.0 \) and the FOV is \( 6' \times 6' \).

At lower redshift, clusters A1835 and AC114 show the same magnification distribution, while A1689 present an higher fraction of high magnified (\( \mu > 2 \)) area; at higher redshift, due to project effect, the difference between the histograms fades since for all the clusters the high magnified percentage of FOV becomes small. We computed the fraction of magnified FOV as a function of the magnification for our clusters in the same condition (i.e., sources at \( z = 8.0 \) and FOV=\( 6' \times 6' \)) and, referring to the last panel with \( z_{cl} = 0.8 \), we estimate which type of clusters they match. Our results are shown
4.4 Expected vs observed number of high-z galaxies

Figure 4.7: Fraction of magnified FOV as a function of the magnification factor for the three cluster A1689, A1835, AC114 for a source at $z=8.0$, FOV=6$'$×6$'$ and for different redshifts of the clusters ($z=[0.1, 0.2, 0.3, 0.8]$). In each panel, all the clusters are standardized to the same redshift values.

...in Fig 4.8: CLG0152 present a trend similar to AC114, with FOV(%) = 1 at $\mu = 1$ mag and 13% of FOV with higher $\mu$; RDCS1252 seems to be more efficient than the template lenses, having a peak in correspondence of $\mu = 0.5$ mag, and can be associated with the A1689-type, having similar FOV% with high $\mu$ (FOV%= 3 at $\mu = 1$ mag and $\sim 20\%$ with $\mu > 1$ mag).

Due to geometrical configuration of the lensing system, the mean magnification as a function of the cluster redshift $z_c$ is expected to reach a maximum and then to decrease: the same trend is expected for the number of high redshift galaxies. In Fig. 4.9 it is shown the number of $z_{\text{source}} = 8$ sources expected to be observed in lensing field for different value of the lensing cluster redshift, for fixed value of depth ($H_{AB} \leq 25.5$) and FOV (6.0$'$ × 6.0$'$) of the survey. The three panels show the expected number of $z_{\text{source}} = 8$ for the three luminosity functions above mentioned and the models of galaxy clusters used in the simulation. The magnification bias grows with increasing cluster redshift, until a maximum is reached. In all the case the maximum fall in the redshift bin $z \sim 0.1 - 0.3$.

Finally, exploring the dependence of the expected number $n_{\text{exp}}$ of observed high-$z$ sources from the depth and the FOV of the lensing survey, they found that it decreases by reducing both the FOV and the depth: in Fig. 4.10 it is shown the expected number as a function of the source redshift for AC114 at $z = 0.184$ for different depths ranging from shallow ($H_{AB} \leq 25.5$) to deep
4.4 Expected vs observed number of high-z galaxies

Figure 4.8: Percentage of magnified FOV for CLG0152 (left) and RDCS1252 (right) as a function of the magnification factor $\mu$ (in magnitudes) for a source at $z = 8.0$ and FOV=$6' \times 6'$.

Figure 4.9: Number of $z_s = 8$ per redshift bin expected to be detected in lensing field as a function of the cluster redshift, for three cluster models (A1689, A1835, AC114), three LF types and for fixed depth ($H_{AB} \leq 25.5$) and FOV ($6.0' \times 6.0'$). (Maizy et al. 2010).
4.4 Expected vs observed number of high-z galaxies

Figure 4.10: Expected number of objects for AC114 at $z_L = 0.184$ in $6' \times 6'$ (red) and $2' \times 2'$ (blue) FOVs, for the limiting magnitudes 26.0, 27.0, 28.0 and 29.0 from top to bottom respectively. The three panels correspond to the three LFs used in the simulation. (Maizy et al. 2010).

Maizy et al. (2010) explored the expected number count of high-z sources in lensing fields for cluster redshift up to $z = 0.8 \pm 0.1$. For a cluster AC114-type at $z = 0.8$ with a $6' \times 6'$ field-of-view and imaging data with magnitude limit $H_{AB} \leq 25.5$, the number of $z = 8$ sources expected to be detected is $\sim 2.4$ assuming as UV luminosity function LF(a), $\sim 1.0$ assuming LF(b) and $\sim 0.2$ with LF(c). We use these values for comparison with our results in CLG0152. For a A1689-type cluster, with the previous FOV and depth, the expected number counts is higher then AC114-type: it is $\sim 2.8$ assuming LF(a), $\sim 1.8$ with LF(b) and $\sim 0.4$ with LF(c). Due to the decreasing trend of the expected number counts of sources at fixed $z_{\text{source}}$ with the growth of the cluster redshift (see Fig. 4.9), we used these number counts estimated at $z = 0.8$ as an upper limit for the number of expected high-z sources for RDCS1252 at $z = 1.23$.

Our final results counts 2 candidates in CLG0152 with $z_{\text{ph}} \sim 8.2$ and $z_{\text{ph}} \sim 9.2$, while 11 in RDCS1252 for which the photometric redshift estimated is $\sim 9$. The observed number in CLG0152 is in agreement with the expected number for LF(a) and LF(b) luminosity functions, while for RDCS1252 we find an observed number of high-z higher then the expected number. The higher number obtained can be related to the lack of contaminant removal: we have to take into account of low-redshift sources showing similar colors and magnitudes as high-z LBGs. In particular there are two kind of sources having similar features as the LBGs at $z > 6$: cool late-type stars, L and T
4.4 Expected vs observed number of high-z galaxies

Dwarfs, that are faint in the optical bands but bright in the near-IR, and old galaxies at \( z \sim 2 \) which Balmer break at 4000 Å can be confused with the Lyman break at \( z > 7 \).

Figure 4.11: **Bottom panel:** Spectra of E0 (in red) and SB (in blue) galaxies at redshift \( z=2 \) and \( z=8 \) respectively. The spectra are taken from the CWW-Kinney library (E0 and SB1 templates), and the SB is corrected to include the Gunn-Peterson effect (i.e. the total absorption at \( \lambda \leq (\lambda_{Ly\alpha}(1+z)) \), see Sec. 1.2.1). **Bottom panel:** transmission curves of the HST/ACS and VLT/ISAAC filters of our datasets. For these sources the Ly\( \alpha \)- and Balmer-break are redshifted at almost the same \( \lambda \) and, using the dropout technique, both the sources are classified as optical dropouts.

In Fig. 4.11 we compare the spectra of an elliptical (E0) early galaxy at \( z=2 \) and of a star burst (SB) late type galaxy at \( z=8 \) (in the bottom panel), and the HST/ACS and VLT/ISAAC filters of our dataset (in the upper panel): through photometric selection both these galaxies would be classified as optical dropout. A consistent removal of such low-redshift contaminants requires both an analysis of the color of these sources, allowing us to identify them through their spectral trend, and the knowledge of the density of these objects along the line of sight, but at present such number counts are far from being robust. However the results obtained in both the clusters seem to exclude any concordance with the LF(c), that is with the luminosity function evolving with redshift, characterized by dimming of the characteristic luminosity \( L^* \) with larger redshift.
We note that in this discussion we have not included the effect due to the cosmic variance, i.e., the variation of galaxy local density along different line-of-sight. Cosmic variance depends on the clustering properties of galaxies, and they are yet to be determined robustly at very high redshift. A more thoughtful analysis of the cosmic variance in our survey requires a much larger number of GTs, and is therefore beyond the aim of the present thesis.

4.5 Star formation rate at \( z > 6 \)

Measuring the star-formation rate at high redshift is one of the main goals in the surveys of the high-\( z \) Universe. In this Section we correct the photometry of our final candidates by the magnification induced by lensing; then we estimate the star-formation rate (SFR) of the selected sample of high-\( z \) galaxies, and compare our findings with some in the recent literature.

In order to estimate the star-formation rate from the UV rest-frame continuum in our sample, it is necessary to correct the photometry of our final high-\( z \) candidates for the lensing magnification.

Given the mass distribution model of our clusters (see Sect. 3.3.2 and Sect. 3.4) by using the software Lenstool (Kneib 1993, Jullo et al. 2007), we computed the magnification maps for our galaxy clusters for sources at different redshifts. Then, over-plotting our high-\( z \) candidates on these maps, we estimated the magnification induced by lensing action of the cluster. For each candidate we measured the average magnification within a circular region with diameter \( 3'' \) centered at the object position.

The magnification map for the galaxy cluster CLG 0152 is shown in Fig. 4.12. The map is computed for a source at \( z = 8 \) (the average redshift expected for our sources) and the high redshift candidates that we selected are overplotted. All the candidates lie more than \( 10'' \) away from the critical lines and none of them is multiply imaged. At such radial distances from the cluster center, the magnification factor is weakly sensitive to source redshift within the range \( 5 \leq z \leq 10 \). We measured the magnification for all candidates in CLG0152 and listed them in Table 4.7.

In Fig. 4.13 it is shown the magnification maps of the galaxy cluster RDCS 1252 computed for a source at \( z = 9 \), with the high-\( z \) candidates overplotted. The lensing magnifications of the candidates are given in Table 4.8. We used the measured magnification factors to correct our magnitudes in each band and to determine the intrinsic luminosity for our high-\( z \) candidates.

In order to estimate the star-formation rate, we converted the unlensed, rest-frame UV continuum luminosity at wavelength \( \lambda = 1500 \ \text{Å} \ (L_{1500}) \) in
4.5 Star formation rate at $z > 6$

Figure 4.12: Magnification map for the galaxy cluster CLG 0152 for a source at redshift $z = 8$. The red circle are our high-$z$ candidate. The gray scale corresponds to different values of the amplification.

Table 4.7: Magnification factors of the high-$z$ candidates in CLG0152.

<table>
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<tr>
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<tr>
<td>9001</td>
<td>1.3 ± 0.2</td>
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<tr>
<td>9003</td>
<td>1.4 ± 0.1</td>
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</table>

Table 4.8: Magnification factor of the high-$z$ candidates in RDCS1252.

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<td>1.8 ± 0.2</td>
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<td>2.5 ± 0.3</td>
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<td>9048</td>
<td>1.9 ± 0.4</td>
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<tr>
<td>9049</td>
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</tr>
<tr>
<td>9078</td>
<td>1.4 ± 0.3</td>
</tr>
</tbody>
</table>
4.5 Star formation rate at $z > 6$

Figure 4.13: Amplification maps for RDCS1252 for a source at redshift $z = 9$. The map is computed from the best SIS mass profile as estimated by Lombardi et al. (2005). The red circles indicate the position of the high-z candidate. The gray scale represents different values of the amplification.
4.5 Star formation rate at $z > 6$

SFR, through the calibration given by Kennicutt (1998):

$$\text{SFR (M}_\odot/$$ yr) = 1.05 \times 10^{-40} L_{1500} \text{ (erg s}^{-1} \text{Å}^{-1})$$

(4.8)

We note that the Kennicutt’s relation has been calibrated by observing nearby (i.e., $z \sim 0$) galaxies, and here we are assuming that the same holds for the primeval galaxies in the young Universe. Unfortunately, we are not yet able to verify directly such correlation in the epoch of reionization. However, as this correlation is widely-used, it also allows to compare straightforwardly our results with those in literature.

To derive the quantity $L_{1500}$ from the photometric data, we used for each candidate the magnitude $m$ at $\lambda_{\text{obs}} = 1500 \times (1 + z_{\text{photo}})$ Å from the best fitting spectra, and then we converted it into flux using the relation from Oke & Gunn (1983):

$$f = 10^{-0.4(mag + 2.406 + 4 \log(\lambda_{\text{obs}}))}$$

(4.9)

By computing the luminosity distance from the photometric redshift within the assumed cosmological model, we converted the computed flux into $L_{1500}$ and through the Eq. 4.8 we finally get the SFR rate for each candidate. We obtained values of the SFR ranging between a few solar mass per year to $\sim 50 M_\odot$/yr. In Fig. 4.14 we compare our results with the results of Richard et al. (2006) that obtained through the study of high-$z$ candidates magnified by the massive clusters A1835 and AC114: the mean SFR rate for the sources in our final sample is $\sim 10 M_\odot$/yr, in good agreement with the results from Richard et al. (2006).

In Table 4.10 we summarize the photometric properties of our final sample of high-$z$ candidates, including their magnitudes, photometric redshift, mean magnification, $L_{1500}$ and the SFR.
4.5 Star formation rate at \( z > 6 \)

Table 4.9: Photometric properties of our high-\( z \) selection.

<table>
<thead>
<tr>
<th>ID</th>
<th>ID</th>
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<th>i</th>
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<th>J</th>
<th>H</th>
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Table 4.10: Summary of the photometric properties of our final high-\( z \) candidates. From left to right: observed magnitudes; photometric redshift, magnification factor, unlensed luminosity at \( \lambda = 1500 \) and star formation rate. For RDCS 1252 the magnitude \( J \) refers to the \( Js \) filter. The symbol - stays for no data.
Conclusions and future work

The nature of the sources that mostly contributed to the end of the so-called 'Dark Ages' (the cosmic epoch between the emission of the CMBR at $z \sim 1100$ and the formation of the first galactic structures, nurseries of the first stars, at $z \sim 20$) is still an open problem. In this thesis we searched for high-redshift ($z > 6$) and low-luminosity galaxies, thought to be the main sources of the UV-radiation field that ionized the neutral hydrogen during the Reionization Epoch. We conducted our search using the so-called Gravitational Telescopes: foreground massive galaxy clusters which magnify the light of more distant sources, by acting as gravitational lenses. As a fact, this method is efficient to investigate the faint end of the luminosity function at redshift $\geq 6$ (see, for instance, Maizy et al. 2010).

We studied two massive galaxy clusters, CLG 0152 (at $z = 0.83$) and RDCS 1252 (at $z = 1.23$), for which we collected a large set of optical, near- and mid-infrared archival data (from the instruments HST/ACS, VLT/ISAAC and Spitzer/IRAC, respectively).

Searching for high-redshift galaxies through Gravitational Telescopes requires a robust knowledge of the mass distribution of the lens cluster, since it is necessary to determine the magnification factor as a function of the position on the image plane, hence allowing us to estimate the intrinsic properties (luminosity and size) of the lensed galaxies.

We determined the mass distribution of the cluster CLG 0152 using the strong lensing technique, adopting parametric mass model for each of the mass components in the cluster. At this aim, we used the software package Lenstool (Kneib 1993, Jullo et al. 2007): through a Bayesian optimization we obtain a model for the cluster with negative entropy $= 10.15$ and $\chi^2_{tot} = 24.34$ (21 degrees of freedom).

The absence of multiply imaged sources makes a strong lensing model for viable for the cluster RDCS 1252. Therefore, in order to estimate its mass distribution we used as starting point the weak lensing map and the best spherical singular isothermal model parameters derived by Lombardi et al. (2005). We computed then the magnification maps for sources at different
redshifts for both the clusters, to employ to estimate the magnification factor of high-redshift lensed sources.

Through the dropout technique we identified the so-called Lyman-break galaxies (LBGs) in our datasets, both as optical and near-infrared dropouts. We then estimated the photometric redshift of these objects. At this aim, we used the template-fitting LePhare tool (Arnouts & Ilbert, in preparation). We collected all the dropouts with photometric redshift $\geq 6$ and through a further visual inspection of these objects we removed spurious selections or possible low-redshift sources (mainly red Galactic stars and earl-type galaxies at $z \sim 2$).

Our final sample of high-$z$ candidates includes 3 objects found in the field of CLG 0152 FOV (with photometric redshift $\sim 6 - 9$) and 11 objects in the FOV of RDCS 1252, with photometric redshift $z \sim 9$. We compared these results with the expected number of high-redshift sources estimated by recent numerical simulations (Maizy et al. 2010). The result from the sky region in direction of CLG 0152 is in agreement with the expected sources with the assumption of the constant (i.e., not evolving) luminosity function, based on measurement in the Hubble Ultra Deep Field at $z \sim 6$ (Bouwens et al. 2008). The number of high-$z$ candidates in RDCS 1252 is higher than the estimated number counts, but we have seen that this cluster seems to be a more efficient lens with respect to the reference clusters used in the simulation.

Anyway, reminding that through gravitational telescope we explore the faint end of the luminosity function at high redshift, both these results are in good agreement with the rest-frame UV luminosity function showing no evolution with redshift at faint luminosities. This is in agreement with the results by Bouwens et al. (2008) that estimated (from blank field survey) an UV luminosity function at $z \sim 8$ with no evolution at low luminosities ($\sim -18$ AB mag) (see Fig 1.8).

We estimated the magnification factor for our final candidates in order to correct their magnitudes and to compute their SFR. From the best fit spectra computed by using LePhare, this time considering the unmagnified magnitudes as input, we computed the intrinsic luminosity at the rest-frame $\lambda = 1500\text{Å}$ from which we finally estimated the SFRs using the Eq. 4.8. We obtained values for the SFR ranging between a few to $\sim 50 M_\odot/\text{yr}$, with mean value $\simeq 10 M_\odot/\text{yr}$, close to the value found by Richard et al. (2006) in a lensing analysis of the clusters A1835 and AC114.

The results presented in this thesis, based only on photometric data, are only the first step in a longer term project. In next future we plan to obtain stronger constraints on the stellar mass and stellar metallicity of the candidate galaxies by including the Spitzer data in the SED fitting technique.
More important, follow-up near-infrared spectroscopic analysis of these candidates is needed, and already planned: first to confirm the photometric redshift we computed above, and better constrain the selection of our final high-redshift sample, and second to obtain more robust information on the stellar properties of the first galaxies from their spectra.

During this thesis project, a complementary observational program has been started. As discussed in Appendix, gamma-ray bursts (GRB) act as powerful lighthouses in the distant Universe, allowing to pinpoint the position of galaxies well beyond $z > 6$. We have obtained 16 hours at VLT, in order to search for the host galaxy of the object GRB 090423 (a GRB observed on April 9th, 2009), at redshift $z = 8.2$: if successful, our observations (planned for January 2011) will allow the observation of the most distant galaxy observed up to date.
Appendix A

Detecting the host galaxy of the most distant gamma-ray burst, GRB 090423

In this Appendix we report about an observational program just started in Spring 2010, aimed at detecting galaxies at the end of the reionization epoch by targeting the host galaxies of distant gamma-ray bursts. As it will be clear in the following, such program allows a complementary way to survey the high-\(z\) Universe, with respect to gravitational lensing by massive clusters.

This work is part of an observational project, already awarded 16 hours of telescope time at VLT, Chile\(^1\). Our goals are:

i) Detecting the host galaxy, which would be the most distant galaxy observed up to date (moreover with a secure spectroscopic redshift), or (in case of no detection) putting a strong upper limit on its global luminosity and, hence, its stellar mass.

ii) Estimating the metallicity of the host galaxy stellar population, and comparing with observations of candidate \(z > 7\) galaxies recently discovered in the HUDF and with recent theoretical results from numerical simulations.

A.1 Gamma-ray burst as lighthouses in the distant Universe

The identification of the first galaxies is one of the most important and challenging tasks in astronomy today. Many different observational techniques are being employed to gather information on the first cosmic structures at

\(^1\)ESO program xxx
beyond $z \sim 7$, mostly based on targeting the Ly-$\alpha$ emission with narrow-band filters (Giavalisco 2002) or using the Lyman break technique (Kudritzki et al. 2000), in blank fields or behind gravitational telescopes (e.g., Zheng et al. 2009, Maizy et al. 2010). Gamma-Ray Bursts (GRBs) provide a complementary and unique view to identify unambiguously source during the reionization epoch, when neutral InterGalactic Medium (IGM) absorbed most of the UV photons emitted by first light sources. As the redshift of the GRBs could be measured from observations soon after the discovery of the event, the host galaxy redshift can be pin down.

GRBs are thought to be associated with the death of very massive stars (see e.g. Piran 2005) so they would be common in star-forming galaxies. Several studies showed that the host galaxies of GRBs are usually bluer, younger, fainter and with a lower mass than the field galaxy population at similar cosmic times (Savaglio et al. 2009). The fact that GRBs could be observed at $z > 6$ can provide a way to observe such star forming galaxies at very high distance from us.

The most distant GRB was observed at redshift $z \approx 8.2$ (Tanvir et al. 2009), corresponding to about 600 million years after the Big Bang. This fact means that the host galaxy of this GRB could belong to the first generation of galaxies formed about 400 million years after the Big Bang.

In the context of the search for the first galaxies, GRBs, albeit rare events, have two important properties: (1) they are the most distant light sources observed and (2) they appear to be more representative than Lyman-$\alpha$ searches, in particular in light of recent results, (Hayes et al. 2010), for which Lyman-$\alpha$ selected samples of high-redshift galaxies require severe revision. GRBs could provide an unbiased sampling of the galaxy population at very high redshift since $\gamma$-rays are not absorbed by interstellar dust and intergalactic medium. Moreover, GBRs are observed up to very high redshift, so they could be thought as one of the possible sources for the reionization process, even if their effective contribution to the reionization of the IGM is still an open question.

Recently, several teams (Labbé et al. 2009, Bouwens et al. 2009) have revealed a sample of candidate $z > 7$ galaxies in the HUDF by means of photometric selection in the near-infrared (NIR). They use the deep NIR data available from HST and several ground telescopes and as a first results they obtain a sample of 16 candidate at redshift $z \sim 7$, (Oesch et al. 2009). However, spectroscopic confirmation of the high redshift of these sources is a very difficult task: spectral lines redder than $[\text{OII}]$ at these distances are at wavelengths larger than $\sim 3.6 \mu m$, while Lyman-$\alpha$ emission line (detectable in the NIR) is often obscured by dust associated with the starburst galaxy. Therefore, at the moment, study of first galaxies at the epoch of reionization
Figure A.1: Spectral energy distribution of the afterglow of GRB 090423 as derived from the GROND data. The top row shows the image of the corresponding filter band. Note the onset of the Lyman-α in the J-band.

is hampered by a number of selection effects: first of all, different classes of foreground objects (e.g., T dwarf stars, red galaxies at \( z \sim 2 \), see, for instance, Stanway et al. 2008) can easily contaminate the photometric sample; second, photometric selection are necessarily sensitive to the very bright-end of the luminosity function, therefore providing a biased view of the galaxy population at such epochs.

GRBs offer the possibility to sample in an unbiased way the population of star-forming galaxies at very high redshift. As they are known to be associated with the death of the first, very massive stars, GBRs effectively act as markers of the starburst galaxy population at high redshift.

Our goal is to detect the host galaxy of the most distant GRB known to date, GRB 090423, at \( z \approx 8.2 \) (Tanvir et al. 2009; Salvaterra et al. 2009), or (in case of null detection) give robust upper limits to its luminosity (and, hence, mass).

From a detailed analysis of the available data, we noted that this GRB is very similar to other GRBs observed at lower redshift (Izzo et al. 2010): therefore, the circumstellar medium around this GRB appears to be not very different from those at lower redshift.

The direct detection of the host galaxy in the H and K bands would allow
the observation of the first galaxy beyond $z \sim 8$ with a secure spectroscopic redshift.

### A.2 The observing program

Our VLT program aims at observing the yet-undetected host galaxy of GRB 090423. We will operate in the near-infrared with the instrument HAWK-I, in the two filters H and Ks. From these observations it will be possible to select $z \sim 8$ candidates in the same cosmic environment of the GRB-host galaxy.

Furthermore, the host-galaxy stellar mass and star-formation rate will be deduced from the measured flux in the H and Ks bands, by adopting specific models for the IMF.

Recently, Savaglio et al. (2009) found that GRB-host galaxies (over a wide redshift range, $1 < z < 6$) have a median stellar mass $M_\ast \sim 10^{9.3} M_\odot$: it would be very interesting to extend such estimates at earlier times, in the first stages of galaxy evolution.

The SFR will be derived from the rest-frame UV luminosity (e.g., Kennicutt 1998) detected in the H band. This would allow us to compare with other estimates (or upper limits) for GRB-hosts at $z \sim 6$ (Berger et al. 2007) and Lyman-break selected galaxies in the HUDF at $z > 7$ (e.g., Bouwens et al. 2009).

Moreover, comparison of the NIR color with theoretical models will allow to put constraints on the metallicity of the galaxy. If we consider as selective extinction $E(B-V) < 0.15$ the values observed in the afterglow of GRB 090423 and a model for the extinction curve $A_V$ (Calzetti 1997), and if we adopt some specific model for the IMF and the IGM absorption (Madau 1995), we could infer the galaxy stellar mass $M_\ast$ and an estimate of the metallicity $Z$ from the flux detected in the H and K band.

HAWK-I is the most efficient ground-based NIR instrument mounted on the Very Large Telescope (VLT), an ideal instrument to detect faint galaxies at $z > 7$. Finally, by using two HAWK-I filters, it will also be possible to select candidate galaxies in the region around GRB 090423, in order to characterize its cosmic environment. At this aim, we will also use optical archival data from VLT and HST.
Bibliography


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