Study of $J/\psi$ production with the LHCb experiment in proton-proton collisions at $\sqrt{s} = 7$ TeV

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Introduction

The LHCb detector is one of the four main experiments installed at the proton-proton Large Hadron Collider (LHC) at CERN, Geneva. The first beams circulated in the collider in September 2008 and the data taking started in April 2010. The LHCb detector is a single-arm forward spectrometer conceived to extensively study the CP violation in the $B$ meson system, looking for constraints of the Standard Model predictions, for possible effects of new physics beyond this theory and generally for rare phenomena in the $b$ and $c$ quark sectors with high precision. Already in the first data recorded by the experiment, a huge amount of $J/\psi$ mesons have been collected, which allow to make an extensive study of the charmonia production. This is a very interesting sector in particle physics. From the experimental point of view, the measurement of the charmonium production cross section and polarization has been carried on by several experiments. At Tevatron, CDF measured the $J/\psi$ and $\psi(2S)$ cross section and the polarization with Run I and Run II data. HERA-B at DESY performed a measurement of the $J/\psi$ polarization with proton collisions on fixed target. Also the PHENIX collaboration at RHIC recently published a measurement of the $J/\psi$ polarization with $pp$ collisions. The experimental results obtained for the $J/\psi$ polarization are hardly comparable among each other because performed in different kinematic ranges and reference frames. For what concerns the theory predictions many models have been proposed in the recent years but none of them is able to fit at the same time the charmonium production cross section and polarization.

The present work aims at giving a preliminary measurement of the prompt $J/\psi$ polarization, binned in $J/\psi$ transverse momentum and rapidity, and of the double differential production cross section (in transverse momentum and rapidity bins) of the $J/\psi$ prompt component: this is done analysing the $J/\psi$ decays into a muon pair. The cross section is calculated taking into account the measured polarization. Moreover the double differential production cross section, in transverse momentum and rapidity, of the $J/\psi$ component coming from the $b$-hadrons decays is presented. From this last result the $b\overline{b}$ production cross section is extrapolated from the LHCb angular coverage to the $4\pi$ solid angle.

The discussion is organized in eight chapters. The first chapter contains an introduction to the physical motivations that drive the interests in the charmonia production and a description of the most recent theoretical models: a presentation of the physical observables and a panoramic view of the experimental situation is also given. In the second chapter an overall description of the
LHCb detector is given. The experimental conditions of the 2010 data taking are also described. The third chapter will give some basic introduction to the track reconstruction and the muon identification procedures in LHCb. The fourth chapter will describe the selection of the interesting signal events and the procedure to disentangle the prompt and from $b J/\psi$ components. In the fifth and sixth chapter the polarization and cross section measurements are described and the obtained results are presented. In the last chapter the conclusions and a summary of the analysis will be drawn.
Chapter 1

Charmonium production mechanism

1.1 The Quantum Chromodynamics frame

The Quantum Chromodynamics (QCD) is the sector of the Standard Model (SM) \[1\] which describes the strong interactions. It can be obtained by the complete Standard Model by setting the weak and electromagnetic coupling constants to zero. It was proposed for the first time in the early 1970s by David Politzer, Frank Wilczek and David Gross, for which they were awarded the Nobel Prize in Physics in the 2004.

The QCD is a $SU(3)$ gauge theory, described by the Lagrange density \[2\]:

$$L_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_q (i\gamma^\mu D_\mu - m_q) q$$  (1.1)

where $q$ can be each one of the quark fields ("flavours") $\{q\} = u,d,s,c,b,t$, $F_{\mu\nu}^a = \partial_\mu A_{\nu}^a - \partial_\nu A_{\mu}^a + g f^{abc} A_{\mu}^b A_{\nu}^c$ is the kinetic term and $D_\mu = \partial_\mu - i T^a A_\mu^a$ is the covariant derivative. Here $f^{abc}$ are the $SU(3)$ structure constants, $T_a$ are the 8 $SU(3)$ generators, $A_\mu^a$ are the 8 gluon fields and $g$ is the strong coupling constant. The QCD Lagrange density is invariant under Poincaré, parity, time reversal and charge conjugation transformations. It is also invariant under $U(1)$ transformations and this implies that the flavour is conserved. Because it is a non-Abelian gauge theory the physical spectrum contains only color singlet states. The simplest among these states are made by two quarks and they are called mesons, or formed by three quarks, named baryons. Also other possibilities can be expected. The strong coupling constant $g$ is related to the QCD coupling constant $\alpha_s$ by the relation $\alpha_s = g^2/4\pi$.

In the 1973 David Gross and Frank Wilczek \[3\] and David Politzer \[4\] discovered that the coupling constant $\alpha_s (q)$ decreases with the increasing of the momentum transfer scale $q$ or, in other words, that the interactions become more and more weak at short distance. This property is known as the asymptotic freedom and is common to a wide range of non-Abelian theories. A direct advantage

\[1\] In $SU(3)$ the commutator rules given by the structure constants are $[T_a; T_b] = i f_{abc} T_c$, where $a, b, c = 1...8$. 

of this behaviour is that at high energies perturbative calculations in $\alpha_s$ are possible. At low energies $\alpha_s$ is close to 1 and the expansion is no more possible. Therefore some non-perturbative technique, as lattice calculations, must be used. Introducing the $\Lambda_{QCD}$ parameter as the energy scale of the theory, the quarks can be divided in light and heavy quarks, according to their mass values: $u,d$ and $s$ are light quarks, with $m_q \ll \Lambda_{QCD}$ ($q = u,d,s$) and $c,b$ and $t$ are heavy quarks, with $m_Q \gg \Lambda_{QCD}$ ($Q = c,b,t$).

1.2 Properties of $J/\psi$ resonance

The $J/\psi$ resonance has been observed for the first time in 1974 at the Brookhaven National Laboratory in $p + \text{Be}$ reaction [5], and at the Stanford Linear Accelerator Center in the $e^+e^-$ annihilation [6] by two groups led by Ting and Richter. The discovery of this state led to the confirmation of the existence of a fourth quark (in addition to the already known $u,d,s$ [7, 8, 9]), the charm, predicted by the theory in 1964 [10] and 1970 [11].

The $J/\psi$ meson is a bound state made by a charm and an anti-charm. Such a state is called charmonium, by analogy with the positronium, a bound state composed by an electron and a positron. Thanks to this simple structure, the charmonium properties, like the mass spectrum, the quantum number and the decay properties and modes, can be well predicted. The charmonium states are summarized in Fig. [1.1] [12]. The charmonium ground state is the spin singlet $\eta_c(1S)$ with quantum numbers $J^{PC} = 0^{-+}$. The $J/\psi$ is instead a spin triplet with quantum numbers $J^{PC} = 1^{--}$ and mass equal to $\sim 3096.916$ MeV/$c^2$. Since the $J/\psi$ state is colourless it can decay only through the emission of three gluons or two gluons plus an additional radiative photon, at the leading order Feymann diagrams. For this reason such a resonance is extremely narrow, with a width of $\sim 92.9$ keV. In Tab. [1.1] the main $J/\psi$ decay modes are listed.

In hadron-hadron collisions three processes can lead to the charmonium production:

- direct production in hadron-hadron interaction;
- feed down from higher charmonium states;
- decay of $b$-hadrons.

In the first two cases the $J/\psi$ is produced promptly at the primary vertex of the interaction. In the third process the $b$-hadrons travel away from the primary vertex before decaying so the $J/\psi$ is produced delayed with respect to the primary interaction. From now on “prompt” $J/\psi$ will refer to the first two processes while $J/\psi$ “from $b$” or “‘delayed’ $J/\psi$ will refer to the third process. In the specific $J/\psi$ case, the feed down can occur from the decays of $\psi(2S)$ and the three states $\chi_{c0}, \chi_{c1}, \chi_{c2}$, through the processes listed in Tab. [1.2]. $J/\psi$ is produced in the decays of $b$-hadrons such as the $B^\pm, B^0$ ad the $B_s$, with the branching fractions listed in Tab. [1.3]
Figure 1.1: Charmonium states and decays.

<table>
<thead>
<tr>
<th>Processes</th>
<th>Branching Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>hadrons + virtual γ</td>
<td>(13.50 ± 0.30) %</td>
</tr>
<tr>
<td>ggg</td>
<td>(64.1 ± 1.0) %</td>
</tr>
<tr>
<td>γgg</td>
<td>(8.8 ± 0.5) %</td>
</tr>
<tr>
<td>e⁺e⁻</td>
<td>(5.94 ± 0.06) %</td>
</tr>
<tr>
<td>μ⁺μ⁻</td>
<td>(5.93 ± 0.06) %</td>
</tr>
</tbody>
</table>

Table 1.1: J/ψ decay modes.

<table>
<thead>
<tr>
<th>Processes</th>
<th>Branching Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>χc₀ → J/ψγ</td>
<td>(1.30 ± 0.11) %</td>
</tr>
<tr>
<td>χc₁ → J/ψγ</td>
<td>(35.6 ± 1.9) %</td>
</tr>
<tr>
<td>χc₂ → J/ψγ</td>
<td>(20.2 ± 1.0) %</td>
</tr>
<tr>
<td>ψ₂S → J/ψX</td>
<td>(56.1 ± 0.9) %</td>
</tr>
</tbody>
</table>

Table 1.2: Main processes in the J/ψ production, involving the decays of higher charmonium states, with the corresponding branching fractions.

<table>
<thead>
<tr>
<th>Processes</th>
<th>Branching Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>B⁺/B⁰ → J/ψX</td>
<td>(1.094 ± 0.032) %</td>
</tr>
<tr>
<td>Bₛ → J/ψφ</td>
<td>(0.13 ± 0.04) %</td>
</tr>
<tr>
<td>b−hadrons → J/ψX</td>
<td>(1.16 ± 0.10) %</td>
</tr>
</tbody>
</table>

Table 1.3: Main b-hadron decays to the J/ψ with the corresponding branching fractions.
1.3 Charmonium production

When the heavy quarkonium, a heavy quark anti-quark bound state, is produced in hard-scattering process, the hard-scattering scale $p$ plays a role in the description of the production process [13]. In the hadroproduction (hadron-hadron collisions) and in the hadron-lepton collisions, $p$ has the same order of magnitude of the quarkonium transverse momentum $p_T$. We can expect that the quarkonium production mechanism can occur in two different steps. At the first stage a quark anti-quark pair $Q\bar{Q}$ is created at the scale $p$: this production can be calculated within the QCD frame as an expansion in series of $\alpha_s(p)$. At the second stage the $Q\bar{Q}$ pair evolves into the final quarkonium state. Referring to the $p$ scale it is possible to introduce the term “short distance” while “long distance” is referring to the typical hadronic momentum scale $m_Qv, m_Qv^2$, where $v$ is the heavy quark velocity in the centre of mass, and $\Lambda_{QCD}$. The factorization theorem establishes that in the physical observables such as the cross section or the amplitude a separation is possible between the short distance terms, which can be calculated perturbatively, and the long distance ones, describing the non-perturbative physics. This means that the physical quantities can be expressed as a sum of products between short distance coefficients and operator matrix elements. The latter contain the long distance, non-perturbative dynamics. Many models and effective theories can be used to describe such a non-perturbative evolution. Among them the Color Singlet Model (CSM) [14, 15], the Color Evaporation Model (CEM) [16] and the Non relativistic QCD (NRQCD) [17] approaches can be mentioned.

1.3.1 Color Singlet Model

The Color Singlet Model has been proposed in the 1975, shortly after the discovery of the $J/\psi$ resonance [14][15]. The model assumes that the $Q\bar{Q}$ pair is produced in a color singlet state and during the evolution in the quarkonium final state the spin and the angular momentum are maintained. So the quarkonium has the same quantum number of the initial $Q\bar{Q}$ bound state. The production rate is given by the evaluation of the $Q\bar{Q}$ wave function at $Q\bar{Q}$ distance equal to zero. The CSM can predict with success the cross section values at low transverse momentum but going higher in $p_T$ large corrections at next-to leading order (NLO) and next-to-next-to leading order (NNLO) must be applied, so that it is not completely clear if the expansion in $\alpha_s$ is converging. The CSM has been extensively used until 1995, when the Tevatron experiments showed an underestimation by more than one order of magnitude of the CSM prediction of the prompt charmonium cross section in the $p\bar{p}$ collisions, as explained in section (1.4.1).

1.3.2 Color Evaporation Model

The Color Evaporation Model [16] was proposed for the first time in 1977. The model assumes that the quarkonium state $H$ cross section is given by a fraction $F_H$ of the production cross section for the $Q\bar{Q}$ pair produced under the $2m_M$ threshold, where $M$ is the lightest meson containing the
1.3.3 Non-Relativistic QCD approach

The NRQCD frame is characterized by an ultraviolet (UV) cut off \( \nu_{NR} = \{\nu_p; \nu_s\} \): here \( \nu_p \) is the UV cut off of the relative three-momentum of the heavy quark and anti-quark, while \( \nu_s \) is the UV cut off of the energy in the center of mass frame of the heavy quark and antiquark.

The NRQCD must describe the dynamics of the quark-antiquark pairs at energy scales in the centre of mass frame smaller than their masses. The high energy processes are instead taken into account through the short distance coefficients: infinite such terms should be included but in practice only few of them give a contribution.

Taking advantage of the property of the asymptotic freedom, \( \alpha_s \) can be written as a function of the momentum transfer \( q \) in the scattering process at the lowest order:

\[
\alpha_s(q^2) = \frac{4\pi}{\beta_1 \ln(q^2/\Lambda_{QCD})}
\]

where \( \beta_1 \) is the beta function of the QCD [18] and \( \Lambda_{QCD} \sim 200 \) MeV. With this expression for \( \alpha_s \) and using \( \Lambda_{QCD} \) as scale, the interactions are “strong” when \( \alpha_s \sim 1 \).

In the specific case of hadroproduction, the heavy quark mass \( m \) and the transverse momentum are higher than \( \Lambda_{QCD} \) and the cut off is such that \( \Lambda_{QCD} \ll \nu_{NR} \ll m \). Thus the corresponding values of the QCD running coupling constant \( \alpha_s \) are significantly lower than one: in the specific case of the \( c \) and \( b \) quark masses \( \alpha_s(m_c) \sim 0.25 \) and \( \alpha_s(m_b) \sim 0.18 \). Moreover the \( c \) quark squared velocity \( v^2 \) is roughly equal to 0.3. Thus \( v \) and \( \alpha_s \) are two small parameters in NRQCD and the expansion can be done in powers of both of them. To have an accuracy of order \( \alpha_s^k v^n \), the Lagrangian must contains terms which contribute to the physical observable under study up to that order. Up to the field redefinition the NRQCD Lagrangian for one heavy flavour of mass \( m \) at \( \mathcal{O}\left(\frac{1}{m^2}\right) \), including the kinetic energy term \( \frac{D^2}{8m^3} \) is [2]:

\[
\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi} \quad (1.3)
\]

where

\[
\mathcal{L}_g = \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a + \frac{1}{4m^2} g f_{abc} F_{\mu\nu}^a F^{\mu\nu\alpha} \quad (1.4)
\]
\[ \mathcal{L}_\psi = \psi^\dagger \left\{ iD_0 + \frac{c_2 D^2}{2m} + \frac{c_4 D^4}{8m^3} + \frac{c_F g \cdot B}{2m} + \frac{c_D g}{8m^2} \mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D} + i c_S \frac{\mathbf{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8m^2} \right\} \psi \]  

\[ \mathcal{L}_\chi = \text{charge conjugate of } \mathcal{L}_\psi \]  

\[ \mathcal{L}_{\psi\chi} = \frac{c_{f_1}}{m^2} O_1(1S_0) + \frac{c_{f_1}}{m^2} O_1(3S_1) + \frac{c_{f_8}}{m^2} O_8(1S_0) + \frac{c_{f_8}}{m^2} O_8(3S_1) = \]  

\[ = \frac{c_{f_1}}{m^2} \psi^\dagger \chi^\dagger \psi + \frac{c_{f_1}}{m^2} \psi^\dagger \sigma \chi^\dagger \sigma \psi + \frac{c_{f_8}}{m^2} \psi^\dagger T^a \chi^\dagger T^a \psi + \frac{c_{f_8}}{m^2} \psi^\dagger T^a \sigma \chi^\dagger T^a \sigma \psi \]  

where \( \psi \) and \( \chi \) are the Pauli spinors which annihilate the quarks and create the antiquarks respectively. Therefore \( iD_0 = i\partial_0 - gA^a_T T^a \) and \( i\mathbf{D} = i\nabla + gA^a_T T^a \) are respectively the scalar and three-vector terms of the covariant derivative. \( E^i = F^{0a_T} T^a \) and \( B^i = -\epsilon_{ijk} F^{jka_T} T^a / 2 \) are the chromoelectric and chromomagnetic fields. The \( O_1(1S_0), O_1(3S_1), O_8(1S_0) \) and \( O_8(3S_1) \) are the four fermion operators: the \( 1 \) and \( 8 \) indices label the colour singlet and octet states. The allowed operators are constrained by the symmetries of the QCD. The matching coefficients \( c_i \) may be calculated in the perturbation theory. The coupling parameters \( g, m \) and \( c_i \) are determined by the request that the NRQCD reproduces the QCD results up to the order \( a_s^k v^n \).

The main assumption of the non-relativistic QCD approach is that it is possible to calculate the production rate using the perturbative theory. To do this the high momentum, perturbative effects (short distance) must be separated from the low momentum, non-perturbative ones (long distance), through a factorization, as described in the following section.

**Factorization in NRQCD**

As introduced in the previous section the NRQCD approach assumes that it is possible to make an expansion in powers of the heavy quark velocity \( v \) in the centre of mass and of the strong coupling constant \( \alpha_s \). The heavy quark state can be decomposed in terms of ortho-quarkonium vector meson state function [19]:

\[ |\Psi(Q)\rangle = \mathcal{O}(1)|\overline{Q}Q_{1}[^3S_1]\rangle + \]  

\[ + \mathcal{O}(v)|\overline{Q}Q_8[^3P_J]\rangle g + \]  

\[ + \mathcal{O}(v^2)|\overline{Q}Q_8[^1S_0]\rangle g + \mathcal{O}(v^2)|\overline{Q}Q_{1,8}[^3S_1]\rangle gg + \mathcal{O}(v^2)|\overline{Q}Q_{1,8}[^3D_J]\rangle gg \]  

The spin, orbital and the total angular momentum are indicated for each Fock component in square brackets. The singlet and color indices indicate the color assignment. If only the order in \( v \) at which the \( q\overline{q} \) pair hadronizes is relevant in eq. (1.8), only the leading Fock component will dominate. In this case the CSM model is retrieved, in which the heavy quarkonium production proceeds through the generation of a colourless \( \overline{Q}Q \) state with the same quantum number of the final state. But considering higher order in \( v \) also other terms with different quantum numbers will enter in the expansion. These octet bound states are associated with the production of hard
gluons and they will evolve non-perturbatively toward the final states. Thus the NRQCD approach is able to include both the Color Singlet and Color Octet (COM, [17]) models. In the NRQCD factorization approach the cross section can be written as sum of products of NRQCD matrix elements and short distance coefficients:

\[ \sigma[H] = \sum_n \sigma_n(\Lambda; Q\bar{Q}^{[2S+1L_J(1,8)]}) \langle O_{1,8}(\Lambda;^{2S+1L_J}) \rangle. \] (1.9)

The \( \Lambda \) parameter is the ultraviolet cutoff of this effective theory, \( \sigma_n \) are the short distance coefficients: because the momentum scale is higher than \( mv \) they can be evaluated as expansion in powers of \( v \) of the \( Q\bar{Q} \) pair production cross section and \( n \) refers to the color, spin and orbital angular momentum state: thus the sum runs over all the possible \( Q\bar{Q} \) states.

The eq. (1.9) includes the factorization between the perturbative short distance term \( \sigma(\Lambda; Q\bar{Q}^{[2S+1L_J(1,8)]}) \) and the non-perturbative long distance evolution to the final \( H \) state \( \langle O_{1,8}(^{2S+1L_J})(\Lambda) \rangle \). The \( \langle O_{1,8}(^{2S+1L_J})(\Lambda) \rangle \) are vacuum expectations of the four-fermions operator in NRQCD. Basically these operators create a \( Q\bar{Q} \) pair, which can evolve in the final heavy quarkonium state. All the non-perturbative physics involved in this evolution is contained in these operators. The vacuum expectation of such an operator is the probability that the \( Q\bar{Q} \) pair forms an heavy quarkonium plus anything. The benefits of such a description is that these matrix elements are process independent. The matrix elements describe both the evolution of a \( Q\bar{Q} \) pair produced in color singlet and color octet state into a color singlet heavy quarkonium.

The short distance terms describe the \( Q\bar{Q} \) production in the interaction between two partons: they are basically the cross section computed in powers of \( \alpha_s \).

### 1.4 Observables in hadroproduction studies

To compare the various theoretical explanations of the production of quarkonium states, physical observables that can be measured experimentally are needed. Possible choices are the inclusive and the differential production cross section of the prompt component and the polarization. The following sections are mainly focused on the quantity related to the \( J/\psi \) production and a panoramic of the experimental results already obtained is given.

#### 1.4.1 Cross section measurements

In Tevatron Run I the CDF Collaboration measured the differential production cross section of the prompt component of several charmonium states, including the \( J/\psi \), in \( p\bar{p} \) collisions at \( \sqrt{s} = 1.8 \) TeV [20]. The measurement has been done looking at the decay of the \( J/\psi \) into a muon pair. The measured cross section is shown in Fig. 1.2 the data refer to the \( J/\psi \) component directly produced in the collisions. The Color Singlet Model predictions are shown as dotted lines: the more steeply falling line is the CSM calculation at the leading order in \( \alpha_s \) while the second dotted
Figure 1.2: Direct $J/\psi$ production cross section as measured by the CDF collaboration. The data points are compared with the different contributions calculated in the CSM and COM frameworks.

The line is the prediction at higher order involving the gluon fragmentation contribution. The diagrams corresponding to the leading-order color singlet

$$gg \rightarrow 3\sigma(3S_1) + g$$

and to the color singlet fragmentation

$$gg \rightarrow [\sigma(3S_1)] + gg + g$$

Figure 1.3: Diagrams for the leading order color singlet term (left) and for the gluon fragmentation to the $3S_1$ color singlet state.
Figure 1.4: Diagrams for gluon fragmentation to the $^{3}S_{1}$ (left) and for the gluon fusion to the $^{3}P_{J}$, $^{1}S_{0}$ (right) color octet states.

are shown in Fig. 1.3 respectively on the left and right. The leading-order term falls like $1/p_{T}^{8}$ while the contribution coming from the gluon fragmentation is proportional to $1/p_{T}^{4}$. Thus the latter contribution renders the cross section shape compatible with the CDF measurement. However the observed rate is more than an order of magnitude higher than the CSM expectation. A similar discrepancy was found also in the $\psi(2S)$ cross section.

In the NRQCD approach the charmonium production cross section contains not only the CSM terms, which are absolutely normalized, but also the COM terms, whose normalization is given by the color octet matrix elements. The most important matrix elements for the $J/\psi$ production are $\langle O_{8}^{H}(^{3}S_{1}) \rangle$, $\langle O_{8}^{H}(^{3}P_{0}) \rangle$ and $\langle O_{8}^{H}(^{1}S_{0}) \rangle$. The Feymann diagrams for the gluon fragmentation to the $^{3}S_{1}$ octet state

$$gg \to c\bar{c}[^{3}S_{1}^{8}] + g \tag{1.12}$$

are shown in Fig. 1.4 (left) and fall like $1/p_{T}^{4}$. The gluon fusion to the $^{1}S_{0}$ and $^{3}P_{J}$ octet states, whose diagrams is also shown in Fig. 1.4 (right)

$$gg \to c\bar{c}[^{3}P_{J}^{8}, ^{1}S_{0}^{8}] + g \tag{1.13}$$

falls instead like $1/p_{T}^{6}$. Thus the first term will dominate in the cross section computation at high $p_{T}$ over the second, which will be significant only below $\sim 10$ GeV/c.

From eq. (1.9) it can be seen that the COM quarkonium production is associated to the emission of gluons. The gluons emitted in the CSM have generally a too high $p_{T}$ to be able to change the quantum numbers of the $Q\bar{Q}$ pair into the ones of the final $H$ state. Since these gluons will carry most of the transverse momentum, the produced quarkonium in the CSM will tend to have a lower $p_{T}$ values. As it can be seen in the figure the CSM expectation, also including the gluon fragmentation, is below the measured data points by a factor $\sim 30$. Only including the COM contributions, the experimental results can be explained.

### 1.4.2 Polarization studies

The $J/\psi$ polarization is, together with the cross section, a crucial quantity to explain the quarkonium production mechanism. For a $1^{--}$ state such as the $J/\psi$ or the $\psi(2S)$ the polarization can be measured by looking at the angular distribution of the two leptons produced in the quarkonium
decay, which depends on the spin state of the quarkonium. The angular decay rate will be:

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \alpha \cos^2 \theta.$$  \hfill (1.14)

The \( \theta \) angle is defined as the angle between the three-momentum of the positive lepton in the quarkonium rest frame and a chosen polarization axis. The choice of the polarization depends on the processes in which the quarkonium is produced. At an hadron collider the most convenient choice is the direction of the boost vector from the quarkonium rest frame to the centre of mass frame of the colliding hadrons. The polarization variable \( \alpha \) can take values between \(-1\) and \(+1\): a pure transverse or pure longitudinal polarization implies \( \alpha = +1 \) or \( \alpha = -1 \) respectively.

The NRQCD gives a simple prediction of the polarization variable \( \alpha \) at large transverse momentum. The production of a quarkonium state with \( p_T \) much larger than its mass is dominated by the gluon fragmentation, in which the quarkonium is formed in the hadronization of a gluon created with larger transverse momentum. The dominant gluon fragmentation process as predicted by the NRQCD factorization is the gluon fragmentation of a \( Q\bar{Q} \) pair into a \( ^3S_1 \) state, at an order of \( \alpha_s \).

At large \( p_T \) the gluon involved in the fragmentation process is nearly on its mass shell and thus it is transversely polarized. Moreover the \( v \) scaling rules predict that the color octet states \( Q\bar{Q} \) keep the transverse polarization while evolving in a \( ^3S_1 \) state, up to \( v^2 \) corrections. There might be some dilution given by various reasons, such as the radiative corrections and the color singlet models. For what concerns the \( J/\psi \) the feed down from higher charmonia states is also important: a 10\% and a 30\% contributions are given to the \( J/\psi \) production by the \( \psi(2S) \) and \( \chi_c \) decays.

On the contrary the Color Evaporation Model gives a zero polarization prediction.

Fig. 1.5 shows the CDF measurement from Run I [21] of the \( J/\psi \) polarization variable \( \alpha \) as a function of the transverse momentum, compared with the NRQCD prediction. The data points are in agreement with the theoretical expectation except for the high \( p_T \) bins. In particular the expected increase of the polarization at high \( p_T \) values is not observed. The measurement has been repeated in Run II by CDF [22] in \( p\bar{p} \) collisions at \( \sqrt{s} = 1.96 \) TeV, giving different results, as showed in Fig. 1.6. The experimental data indicates a slightly longitudinal polarization, increasing in magnitude with increasing \( p_T \).

For what concerns the fixed target experiments, the most convenient choice of the polarization axis is the direction of the boost vector from the \( J/\psi \) rest frame to the laboratory frame. The \( J/\psi \) polarization was measured by the HERA-B experiment [23] at DESY in proton-nucleus collisions at a centre of mass energy \( \sqrt{s} = 41.6 \) GeV, showing a longitudinal polarization, increasing with the increasing \( p_T \). Since there is no overlap between the CDF and HERA-B kinematic regions is not possible to directly compare the results from the two experiments. Thus it is not possible yet to draw a strong conclusion in the present situation.

\footnote{This could be different from the laboratory frame because in an hadron collider the two beams can collide with an angle.}
Figure 1.5: Run I measurement of the $J/\psi$ polarization variable $\alpha$ as a function of the $J/\psi$ transverse momentum by the CDF collaboration, in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ GeV. The band is the NRQCD factorization prediction, while the other curves are the values for the different feed down contributions.

Figure 1.6: $J/\psi$ polarization variable $\alpha$ as a function of the $J/\psi$ transverse momentum, measured by the CDF collaboration, in Run II $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ GeV. The experimental results are compared with the NRQCD expectations.
Chapter 2

The LHCb experiment

The LHCb detector is one of the four experiments that are installed at the Large Hadron Collider (LHC) at CERN in Geneva, Switzerland. The LHC project has been approved in December 1994. The first beams have circulated in September 2008 and the first collisions at high energy, 7 TeV in the centre-of-mass frame, took place in April 2010. Up to now an integrated luminosity of 1 fb$^{-1}$ has been recorded by the LHCb experiment.

In this chapter an overall description of the accelerator and of the experiment will be given. Each sub-detector will also be described in the following sections.

2.1 The Large Hadron Collider

LHC [24] is a circular proton-proton collider with a circumference of 26.7 km, located underground in the tunnel which also housed the LEP accelerator. The beams travel in opposite directions in two pipes enclosed within superconducting magnets cooled by liquid helium. The design energy of the collisions and the peak luminosity are respectively $\sqrt{s} = 14$ TeV in the centre-of-mass and $\mathcal{L} = 10^{34}$ cm$^{-2}$s$^{-1}$. To reach such values of energy the beams are accelerated in several steps by the CERN accelerator complex, shown in Fig. 2.1. Four main experiments are installed at LHC, ALICE [25], ATLAS [26], CMS [27] and LHCb [28] on the four interaction regions IP2, IP1, IP5 and IP8 respectively (see Fig. 2.2). ATLAS and CMS are required to work at the maximum peak luminosity delivered by LHC while the dedicated ion experiment ALICE needs to work at $\mathcal{L} = 2 \times 10^{27}$ for the nominal Pb-Pb collisions.

The instantaneous working luminosity of LHCb in 2011 is $\mathcal{L} = 3.5 \times 10^{32}$ cm$^{-2}$s$^{-1}$, while during the 2010 data taking a value $\mathcal{L} = 1.6 \times 10^{32}$ cm$^{-2}$s$^{-1}$ was reached. This is almost two orders of magnitude lower than the project value of LHC. At this luminosity the events containing $pp$ interactions ($\sim 30\%$ of the total number of events) are dominated by single interactions, as shown in Fig. 2.3. These are the optimal conditions for data taking given the kind of physics which LHCb aims to study. In fact to investigate the $B$ meson production, LHCb needs to identify their decay vertices with very high precision and this goal can be reached if a maximum number of two $pp$
Figure 2.1: Sketch of the CERN accelerator complex.

Figure 2.2: Layout of the LHC collider, with the four main experiments, ATLAS, ALICE, CMS, and LHCb.
interactions occurs in each bunch crossing. In order to fit the demand of LHCb, the luminosity is reduced by defocusing the beams in correspondence of IP8. At $\sqrt{s} = 14$ TeV the $b\bar{b}$ cross section is estimated to be 500 mb while at $\sqrt{s} = 7$ TeV it is about 300 mb; given a luminosity of $10^{32}$ cm$^{-2}$s$^{-1}$ $10^{12}$ $b\bar{b}$ pairs are expected to be produced within a 2 fb$^{-1}$ integrated luminosity (which was the expected statistics at the end of the 2011 data taking). This means a three orders of magnitude improvement with respect to the $B$ factories such as Babar [29] and Belle [30]. At LHC all the $b$-hadrons are produced with hadronization probability of 39.9%, 39.9%, 10.2%, 0.1% and 10% for $B_u$, $B_d$, $B_s$, $B_c$ and the $b$-baryons respectively, contrary to the $b$-factories where only $B_u$ and $B_d$ mesons can be produced.

2.2 The LHCb detector

The LHCb detector is a single-arm forward spectrometer, with an angular coverage of 10-300 mrad in the bending plane and 10-250 mrad in the non-bending plane. Introducing the LHCb laboratory frame, the $z$ axis lies along the beam axis, with positive direction going from the Vertex Locator to the Muon detector, the $y$ axis is along the magnet field direction, pointing upwards and the $x$ axis is chosen in such a way that the three axes are right handed. With this definition the bending plane corresponds to the $z-x$ plane and the non bending plane is the $z-y$ plane. The particular geometry of LHCb is optimal to study the $B$ mesons system because at the high energy reached at LHC the $b$ and $\bar{b}$ hadrons are mainly produced within a small polar angle cone around the beam axis, as shown in Fig. 2.4.

The detector layout is shown in Fig. 2.5 as seen from the $y-z$ plane. The main characteristics of the detector are listed below.

- High precision vertex reconstruction. In particular for quarkonium studies a very good...
Figure 2.4: Polar angle of $b$ and $\bar{b}$ quarks at LHC collisions energy, as simulated by PYTHIA event generator.

Figure 2.5: A view of the LHCb geometry in the $y - z$ plane.
Figure 2.6: $z - x$ and $z - y$ view of the VELO detector. The three lines indicate the maximum and minimum angular coverage and the average angle of tracks in minimum bias events, respectively.

proper time resolution is needed and for LHCb is between 30-50 fs.

- Excellent particle identification, to separate the different final states. For what concerns the muon identification it is $\epsilon(\mu \rightarrow \mu) \sim 97\%$, with a mis-identification of $\epsilon(\pi \rightarrow \mu) \sim 2\%$.
- High momentum resolution, $\delta p/p \sim 0.35 - 0.55\%$ for charged tracks.

The LHCb experiment consists of a vertex locator, a dipole magnet, two Ring Imaging Cherenkov detectors (RICH1 and RICH2), a tracking system, two calorimeters (electromagnetic and hadron) and a muon detector.

2.2.1 The vertex locator

The main task of the vertex locator (VELO) [31] is to reconstruct the primary and secondary vertexes. Its layout, shown in Fig. 2.6 consists of 25 circular stations perpendicular to the beam axis. Each station is composed by two disks of silicon microstrip detectors segmented along the radial and the azimuthal direction (see the sketch in Fig. 2.7), to cover approximately an angle of $182^\circ$ and almost 200000 channels. As their inner radius is only of 8 mm, the half-disks are retractile, with a Roman Pot geometry, to avoid damages during the LHC beam injection. The resolution for the reconstruction of the primary vertexes is $\sim 42 \mu$m along z axis and $\sim 11 \mu$m on the perpendicular plane. The VELO is equipped also with a PILE UP VETO system, composed by two disks located upstream the beam interaction point. The task of the PILE UP VETO is to count the number of primary vertexes in each bunch crossing and it is used by the first step of the trigger to identify and possibly reject the events with more than one interaction.
2.2.2 The magnet

LHCb is equipped with a dipole magnet generating a field vertically oriented (along \( y \) axis) with a maximum value of 1.1 T. During the data taking the field direction is periodically flipped to avoid systematic uncertainties due to a possible asymmetry of the detector along the \( x \) axis. The integrated field on the tracks crossing the magnet is around 4 Tm.

2.2.3 The tracking system

The LHCb tracking system is composed by four stations. The first one, referred to as the Trigger Tracker (TT), is in front of the Magnet and behind the RICH1 while the last three (T1-T3) are placed behind the Magnet with equal spacing.

The Trigger Tracker is formed by four layers of silicon microstrip detectors and it has two main purposes. The first one is to reconstruct the tracks of low momentum particles, which will not reach the following three stations. The latter is to give to the High Level Trigger a fast information about the momentum of the particle with large impact parameter.

The surface of last three stations is divided in two parts. The inner region (around the beam pipe) is the Inner Tracker (IT) and it is made by silicon microstrip detectors. The external region, surrounding the IT, is the Outer Tracker (OT) and it is equipped with straw tubes with a
spatial resolution of 200 $\mu$m. Both the Inner and Outer Tracker are formed by four layers. The external and internal layers are read out respectively along the $x$ axis and along the $\pm 5^\circ$ directions with respect to the $y$ axis, following the $xuvx$ geometry (i.e. the two $x$ layers have vertical detection cells, the $u$ and $v$ layers have detection cells rotated clock-wise and counter clock-wise respectively by a $5^\circ$ stereo angle). This particular layout allows to measure the $y$ coordinate and at the same time solve the ambiguity on the position of which the particle has crossed the stations.

A schematic view of one of the tracking stations is shown in Fig. 2.8.

### 2.2.4 The Cherenkov detector

The purpose of the two Ring Imaging Cherenkov [35] detectors of LHCb, whose schematic view is given in Fig. 2.9, is to identify the charged particles with momentum reaching 150 GeV/$c$ down to 1 GeV/$c$.

The first Cherenkov detector RICH1 is located between the VELO and the Magnet. Its angular coverage is $300 \times 250$ mrad, to match the full LHCb acceptance. It uses as radiator a combination of silicon aerogel and $C_4F_{10}$, which is well suited to detect and identify particles with low momentum (below 60 GeV/$c$). The RICH2 is located in front of the two calorimeters and it has a reduced angular acceptance, $120 \times 100$ mrad. It uses only $CF_4$ as radiator gas to identify high momentum particles.

For both RICH1 and RICH2 the Cherenkov light emitted by the particles is focused by a system of mirrors on two planes equipped with Pixel Hybrid Photon Detector (HPD), placed out of the LHCb acceptance. The anode of this photomultipliers is a silicon pixel sensor on which the electrons (produced by the photocathode) are focused by electric fields.

### 2.2.5 The calorimeters

LHCb is equipped with an electromagnetic and an hadronic sampling calorimeter (ECAL and HCAL) [36], for the measurement of the energy and the position of the particles. All the information provided by the calorimeters is used by the low level trigger. Both the calorimeters are
Figure 2.9: Cross-section view of RICH1 (left) and RICH2 (right) detectors.

Figure 2.10: Lateral segmentation of the SPD/PS and ECAL. The dimension of the cells are given for the ECAL (reduced by 1.5% for the SPD and PS). A quarter of the detector is shown.
segmented and their granularity is finer going toward the inner region: a sketch of the segmentation is given in Figs. 2.10 and 2.11.

The electromagnetic calorimeter is constituted by a 4 mm thick plane of scintillators interspersed with 2 mm thick lead planes, covering 25 radiation lengths. The resolution provided on the measurement of the energy of photons and electrons is

$$\frac{\sigma(E)}{E} = \frac{10\%}{\sqrt{E}} \oplus 1.5\%$$

(2.1)

where $E$ is given in GeV.

The hadronic calorimeter is formed by scintillator planes alternated with iron planes, respectively 4 mm and 16 mm thick, for a total of 5.6 interaction lengths. The energy resolution is

$$\frac{\sigma(E)}{E} = \frac{80\%}{\sqrt{E}} \oplus 10\%$$

(2.2)

with $E$ in GeV.

The calorimeter system is equipped also with a Preshower (PS) and a Scintillators Pad Detector (SPD), which are two scintillator layers 15 mm thick, separated by a lead plane 12 mm thick and divided in square shaped pads, read out by optical fibers. Their main purpose is to generate the electromagnetic showers before the calorimeter for a better separation of the electrons from the other particles.
Table 2.1: The table shows the distance of each station from the interaction point, the R1-R4 surface and the logical pad dimensions. The latter are projected on the M1 station: to obtain the real dimensions they need to be multiplied by $z_{M_i}/z_{M1}$. The readout method (anode or cathode) is also described for the different regions.

<table>
<thead>
<tr>
<th>Stn.</th>
<th>$z$ (mm)</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>Surface (m$^2$)</td>
<td>0.6</td>
<td>0.9</td>
<td>1.0</td>
<td>1.2</td>
<td>1.4</td>
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<td></td>
<td>Dim. pads (cm$^2$)</td>
<td>$1 \times 2.5$</td>
<td>$0.5 \times 2.5$</td>
<td>$0.5 \times 2.5$</td>
<td>$2 \times 2.5$</td>
<td>$2 \times 2.5$</td>
</tr>
<tr>
<td></td>
<td>Readout</td>
<td>anode</td>
<td>anode-cathode</td>
<td>anode-cathode</td>
<td>cathode</td>
<td>cathode</td>
</tr>
<tr>
<td>R2</td>
<td>Surface (m$^2$)</td>
<td>2.3</td>
<td>3.6</td>
<td>4.2</td>
<td>4.8</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>Dim. pads (cm$^2$)</td>
<td>$2 \times 5$</td>
<td>$1 \times 5$</td>
<td>$1 \times 5$</td>
<td>$4 \times 5$</td>
<td>$4 \times 5$</td>
</tr>
<tr>
<td></td>
<td>Readout</td>
<td>anode</td>
<td>anode-cathode</td>
<td>anode-cathode</td>
<td>cathode</td>
<td>cathode</td>
</tr>
<tr>
<td>R3</td>
<td>Surface (m$^2$)</td>
<td>9.2</td>
<td>14.4</td>
<td>16.8</td>
<td>19.3</td>
<td>22.1</td>
</tr>
<tr>
<td></td>
<td>Dim. pads (cm$^2$)</td>
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<td>$2 \times 10$</td>
<td>$2 \times 10$</td>
<td>$8 \times 10$</td>
<td>$8 \times 10$</td>
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<tr>
<td></td>
<td>Readout</td>
<td>cathode</td>
<td>cathode</td>
<td>cathode</td>
<td>cathode</td>
<td>cathode</td>
</tr>
<tr>
<td>R4</td>
<td>Surface (m$^2$)</td>
<td>36.9</td>
<td>57.7</td>
<td>67.2</td>
<td>77.4</td>
<td>88.3</td>
</tr>
<tr>
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<td>Dim. pads (cm$^2$)</td>
<td>$8 \times 20$</td>
<td>$4 \times 20$</td>
<td>$4 \times 20$</td>
<td>$16 \times 20$</td>
<td>$16 \times 20$</td>
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<tr>
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<td>anode</td>
<td>anode</td>
<td>anode</td>
<td>anode</td>
</tr>
</tbody>
</table>

2.2.6 The Muon detector

The purpose of the muon detector [37] is to provide a fast identification of the muons and a measurement of their momentum to the low level trigger. It is constituted by five stations M1 to M5, located along the beam axis. Its angular acceptance is of 306 mrad and 258 mrad in the bending and non-bending plane respectively.

The first station is located in front of the calorimeters in order to improve the resolution on the transverse momentum measurement provided to the low level trigger. The remaining four stations are placed after the calorimeters and interspersed with iron absorbers 80 cm thick. The detector geometry is completely projective so the dimensions of the stations scale proportionally to the distance from the interaction point.

Each station is divided in four regions numbered from R1 to R4 going from the beam pipe to the external zone. Their linear dimensions scale with proportion 1:2:4:8, in such a way that the expected particle flux is almost the same in the four regions. Each region is in turn divided in four quadrants, going from Q1 to Q4 counterclockwise from the upper left corner looking in the positive $z$ direction.

All the stations are equipped with Multi Wire Proportional Chambers (MWPCs), except for the inner region of M1, which is exposed to high fluxes of particles, where Gas Electron Multipliers detectors (GEM) [38] are used.

All the chambers are segmented into physical pads, which are connected to one readout electronics channel. In the MWPCs the readout can be done either on the anode or on the cathode, so the physical pads can be constituted by a cathode pad or by a group of anode wires. Instead in the GEM
chambers the readout can be done only on the anode. The front-end (FE) electronic channel is based on an integrated circuit, tested to be radiation resistant, which is constituted by an amplifier-shaper-discriminator chain, so that the output is a binary signal. A certain number of FE cards are then logically ORed, depending on the station and region of the detector.

Thus each station is divided in logical pads, obtained from the combinations of physical pads, with different dimensions depending on the region of the muon detector and on the required muons transverse momentum resolution provided to the first trigger level. A summary of the logical pad dimensions for each region and station is shown in Tab. [2.1] In some regions the logical pads are also obtained offline by crossing horizontal and vertical strips, named logical channels, formed from FE channels in logical OR. This allows a reduction of the channels to be handled by the off-chamber electronics. The layout of a quadrant of M2 station is shown in Fig. [2.2.6] with the segmentation in logical channels and pads.
2.2.7 The trigger system

At the project LHC luminosity the rate of $pp$ interactions is 40 MHz, out of which only 10 MHz contain visible interactions. The production rate of the $b\bar{b}$ pairs is roughly 100 kHz, but only 15% is within the detector acceptance. Moreover the interesting decay rates for the CP violation studies are within $10^{-3}$ and $10^{-5}$. For all this reasons the trigger system must be able to select only the interesting channels, rejecting the background in the most efficient way. The LHCb trigger system has two different steps.

At the first level, named Level 0 (L0), the event rate is reduced from 40 MHz to 1 MHz. At this rate it is possible to store in the memory all the information coming from the LHCb subdetectors. This is performed selecting only events containing high transverse momentum particles (hadrons, leptons or photons), which are a tag of a $b$-hadron decay. The L0 trigger uses the information collected from four different subsystems:

- PILE UP VETO: it rejects the events with more than one $pp$ interaction, as described in section (2.2.1);

- calorimeters: the high energy hadrons, leptons and photons are identified by looking at the isolated showers in the ECAL and HCAL and measuring their transverse energy $\mathbf{E}_T^2$. The information coming from the SPD are used to validate the detected showers;

- muon detector: the L0 muon trigger looks for high transverse momentum muon tracks. Hits are searched, on the five muon stations, along a straight line pointing to the interaction region. Finally using the hits in the first two stations, M1 and M2, the transverse momentum is measured;

- Level 0 Decision Unit (L0DU): it collects all the information from the other subsystems and computes one decision per crossing.

The latency of the L0 trigger is below 4 $\mu$s, including the flight time of the particles, the cables length, the response of the electronics and the time needed to take the decision. Moreover all the L0 components are synchronous and allow the L0DU to deliver a decision every 25 ns, the time between two consecutive crossings of beams.

The second step of the trigger system, or High Level Trigger (HLT), is able to reduce the rate from 1 MHz, out of the L0 trigger, to 2 kHz, the rate at which data are written on disk. The HLT has access to all the information provided by all the subdetectors but it aims at rejecting the bulk of the events using only few of them, given the high input rate of 1 MHz and the limited CPU resources available. It consists of two different steps, the HLT1 and HLT2.

---

1 An interaction is visible when it produces at least two charged particles detected by the VELO or the tracking system to allow their reconstruction

2 $\mathbf{E}_T$ is defined as the projection on the $xy$ plane of a vector pointing from the interaction region to the energy deposition on the calorimeters, whose module is the energy deposition itself.
The HLT1 applies different sequences of algorithms, depending on the type of candidate which has passed the L0 trigger. It basically confirms the L0 decision, adding the information coming from VELO and the tracking system and applying cuts on the transverse momentum and the impact parameter. Eventually it searches for additional particles other than the L0 candidate to improve identification of the $B$ decays.

The HLT2 is applied on all the events passing the HLT1, independently from the algorithm that has been passed. A full pattern recognition is performed to find all the particles in the events, using the VELO tracks segments as seeds, and different sets of selections (inclusive or exclusive) are applied.

The full set of selection algorithms and cuts applied by the three trigger steps is defined by a Trigger Configuration Key (TCK).

### 2.2.8 The LHCb software

All the software applications used in LHCb are included in a general framework named GAUDI and written in C++. Each one of the applications is built to manage a particular step in the data analysis: the Monte Carlo generation, the description of the detector, the reconstruction of the events and finally the analysis of the processed data. The main applications are:

**Gauss** [40] is the application which manages the generation and the simulation in LHCb. In particular it generates the $pp$ interaction, using PYTHIA [41] and EvtGen [42] (for the $B$ mesons decays) as interface, and it simulates with GEANT4 [43] the trajectories of the particles inside the detectors, considering the interaction with the materials, the geometry and in general all the physical processes.

**Boole** [44] manages the last part of the simulation, generating the response of the detector to the hits coming out from GEANT4. At this step all the instrumental effects and the read out electronics are included, as the spillover, the inefficiencies, the cross talk and the electronic noise. The digitalized signal are then produced by BOOLE, simulating the real response of the instruments.

**Brunel** [45] is built to process real and simulated (coming from BOOLE) data. This is the reconstruction application: using the information coming from all the subdetectors it reconstructs the tracks of the particle in each event. In this way it defines a sort of proto-particle, which are not identified but contain all the kinematic informations.

**DaVinci** [46] is the analysis application. At this step the full decay is reconstructed applying a set of selection algorithms to identify the proto-particle reconstructed by BRUNEL. The identified particles in each event are then combined and the set of selection cuts is applied to finally get the decay channel of interest.
2.3 Running condition in 2010 data taking

The integrated luminosity recorded by the LHCb experiment during the 2010 data taking, from March to October was \( \sim 38 \text{ pb}^{-1} \) with an efficiency of \( \sim 90\% \), as shown in Fig. 2.13. An instantaneous luminosity of \( 1.7 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1} \), 80% of the nominal value, was reached but with 344 colliding bunches and with almost 1/3 of the design \( \beta^* \) value. This leads to an increasing value of the average number \( \mu \) of visible \( pp \) interactions per bunch crossings. The design value is \( \mu \sim 0.4 \): this choice allows to simplify the event reconstruction and to reduce the radiation level (see section 2.1). During the 2010 \( \mu \) reached 2.5, as shown in Fig. 2.14. Direct consequences of the increase of \( \mu \) are a larger number of vertices and tracks to be reconstructed, a higher readout rate per bunch crossing, a larger event size and a longer processing time but with the gain of a higher luminosity that was a great advantage especially in the study of very rare decays such as \( B_s \rightarrow \mu^+\mu^- \). To avoid this pile-up problems, for the present analysis, only the fraction of events recorded with lower \( \mu \) value have been used. With the current conditions of collision energy and instantaneous luminosity an order of \( 10^{11} b\bar{b} \) pairs has been collected in 1 fb\(^{-1} \) integrated luminosity, collected by the experiment at the end of the 2011 data taking.
Figure 2.14: Number of visible proton-proton interactions per bunch crossings during 2010 operation.
Chapter 3

Muon identification and tracking

The reconstruction and identification of the muon tracks is a crucial aspect of the present analysis. To determine the trajectories of the particles the hits in various subdetectors (the VELO, the tracking system, the muon detector) need to be collected and combined. The average track multiplicity (number of tracks per event) in 2010 was up to several dozens and the challenge of the reconstruction procedure is to estimate their correct parameters with good accuracy. Once the track has been reconstructed it needs to be identified as a muon, by applying a list of selection criteria on the information provided by the muon detector. In the following sections the tracking strategies in LHCb will be presented and the muon identification will be described.

3.1 Tracking strategies in LHCb

The track parameters are estimated by a fit procedure, together with the corresponding covariances. The parameters are then used to match the tracks with the particle identification informations, provided by the RICH rings, the calorimeter clusters and the muon hits. Moreover the offline reconstruction uses the parameter values to reconstruct the primary and secondary vertices and the invariant mass of the particle combinations.

A track is basically a collection of straight segments, tangent to the particle trajectory: other shapes can be chosen for the track segments, such as helix or parabola, but since most of the hits in LHCb are found in the low magnetic field region, the straight line is a good approximation. The tracks are parametrized as a function of the $z$ coordinate and their segments are defined by the position along the $z$ axis and the tangent direction. Also the curvature introduced by the magnetic field has to be taken into account, for the large extrapolation between hits: the curvature provides information about the particle momentum, which can be added to the other track parameters.
3.2 Pattern recognition

The first step of the track reconstruction is to find all the tracks in the events. This is achieved by the pattern recognition algorithms, whose main purpose is to assign to each track the correct hits. According to their trajectories the LHCb tracks are classified in different types, schematically shown in Fig. 3.1.

**VELO tracks.** They are only reconstructed in the VELO detector. Thus they tend to have a large polar angle, with respect to the beam line, and for this reason they allow to reconstruct the primary vertices with high accuracy.

**T tracks.** They are only reconstructed in the T1, T2 and T3 stations of the tracking system and they are used for the RICH2 reconstruction.

**Upstream tracks.** They only traverse the VELO and the TT stations, since they are bent out of the detector acceptance by the magnetic field before reaching the following stations of the tracking system.

**Downstream tracks.** They have hits only in the tracking system (TT and T1, T2, T3) but they are not reconstructed in the VELO. Thus they can be used to reconstruct the particles decaying outside the VELO acceptance.

**Long tracks.** They are reconstructed in the whole detector, from the VELO to the tracking system. They have an accurate momentum resolution and for this reason they are the most useful in the physics analysis.

![Figure 3.1: Sketch of the tracks types in LHCb detector.](image)

Several algorithms are used in the pattern recognition, each one dedicated to reconstruct one of the track types previously listed. First of all three-dimensional points are created from the $r$ and $\phi$
VELO clusters and are used to reconstruct VELO tracks, pointing to the interaction point. These VELO segments constitute the seeds for the other track finding algorithms which, starting from them, look for a continuation in the TT and T1, T2, T3 stations.

In particular for the long tracks, given a VELO seed, tracks are searched in the tracking stations. This is done by parametrizing the expected position in T1, T2 or T3 as function of the VELO seed track parameters and the position of a single hit on the T stations. Further hits in the tracking stations in a window around the expected position are then picked up. Some quality cuts are applied to the combination of the VELO seeds and the tracking station hits: tracks satisfying these criteria are promoted to long tracks. Hits in TT are picked up if they are close enough to a track through VELO and T station hits.

An alternative method starts from track segments reconstructed in the VELO and in the tracking stations, using both as seeds and trying to match them. Both the seeds are extrapolated to the bending plane of the magnetic field and several quantities are evaluated (position in the bending plane, slope change, number of compatible hits in TT) to determine if the two segments belong one to each other. The TT hits closer to the tracks are added afterwards.

For what concerns the downstream tracks the starting point of the pattern recognition algorithm is constituted by the track segments in the tracking stations. Using them as seeds, TT hits are looked for and added to the track. The efficiency of the track reconstruction can be determined on a Monte Carlo sample asking for the Monte Carlo truth. To cross check the results with the data, the *tag and probe* method is used: both the *tag* and *probe* tracks are muon tracks forming a $J/\psi$ candidate. For long tracks the *tag* track is a long track identified as a muon, while the *probe* track is a muon track (a track segment reconstructed in the muon detector) with hits in the Trigger Tracker. The efficiency of long track finding is shown in Fig. 3.2 as a function of the track $\eta$ and $p$. The discrepancy of the two results is taken as a systematic uncertainty due to the tracking efficiency, as described and calculated in section 6.7.1.

Figure 3.2: Efficiency of long track reconstruction in the Monte Carlo and 2010 data sample as a function of track $\eta$ (left) and $p$ (right).
3.3 Primary vertex reconstruction

The offline reconstruction of the primary vertices (PV) is performed in two main steps \[48\]. In the first one the PV seeds are searched, providing the \( z \) coordinates of the PV candidates. In the second step the primary vertex is reconstructed, with an iterative fit using the tracks extrapolated to the \( z \) coordinates of the seeds.

The first estimate of the PV candidates, performed with the seeding procedure, is provided by the clusterization. The initial cluster is determined by the closest approach of the tracks with respect to the \( z \) axis and is determined by its \( z \) coordinate and its uncertainty. Among the list of all the pairs of clusters a single pair is considered and their distance along the \( z \) axis is computed: if this distance is below a certain value the two clusters are merged. A list of quality requirements are applied to the list of clusters.

For each PV seed a fitting procedure is applied using the Long Upstream and Velo tracks: the selection of the tracks is based on the \( z \) position of their closest approach to the \( z \) axis. The position of the primary vertex is determined with the least square method, minimizing the \( \chi^2_{PV} \):

\[
\chi^2_{PV} = \sum_{i=1}^{n_{tracks}} \frac{d_{0i}^2}{\sigma_{d_{0i}}^2}
\]  

(3.1)

being \( d_{0i} \) the impact parameter of the track and \( \sigma_{d_{0i}} \) the corresponding uncertainty. The position of the PV candidate is determined iteratively. At each iteration a new position of the primary vertex is determined. The tracks are extrapolated to the position of the new PV and the impact parameters are recalculated. The tracks with highest impact parameter are removed from the set of tracks used for the fit of the primary vertex. When no more tracks can be discarded the iterative procedure stops. Finally some quality cuts are applied, based on the multiplicity of the PV candidates, i.e. the number of reconstructed tracks coming from the vertex. To calculate the next candidate the procedure starts from a new seed removing those tracks which already belong to any previous primary vertex.

3.4 Muon identification technique

The muon identification is one of the main aspects of the analysis discussed in this thesis which is based on the reconstruction of the decays of the \( J/\psi \) resonance into a muon pair. The identification procedure \[49\] is applied to long or downstream tracks defined in the previous section adding the information provided by the LHCb muon detector. A muon crossing the muon detector fires the pads of the muon chambers, leaving an hit. Taking advantage of the information about these hits provided by the muon detector the procedure is carried on by a three-steps algorithm.

1. The candidate track is linearly extrapolated to the muon detector and on each station a “Field of Interest” (FoI) is opened around the extrapolated position. The dimensions of the
Foil depend on the track momentum and on the muon station to which it is extrapolated.

2. Hits are looked for in the FoI. A boolean parameter, IsMuon, is switched to true by the identification algorithm if at least one hit is found in the opened FoI on a certain number of stations, depending on the track momentum, according to Tab. 3.1. The requirements of the IsMuon decision assume a high muon detector efficiency: this has been measured with the first data acquired by the experiment and it has been found as satisfactory as expected, above 99% [50, 51].

3. To enhance the rejection of fake candidates, an hypothesis test is performed on the tracks satisfying the IsMuon decision, by calculating the probability of being a muon. The hypothesis test is based on the distribution of the squared distance between the extrapolated position \((x_{\text{track } i}; y_{\text{track } i})\) of the track and the closest hit associated \((x_i; y_i)\) in the FoI, normalized to the granularity of the detector:

\[
D = \frac{1}{N} \sum_{i=1}^{5} \left\{ \left( \frac{x_i - x_{\text{track } i}}{\text{pad}_x} \right)^2 + \left( \frac{y_i - y_{\text{track } i}}{\text{pad}_y} \right)^2 \right\}
\]  

(3.2)

where the sum runs over all the five muon stations.

The distribution of the \(D\) quantity depends on the multiple scattering and therefore on the momentum or transverse momentum of the muon candidates. For this reason the hypothesis test is performed binning the distributions in momentum bins and according to the muon detector regions. Using the \(D\) distribution the \(p_\mu\) and \(p_{\text{non-}\mu}\) probability of being a muon or not being a muon respectively, is built. The logarithm of the ratio of these two quantities \(D\mathcal{L} = \log(p_\mu/p_{\text{non-}\mu})\) is giving the final discriminant variable of the method.

Generally the efficiency of this procedure can depend on several kinematic variables, like the muon momentum or transverse momentum and the impact point on the muon detector. The efficiency is defined as the ratio of the number of muon reconstructed tracks identified as muons and the total number of muon reconstructed tracks:

\[
\varepsilon_{\mu\text{ID}} = \frac{\mu(\text{track} & \mu\text{ID} = \text{TRUE})}{\mu\text{track}}
\]

(3.3)

It can be evaluated in data with the tag and probe method on a control sample of \(J/\psi \rightarrow \mu^+\mu^-\) which provides a pure source of muon tracks. In the tag and probe method the first muon \(\mu_{\text{tag}}\) is required to satisfy the muon identification criteria, while the second one (\(\mu_{\text{probe}}\)) is selected without using any information provided by the muon detector. The probe is then used to estimate the efficiency of the identification requirements. The obtained efficiency in data and Monte Carlo is shown in Fig. 3.3.

The muon identification efficiency reached with this method is up to 97% with a corresponding misidentification (probability of wrongly identify a particle as a muon) of 2%. This value is av-
averaged over all the values of the track momenta: above 8 GeV/c the performance is even better, higher than 97%.

<table>
<thead>
<tr>
<th>Track momentum range</th>
<th>Muon stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 GeV/c &lt; p &lt; 6 GeV/c</td>
<td>M2+M3</td>
</tr>
<tr>
<td>6 GeV/c &lt; p &lt; 10 GeV/c</td>
<td>M2+M3+(M4.or.M5)</td>
</tr>
<tr>
<td>p &gt; 10 GeV/c</td>
<td>M2+M3+M4+M5</td>
</tr>
</tbody>
</table>

Table 3.1: Muon stations required to have an hit in the FoI, to identify a muon candidate, as a function of momentum range.

Figure 3.3: Muon identification efficiency as a function of muon momentum, evaluated in data and Monte Carlo sample.
Chapter 4

Selection of $J/\psi$ candidates

This chapter will go through the identification of the muon candidates and their combination to reconstruct the $J/\psi \rightarrow \mu^+\mu^-$ decay. The trigger and the offline selection of the events for the polarization and the cross section measurement will also be presented. Finally the procedure to extract the signal and to disentangle the prompt $J/\psi$ from the $J/\psi$ coming from $b$ decays will be described.

The cross section and polarization are performed in bins of $J/\psi$ transverse momentum $p_T$ and rapidity $y$, defined as:

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \quad (4.1)$$

being $E$ and $p_z$ the $J/\psi$ energy and momentum component along the beam axis respectively. The distributions of these quantities are shown in Fig. 4.1. In the limit where a particle has a mass close to zero, or if it is travelling with a velocity close to $c$, the rapidity is a good approximation of the pseudorapidity, defined as:

$$\eta = -\ln \left( \tan \left( \frac{\theta}{2} \right) \right) \quad (4.2)$$

being $\theta$ the angle of the particle with respect to the beam axis.

4.1 The trigger

The signal events are selected for both the cross section and polarization measurement, by the trigger requiring they pass all the three trigger steps L0, HLT1 and HLT2, described in section 2.2.7.

For this kind of analysis two different requirements, the so called trigger lines, are applied at the L0 trigger. The first is the single muon line, which asks for one muon candidate with $p_T$ larger than 1.4 GeV/c. The second is the di-muon line which requires two muon candidates, having $p_T$ larger than 0.56 GeV/c and 0.48 GeV/c, respectively. To be selected an event must be accepted by
at least one of the two trigger lines, i.e. the L0 requirement consists in the OR of the single muon and di-muon line. The events selected by these L0 requirements provide the input to the second step, the HLT1 lines. Two different HLT1 sets of requirements have been used for the polarization and cross section analysis.

- For the cross section measurement two HLT1 lines have been chosen. The first one asks a confirmation of the single muon candidate, selected at the L0, and applies a harder cut on the transverse momentum, requiring it to be higher than $1.8 \text{ GeV}/c$. The second line confirms the di-muon candidates which passed the corresponding L0 line and adds a cut on the invariant mass, asking it to be greater than $2.5 \text{ GeV}/c^2$. As at the L0 step the single muon and di-muon line are in logical OR, so that an event must be selected by at least one of the two to be accepted by the HLT1 trigger.

- In the full 2010 data sample (used in the polarization measurement), selected by the HLT1 di-muon line, it has been found a discrepancy between data and Monte Carlo efficiency, in the overlap region between the two half sides of the Inner and Outer Tracker. For this reason the di-muon line has been discarded in the polarization analysis, asking only for the single muon line at the HLT1.

The requirement of the di-muon line on the invariant mass is tightened by the HLT2 line, which asks for two muons candidates with invariant mass higher than $2.9 \text{ GeV}/c$. In Tab. 4.1 the trigger lines and the main corresponding cuts, relevant for this kind of analysis, are summarized.

### 4.2 J/ψ selection

To better select the signal from the background, a list of cuts is applied offline to the events selected by the trigger. An event is kept if it contains at least one reconstructed primary vertex (PV): the reconstruction of the primary vertices is described in section 3.3. If it contains more than one
primary vertex the closest to the $J/\psi$ decay vertex, along the $z$ axis, is kept. The first step of the analysis is the reconstruction and selection of the $J/\psi \rightarrow \mu^+ \mu^-$ decay. Referring to the invariant mass distribution (shown for example in Fig. 4.5, it is possible to define a signal and two sideband regions. The signal region includes the events whose mass is within $M_{J/\psi} \pm 3\sigma$ where $M_{J/\psi}$ is the $J/\psi$ mass and $\sigma$ is the measured width of the mass peak. The sideband regions instead, include the events whose mass is within the ranges $[M_{J/\psi} - 7\sigma, M_{J/\psi} - 4\sigma]$ and $[M_{J/\psi} + 4\sigma, M_{J/\psi} + 7\sigma]$. As it will be more clear in the following, the sideband regions are essentially populated only by background events and they can be used to estimate background contributions to the various physical observables used in the analysis.

The mass distribution is obtained by first computing the invariant mass of two muon tracks of opposite charge, reconstructed in the full LHCb tracking system and accepted by the L0, HLT1 and HLT2 trigger levels. The two tracks must be identified as muons (with positive IsMuon decision) and have the transverse momentum higher than 750 MeV/c. The track fit $\chi^2$, normalized to the number of degrees of freedom, must be less than 4. In Fig. 4.2 the $J/\psi$ candidate $\chi^2$ distribution is compared with the background. The comparison is made with the sPlot technique described in Appendix C. In this framework, the background is estimated from the sideband distribution and then statistically subtracted to the signal. The distribution is built using the events passing all the cuts in the selection. The $J/\psi$ candidate is formed requiring the two tracks to originate from a common vertex. In order to calculate the position of the decay vertex a fit is performed on the tracks originating from a common point. The $\chi^2$ probability associated to this fit is shown in Fig. 4.3 and must be higher than 0.5%. The two signal and background distribution shown in Fig. 4.3 are again obtained with the sPlot technique. The distribution is built using the events passing all the cuts in the selection, except the one on the vertex $\chi^2$. The reconstruction algorithm assigns to the tracks a likelihood of being a certain particle, i. e. a muon, a pion, a kaon, etc. In section 3.4 the definition of the probability of being a muon $p_{\mu}$ is given. For this particular analysis the likelihood discriminant is built using the probability of being a muon with respect to

<table>
<thead>
<tr>
<th>Trigger lines</th>
<th>Cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0SingleMuon</td>
<td>$p_T &gt; 1.4 \text{ GeV}/c$</td>
</tr>
<tr>
<td>L0DiMuon</td>
<td>$p_{T,1} &gt; 0.56 \text{ GeV}/c, p_{T,2} &gt; 0.48 \text{ GeV}/c$</td>
</tr>
<tr>
<td>Hlt1SingleMuon</td>
<td>L0SingleMuon and $p_T &gt; 1.8 \text{ GeV}/c$</td>
</tr>
<tr>
<td>Hlt1DiMuon</td>
<td>L0DiMuon and $M_{\mu\mu} &gt; 2.5 \text{ GeV}/c$</td>
</tr>
<tr>
<td>Hlt2DiMuon</td>
<td>$M_{\mu\mu} &gt; 2.9 \text{ GeV}/c$</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of the trigger lines used for this analysis, with the correspondent cuts which are relevant for the analysis.
the pion hypothesis, in the following way:

\[ DLL_{\mu\pi} = \ln \left( \frac{p_\mu}{p_\pi} \right) \]  

(4.3)

The distribution of the muon and pion likelihood is shown in Fig. 4.4 in red and black respectively. A cut is applied on this variable requiring it to be higher than zero. It can happen that one single particle is reconstructed twice by mistake by the reconstruction software using overlapping sets of hits (“clone tracks”). Two clone tracks are identified looking at the angle between them: if the cosine of this angle is higher than 0.9999 they are labelled as clones. When two \( J/\psi \) candidates are originated by clone tracks only one of the two candidates is kept.

The cuts are summarized here (when the cut is on the muon, it is applied on both muons):

- Select events with at least one reconstructed primary vertex
- IsMuon flag for both muons
- \( p_T(\mu) > 750 \text{ MeV}/c \)
- \( \chi^2(\mu \text{ track})/\text{ndof} < 4 \)
- MuonPID(\( \mu \)) > 0
- clone track cuts: \( \cos \theta(\mu_1^+, \mu_2^+) > 0.9999 \) and \( \cos \theta(\mu_1^-, \mu_2^-) > 0.9999 \)
- prob(\( J/\psi \) vertex \( \chi^2 \)) > 0.5%
- \( \mu^+\mu^- \) invariant mass > 2900 MeV/c\(^2\)

### 4.3 Invariant mass distribution

The number of signal events can be extracted from a fit to the invariant mass distribution, modellng the signal peak with a Crystal Ball function and the background with a negative exponential. The Crystal Ball function is defined as:

\[
 f_{\text{CB}}(x; M, \sigma, a, n) = \begin{cases} 
 \frac{\left(\frac{n}{|a|}\right)^n e^{-\frac{x-a}{a}}}{\left(\frac{n}{|a|} - |a| - \frac{x-M}{\sigma}\right)^n} & \frac{x-M}{\sigma} < -|a| \\
 \exp\left(-\frac{1}{2} \left(\frac{x-M}{\sigma}\right)^2\right) & \frac{x-M}{\sigma} > -|a| 
\end{cases} 
\]  

(4.4)

and consists of a gaussian core with a power law tail: the gaussian and the power law functions are connected together at a certain threshold \( a \). It is commonly used in particle physics to take
into account radiative process. In this case the power law tail is particularly suitable to model the radiative decays of the $J/\psi$, like $J/\psi \rightarrow \mu^+\mu^-\gamma$, $J/\psi \rightarrow \mu^+\mu^-\gamma\gamma$ etc. The background fit function is:

$$f_{\text{Bk}}(M; p_0, a_0) = a_0 e^{-p_0 M}$$

(4.5)

where $a_0$ and $p_0$ are free parameters.

Thus the total fit function to the invariant mass distribution is:

$$F_{\text{mass}}(M_{J/\psi}; M, \sigma, f_{J/\psi}, p_0, a_0) = f_{J/\psi} f_{\text{CB}}(M_{J/\psi}; M, \sigma) + (1 - f_{J/\psi}) f_{\text{Bk}}(M_{J/\psi}; p_0, a_0).$$

(4.6)

where $f_{J/\psi}$ is the fraction of signal events over the total.

The invariant mass distribution of the two muons is shown in Fig. 4.5 between 2900 MeV/$c^2$ and 3300 MeV/$c^2$, with the signal and background components. The width of the Crystal Ball function extracted from the fit, $\sigma_M = 14.8$ MeV/$c^2$, is very close to the Monte Carlo expectation, $\sim 13$ MeV/$c^2$. Almost $10^5$ $J/\psi$ per pb$^{-1}$ are detected and reconstructed in LHCb.

### 4.4 Distinction between prompt $J/\psi$ and $J/\psi$ from $b$

To discriminate between prompt $J/\psi$ and those from $b$-hadron decays it is possible to use the fact that the $J/\psi$ coming from $b$ decays tend to be produced detached from the primary vertex of the
events, contrary to the prompt $J/\psi$. Therefore, as a discriminant variable, the projection along the $z$ axis of the $J/\psi$ pseudo-proper time $t_z$ is used, which is defined as:

$$t_z = \frac{(z_{J/\psi\text{vertex}} - z_{J/\psi PV}) \times M(J/\psi)}{p_z}.$$  \hspace{1cm} (4.7)

This variable takes advantage of the fact that the lifetime of the $b$-hadrons is longer than the $J/\psi$ lifetime. Referring to Fig. 4.5, $z_{J/\psi\text{vertex}}$ is the position along the beam axis of the $J/\psi$ decay vertex and $z_{J/\psi PV}$ is the position along the beam axis of the primary vertex. $M(J/\psi)$ and $p_z$ are respectively the mass and the momentum component along the beam axis of the $J/\psi$. The pseudo proper time is a good approximation of the decay proper time of the $b$-hadrons. Since the decays studied in this analysis are inclusive, the proper time of the $b$-hadrons is unknown and is replaced by the pseudo proper time. The distribution of this variable is centered around zero for the prompt component and it follows an exponential decay for the $J/\psi$ coming from the $b$-hadron decays. The decay parameter of the exponential distribution is close to the lifetime of a $b$-hadron mixture $\tau_{b-hadr} = 1.568 \pm 0.009$ ps \cite{12}. Since the particles detected are in the forward rapidity region, the main contribution to the $t_z$ is given by the component along the beam axis. Thus, without losing information, the discriminant variable can be chosen to be the only $z$ component. The distribution of the $z$ component of the pseudo proper time of the data is shown in Fig. 4.7.
Figure 4.4: Difference between the muon and non-muon likelihood for background subtracted samples of protons (blue), muons (red) and pions (black).

The separation between the prompt and the delayed $J/\psi$ component can be achieved also by using an alternative variable which is the pseudo proper time significance defined as:

$$t_{zs} = \frac{t_z}{\sigma_{t_z}}$$

(4.8)

The contributions to the uncertainty $\sigma_{t_z}$ come from the reconstructed quantities $z_{J/\psi\text{ vertex}}$, $z_{J/\psi\text{ PV}}$ and $p_z$. For each $J/\psi$ candidate the uncertainty $\sigma_{t_z}$ on the $t_z$ is evaluated:

$$\sigma_{t_z} = \frac{\sigma^2_{J/\psi\text{ vertex}} + \sigma^2_{J/\psi\text{ PV}}}{(z_{J/\psi\text{ vertex}} - z_{J/\psi\text{ PV}})^2} + \frac{\sigma^2_{p_z}}{p_z^2}.$$  

(4.9)

The distribution of the pseudo proper time significance is shown in Fig. 4.8.

The number of prompt and delayed $J/\psi$ is determined by a simultaneous fit to the $\mu^+\mu^-$ invariant mass and to the pseudo proper time distributions.

The pseudo proper time distribution of prompt $J/\psi$ is described by a $\delta$-function and the $J/\psi$ from $b$ are modelled by an exponential decay. Both the functions are convoluted with the detector resolution. Moreover a long tail is also present in the data distribution, coming from the events in which the wrong primary vertex is associated to the $J/\psi$. The wrong association is basically due to two reasons:

- the right primary vertex is not reconstructed by the reconstruction algorithm because the
number of tracks originating from it is not enough to perform the fit;

- several primary vertices close to each other are reconstructed in the event but the wrong one is chosen because of the finite resolution of the primary vertex measurement. Thus for the prompt $J/\psi$ this category of events is distributed on $t_z = 0$ ps and is automatically taken into account by the resolution of the detector.

The long tail is mainly due to the first category of events and because of its shape, it will affect the determination of the number of $J/\psi$ from $b$. To model the tail the following procedure has been followed. The wrong association between the primary and the $J/\psi$ decay vertex is simulated taking the primary vertex from the event subsequent the one taken into account. Being $z_{\text{next PV}}$ the position along the beam direction of the primary vertex in the subsequent event, the $t_{z\text{next}}$ variable is defined as:

$$t_{z\text{next}} = \frac{(z_{J/\psi\text{ vertex}} - z_{\text{next PV}}) \times M(J/\psi)}{p_z}.$$  

(4.10)

In Fig. 4.9 the $t_{z\text{next}}$ is shown superimposed to the $t_z$ distribution. The $t_z$ distribution is built subtracting the background events from the total distribution, in order to select only signal events, with the sPlot technique (see Appendix C). From the figure it can be seen that the agreement between the tail and the $t_{z\text{next}}$ distribution is satisfactory.
Figure 4.6: Scheme of $J/\psi$ decay, showing all the quantities involved in the definition of pseudo proper time.

Figure 4.7: $t_z$ distribution for the selected $J/\psi$ candidates (data). The exponential tail due to the $J/\psi$ from $b$-hadron decay can be identified clearly in the right side of the distribution.
Figure 4.8: Distributions of $t_{zs}$ significance of the selected $J/\psi$ candidates, in the full range and zoomed in the region of interest, between $-10$ and 10.

Figure 4.9: $t_z$ distribution of background subtracted $J/\psi$ selected candidates in data (full dots) superimposed to the tail estimated with the “next” event (histogram), as described in the text.
The pseudo proper time distribution of signal events is modelled with the function:

\[ f_{\text{signal}}(t_z; f_p, f_b, \tau_b) = f_p \delta(t_z) + f_b \frac{e^{-t_z/\tau_b}}{\tau_b} + (1 - f_b - f_p) f_{\text{tail}}(t_z) \]  \hspace{1cm} (4.11)

where \( f_b \) and \( f_p \) are respectively the fraction of \( J/\psi \) candidates coming from \( b \)-hadrons decays and prompt for which the primary vertex has been correctly reconstructed, over the total number of signal events. \( \tau_b \) is the \( b \)-hadron pseudo-lifetime and \( f_{\text{tail}} \) is the probability density function describing the tail, built from the \( t^\text{ext}_z \) histogram. The fraction of \( J/\psi \) from \( b \) is then given by:

\[ F_b = \frac{f_b}{f_b + f_p} \]  \hspace{1cm} (4.12)

The \( \delta \) and exponential part of the \( f_{\text{signal}} \) function are then convoluted with the detector resolution function, given by the sum of two gaussian distributions:

\[ f_{\text{res}}(t_z; \mu, \sigma_1, \sigma_2, \beta) = \frac{\beta}{\sqrt{2\pi} \sigma_1} e^{\frac{(t_z - \mu)^2}{2\sigma_1^2}} + \frac{1 - \beta}{\sqrt{2\pi} \sigma_2} e^{\frac{(t_z - \mu)^2}{2\sigma_2^2}} \]  \hspace{1cm} (4.13)

where \( \mu \) is the mean value of the resolution function, common to the two gaussians, and \( \beta \) is the fraction of the gaussian with the largest width, \( \sigma_1 \). The \( \sigma_1 \) value got from the fit is close to the Monte Carlo expectation, which is \( \sim 60 \) fs.

Two main sources contribute to the background of the pseudo proper time distribution. The first one comes from the random combination of reconstructed muons produced in the semileptonic decays of the \( b \) and \( c \)-hadrons. The second is given by the misreconstructed tracks coming from the decays in flight of the pions and kaons. These events are distributed in a wide range of invariant mass around the \( J/\psi \) mass peak. Therefore the background distribution is built with the events contained in the so called sideband regions of the invariant mass distribution, defined in this case as \([2900; 3050]\) \( \text{MeV}/c^2 \) and \([3150; 3300]\) \( \text{MeV}/c^2 \). For these events the background is modelled with an empirical function, built with the sum of a \( \delta \)-function, three positive and two negative exponentials. The sum of the three contributions are convoluted with the same resolution function as the signal. Thus the total function is:

\[ f_{\text{bkg}}(t_z) = \left[ (1 - f_1 - f_2 - f_3) \cdot \delta(t_z) + \theta(t_z) \left( f_1 \frac{e^{-t_z/\tau_1}}{\tau_1} + f_2 \frac{e^{-t_z/\tau_2}}{\tau_2} \right) + \theta(-t_z) \left( f_3 \frac{e^{-t_z/\tau_3}}{\tau_3} + f_4 \frac{e^{-t_z/\tau_4}}{\tau_4} \right) \right] \otimes \left( \frac{\beta'}{\sqrt{2\pi} \sigma_1} e^{\frac{(t_z - \mu)^2}{2\sigma_1^2}} + \frac{1 - \beta'}{\sqrt{2\pi} \sigma_2} e^{\frac{(t_z - \mu)^2}{2\sigma_2^2}} \right) \]  \hspace{1cm} (4.14)

The choice of this particular function is mainly empirical, based on the experimental distribution of \( t_z \) from sideband events. It is however partially justified following the arguments explained by the CDF study in the measurement of the \( J/\psi \) cross section \([52]\). The positive exponentials correspond to the random combinations of muons coming from the \( b \) and \( c \) decays while the
misreconstructed tracks coming from the decays in flight of pions and kaons contribute to both positive and negative exponentials. The parameters of $f_{bkg}(t_z)$ are extracted from the fit to the sideband distributions and are fixed in the final fit. The result of the fit is shown in Fig. 4.10 a very good agreement between the data and the fit is obtained.

The function used for the fit to the pseudo proper time is therefore:

$$f(t_z; f_{J/\psi}, f_p, f_b, \tau_0, \mu, \sigma_1, \sigma_2, \beta) = f_{J/\psi} \left[ \left( f_p \delta(t_z) + f_b e^{-t_z/\tau_0} \right) \otimes f_{res} + \right.$$

$$\left. (1 - f_p - f_b) \cdot f_{tail} \right] + (1 - f_{J/\psi}) f_{bkg} \quad (4.15)$$

and the product of $f(t_z; f_{J/\psi}, f_p, f_b, \tau_0, \mu, \sigma_1, \sigma_2, \beta)$ with the invariant mass probability density function is used for the final global fit:

$$F(t_z, M_{J/\psi}; f_{J/\psi}, f_p, f_b, \tau_0, \mu, \sigma_1, \sigma_2, \beta, M, \sigma, f_{J/\psi}, p_0, a_0) =$$

$$F(t_z; f_{J/\psi}, f_p, f_b, \tau_0, \mu, \sigma_1, \sigma_2, \beta) \times F_{mass}(M_{J/\psi}; M, \sigma, f_{J/\psi}, p_0, a_0). \quad (4.17)$$

The free parameters are listed explicitly in parenthesis for both the mass and the $t_z$ functions. In Fig. 4.11 the $t_z$ distribution of data is plotted, superimposed with the result of the fit. The different components contributing to the signal, the background and the tail are shown separately.
Figure 4.10: $t_z$ background distribution in data. The histogram is superimposed with the fit function used to model the background, defined in eq. (4.14).
Figure 4.11: $t_z$ distribution in data (full dots), fitted with $F(t_z; f_{J/\psi}, f_p, f_b, \tau_b, \mu, \sigma_1, \sigma_2, \beta)$ (blue line). The different components contributing to the fit function are explicitly shown in the figure.
Chapter 5

$J/\psi$ polarization measurement

The three polarization states of a $J = 1$ quarkonium can be specified in terms of a given coordinate system in the rest frame of the quarkonium itself. This coordinate system is often called the “spin-quantization frame”. In a hadron collider, the $J/\psi$ are actually reconstructed through their electromagnetic decays into a lepton pair. In this case, the information about the polarization of the quarkonium state is encoded in the angular distribution of the two leptons which is usually described in the quarkonium rest frame with respect to a particular spin-quantization frame. In that case, the angular distribution of the quarkonium can be expressed in terms of three real parameters that are related to the spin-polarization amplitudes of the $J = 1$ quarkonium state.

Several polarization frames definitions can be used among which the most widely used are listed below.

**Helicity frame:** referring to Fig. 5.1 the polar axis $z_{H_X}$ coincides with the flight direction of the quarkonium $Q\overline{Q}$ in the center of mass of the colliding hadrons $h_1$ and $h_2$.

**Collins-Soper:** referring to Fig. 5.2 the polar axis $z_{CS}$ is the direction of the relative velocity of the colliding partons, as seen in the quarkonium rest frame. The approximation is satisfactory if it is possible to neglect the smearing effect of the intrinsic parton momentum.

**Gottfried-Jackson:** as sketched in Fig. 5.2 the polar axis $z_{GJ}$ is chosen to be the direction of the beam momentum in the quarkonium rest frame.

It is possible to define the polar and azimuthal angle, respectively $\theta$ and $\phi$, between the positive lepton and the chosen polar axis, with respect to the “production plane”, which contains the momenta of the colliding hadrons in the quarkonium rest frame. In particular for the helicity frame the $\theta$ and $\phi$ angles will be the polar and azimuthal angle of the momentum of the positive lepton (measured in the $J/\psi$ rest frame) with respect to the $J/\psi$ flight direction (measured in the
Figure 5.1: Definition of the Helicity frame. The polarization axis $z_{HX}$ is the quarkonium $Q\bar{Q}$ flight direction in the centre of mass of the two colliding hadrons $h_1$ and $h_2$.

Figure 5.2: Comparison between the three polarization axis $z_{HX}$, $z_{GJ}$ and $z_{CS}$, respectively in the helicity, the Gottfried-Jackson and the Collins-Soper frames, as seen in the quarkonium $Q\bar{Q}$ rest frame. The two beam axis $h_1$ and $h_2$ are also shown.
laboratory frame). The angular decay distribution is usually defined as:

\[
P(\cos \theta, \phi; \lambda) = \left( \frac{3}{4\pi(3 + \lambda)} \right) (1 + \lambda \cos^2 \theta + \lambda_{\theta\phi} \sin 2\theta \cos \phi + \lambda_{\phi} \sin^2 \theta \cos 2\phi)
\]

(5.1)

where \(\lambda_\theta\), \(\lambda_{\theta\phi}\), and \(\lambda_\phi\) are the three parameters which characterize the polarization. The distribution in eq. (5.1) is normalized to 1. The values of \(\lambda_\theta\), \(\lambda_{\theta\phi}\), and \(\lambda_\phi\), and thus the polarization, vary depending on the polarization frame. This happens because the frames differ one from each other according to the analysed phase space of the quarkonium. The best choice is a polarization frame which maximize the \(\lambda_\theta\) parameter, or in which the polarization naturally arises. It is possible to define a quantity, a combination of these parameters, which is invariant with respect to the frame [55, 56]:

\[
\lambda = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi}
\]

(5.2)

It is important to notice that if \(\lambda_\phi = 0\) the \(\lambda\) invariant gives exactly the \(\lambda_\theta\) value. The value of this invariant parameter gives the real amount of polarization (i.e. not artificially caused by the frame). Using this quantity it is possible to cross check the measurements obtained in the different frames.

Integrating over one of the two variables the single variable distributions are obtained.

\[
\frac{dN}{d \cos \theta} \propto 1 + \lambda_\theta \cos^2 \theta
\]

(5.3)

\[
\frac{dN}{d \phi} = 1 + \frac{2\lambda_\phi}{3 + \lambda_\theta} \cos(2\phi)
\]

(5.4)

from which it is possible to extract two of the three polarization parameters.

For the present analysis the helicity frame has been chosen.

The angular analysis is performed in two different way.

- **Full angular analysis**: the bidimensional distribution in \(\cos \theta\) and \(\phi\) is fitted with the function defined in eq. (5.1), in order to get all the three parameters \(\lambda_\theta\), \(\lambda_{\theta\phi}\), and \(\lambda_\phi\). The fit is performed with an unbinned maximum likelihood approach (Discriminant Log Likelihood, or DLL, method), described later on. A binned maximum likelihood fit has also been used to cross check the results.

- **Analysis of single angular variables**: the two single variable distributions in \(\cos \theta\) and \(\phi\), of eqs. (5.3) and (5.4), are fitted to extract \(\lambda_\theta\) and \(\lambda_\phi\). To get the third parameter \(\lambda_{\theta\phi}\) another distribution is needed. For this purpose a new angular variable can be defined as combination of \(\cos \theta\) and \(\phi\). Following this approach the acceptance and detector efficiency function, which appears in the angular distribution of the two leptons and which depends in general on both \(\cos \theta\) and \(\phi\), has to be integrated. In particular to get \(\lambda_\theta\) the function defined in eq. (5.3) is obtained just assuming that the efficiency is flat in \(\phi\). Any dependence
introduces a bias in the results, which can be neglected only if $\lambda_\theta, \lambda_\phi \ll 1$. As it will be shown in the final results this is actually verified for the $J/\psi$ polarization in the helicity frame.

In the following sections the above methods will be described in details and the final results will be presented and compared.

## 5.1 Data and Monte Carlo samples

The polarization measurement is based on a data sample of 23 pb$^{-1}$ integrated luminosity, acquired by the LHCb experiment during the 2010 data taking, at $\sqrt{s} = 7$ TeV and with both magnet polarities. This corresponds roughly to 63% of the full 2010 data sample and is a sub-sample acquired with similar trigger conditions.

## 5.2 Monte Carlo simulation

A sample of 20 millions Monte Carlo inclusive $J/\psi \rightarrow \mu^+\mu^-$ events have been used to study the detector performance (in particular the acceptance and the efficiency). The Monte Carlo sample was generated, with the same trigger conditions as the data, with the 2010 version of simulation software Gauss (mentioned in section 2.2.8). To simulate the prompt $J/\psi$ production the Leading Order Color Singlet Model and Color Octet Model processes are activated in PYTHIA, including also the $\psi(2S)$ production with the Color Singlet processes. The production of $b$-hadrons is given by $2 \rightarrow 2$ processes at leading order, like: $q\bar{q} \rightarrow q\bar{q}$, $qq' \rightarrow qq'$, $q\bar{q} \rightarrow gg$, $gg \rightarrow gg$, $gg \rightarrow q\bar{q}$ and $gg \rightarrow gg$. The parton density functions (from CTEQ6) used in the simulated cross sections are obtained from a fit with the leading order hard cross section and using $\alpha_s$ at the leading order. The EvtGen package takes care of the decays of charmonium states, using the branching fractions shown in Tab.5.1.

For what concerns the $J/\psi$ coming from $b$, the production and decays of $b$-hadrons are simulated with PYTHIA and EvtGen respectively. The inclusive branching ratios and the corresponding hadronization fraction used in the simulation are shown in Tab.5.2. The PHOTOS package is also used to include the QED radiative corrections to the decay of the $J/\psi$ into muon pairs [57].

<table>
<thead>
<tr>
<th>decay mode</th>
<th>branching fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi c_0 \rightarrow J/\psi \gamma$</td>
<td>1.28 %</td>
</tr>
<tr>
<td>$\chi c_1 \rightarrow J/\psi \gamma$</td>
<td>36 %</td>
</tr>
<tr>
<td>$\chi c_2 \rightarrow J/\psi \gamma$</td>
<td>20 %</td>
</tr>
<tr>
<td>$\psi(2S) \rightarrow J/\psi X$</td>
<td>52.7 %</td>
</tr>
</tbody>
</table>

Table 5.1: Charmonium branching fractions of decays to $J/\psi$ assumed in the simulation.
Table 5.2: Branching fractions of inclusive $b$-hadron decays to $J/\psi$ and hadronisation fractions in the LHCb simulation.

<table>
<thead>
<tr>
<th>$b$-hadron</th>
<th>branching fraction in LHCb simulation</th>
<th>hadronization fraction in LHCb simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0$</td>
<td>$\mathcal{B} (\overline{B}^0 / B^0 \to J/\psi \ X) = (1.59 \pm 0.04)%$</td>
<td>40.5%</td>
</tr>
<tr>
<td>$B^-$</td>
<td>$\mathcal{B} (B^- / B^+ \to J/\psi \ X) = (1.48 \pm 0.04)%$</td>
<td>40.5%</td>
</tr>
<tr>
<td>$B^0_s$</td>
<td>$\mathcal{B} (\overline{B}^0_s / B^0_s \to J/\psi \ X) = (1.39 \pm 0.04)%$</td>
<td>10%</td>
</tr>
<tr>
<td>$\Lambda^0_b$</td>
<td>$\mathcal{B} (\Lambda^0_b / \overline{\Lambda}^0_b \to J/\psi \ X) = (0.81 \pm 0.03)%$</td>
<td>9%</td>
</tr>
</tbody>
</table>

5.3 Signal selection

The signal events are selected applying the trigger selection described in section 4.1 and the cuts listed in section 4.2.

The prompt $J/\psi$ candidates are selected by requiring the absolute value of the pseudo proper time significance $t_{zs}$, defined in eq. (4.8), smaller than 4 to separate prompt $J/\psi$ from $J/\psi$ from $b$-hadron decays.

To extract the signal events a fit is performed to the $\mu^+\mu^-$ invariant mass distribution modelling the signal peak and the background with a Crystal Ball and an exponential function respectively, as described in section 4.3. The fit is performed separately in 6 bins of $p_T$ and 5 bins of $y$. When fitting in the different $p_T$ and $y$ bins a dependence of the $\eta$ and $\phi$ parameters of the Crystal Ball function from $\sigma$ is observed. This effect has been studied but this dependence is found to have no influence on the results values of the $\lambda_\eta$, $\lambda_{\beta\phi}$ and $\lambda_\phi$ parameters, obtained with the three methods. Instead this dependence is found to be important and it is taken properly into account for the cross section measurement, as it will be explained in the following chapter.

In Fig. 5.3 the 2-dimensional angular distribution is shown for the selected $J/\psi$ candidates for a particular $p_T$ and $y$ bin.

5.4 Detection efficiency

The detection efficiency includes three terms:

- the geometrical acceptance of the detector $\varepsilon_{\text{acc}}$. Some fiducial cuts have been applied to remove areas of the detector with known hardware inefficiency issues: in fact these are present only in the data and they are not described in the Monte Carlo, causing a discrepancy between the real and simulated sample. The tracks crossing these problematic regions are then excluded from the analysis in both the data and the Monte Carlo sample;

- the reconstruction and selection efficiency $\varepsilon_{\text{rec}}$, given by the probability for a signal event to be reconstructed and accepted by the selection cuts;
Figure 5.3: 2-dimensional angular distribution for the selected $J/\psi$ candidates for a particular $p_T$ and $y$ bin.
• the trigger efficiency $\varepsilon_{\text{trig}}$, which is the efficiency for an event to pass the Level 0, HLT1 and HLT2 trigger.

The efficiency is calculated using the unpolarized Monte Carlo sample and the Monte Carlo truth is used to decide if the reconstructed $J/\psi$ candidate is a real $J/\psi$. In general the geometrical, reconstruction and trigger efficiencies can depend on the azimuthal and polar angles and also on the kinematic quantities, like the $J/\psi$ transverse momentum and rapidity. Thus the efficiency is studied in four kinematic variables: the $p_T$ and $y$ of the $J/\psi$ and the $\cos \theta$ and $\phi$ of the positive muon measured in the chosen polarization frame. In principle, since the efficiency is completely defined as a function of these four kinematic variables it doesn’t depend on the polarization parameters $\lambda_{\theta}$, $\lambda_{\theta\phi}$ and $\lambda_\phi$. This is true in the limit of a punctual description of the efficiency distribution but, since it is binned in $p_T$, $y$ and in $\cos \theta$ and $\phi$, an integration is performed in each bin. This could lead to a bad description of the efficiency and, consequently, to a wrong results of the polarization parameters.

The efficiency distribution has been built in 20 bins of both $\cos \theta$ and $\phi$. A cross check has been performed to demonstrate that this binning is the best compromise between a good description of the efficiency and the statistics, which should be enough in each bin to perform the fit. The cross check has been done using a toy Monte Carlo \textsuperscript{1} to simulate with different binning an angular distribution representing the one extracted from the data, and an efficiency distribution. For both the angular and the efficiency distribution a parabolic shape has been chosen. The efficiency corrected angular distribution is found dividing the first one by the efficiency. The resulting distribution is fitted with the parabolic function and the fit results obtained for the different binning choices have been compared. It has been verified that the results fluctuate for coarse binning choices but they tend to stabilize as the binning becomes more and more fine. For the nominal 20 binning choice the fit results are completely stable, inside the statistical errors of the generated sample.

For what concerns the $J/\psi$ $p_T$ and $y$ variables six bins in transverse momentum and five bins in rapidity have been chosen. The edges of the bins are defined in the following way:

\[
p_T : [2, 3, 4, 5, 7, 10, 15] \text{GeV/c}
\]

\[
y : [2.0, 2.5, 3.0, 3.5, 4.0, 4.5]
\]

The polarization is finally measured in these $6 \times 5$ $p_T$ and $y$ bins. This allows to reduce the uncertainty due to the possible differences in the kinematic spectra between the data and the Monte Carlo. The data below 2 GeV/c in transverse momentum are not used for several reasons. The statistics both in the data and Monte Carlo in this regions is relatively poor giving very high statistical uncertainty (depending on the rapidity bin). In addition, the background in this region

\textsuperscript{1}A toy Monte Carlo is a Monte Carlo simulation where one or a small set of kinematic variables are extracted randomly from a measured or simulated distribution. This technique is generally used to simulate a large number of “experiments” in order to study the effect on a certain measurement due to the statistical fluctuations of the variables under study.
is relatively high.

5.5 Polarization determination with the DLL approach

In this section the procedure used to measure the polarization with the DLL fit is described in detail. The first important input to the likelihood function that is described in sections 5.5.1 and 5.5.2 is the detection efficiency which is discussed in the following section.

5.5.1 Likelihood estimator construction

The log likelihood function (estimator) for data in each $p_T$ and rapidity bin is defined as:

$$
\ln L = \ln \prod_{i=1}^{N_s} \left[ \frac{P(\cos \theta_i, \phi_i) \times \varepsilon_{\text{tot}}(\cos \theta_i, \phi_i)}{\text{Norm}(\lambda_\theta, \lambda_{\theta\phi}, \lambda_\phi)} \right]
$$

where $P(\cos \theta, \phi)$ represents the angular distribution defined in eq. (5.1), and Norm($\lambda_\theta, \lambda_{\theta\phi}, \lambda_\phi$) is the normalization, which generally depends on the three parameters $\lambda_\theta$, $\lambda_{\theta\phi}$ and $\lambda_\phi$. In this definition the sum runs over the signal events whose angular distribution is parametrised by $P(\cos \theta, \phi)$. To take into account the background contribution, the following procedure is applied. The di-muon invariant mass distribution is fitted, modelling the signal peak with a Crystal Ball function and the background with a first order polynomial. The $n$ parameter is fixed to 1: this value follows directly from the Quantum Electrodynamics description of the final state radiation. In fact the probability to radiate a photon for each muon, calculated in the QED frame, is proportional to $1/M_{J/\psi}$. Since the only reason of the mass distribution asymmetry is the photon emission, the Crystal Ball function must be continued by a $1/M_{J/\psi}$ tail, fixing $n = 1$. Instead the $a$ parameter is fixed to the Monte Carlo expectation. The fit is performed in each $p_T$ and rapidity bin.

A signal region is then defined by $\mu \pm 3\sigma$ where $\mu$ and $\sigma$ are the mass and width of the $J/\psi$ peak as obtained from the fit. Sideband regions are then defined in the invariant mass distribution by $[\mu - 7\sigma, \mu - 4\sigma]$ and $[\mu + 4\sigma, \mu + 7\sigma]$. The sidebands are chosen to be far enough from the signal peak to avoid contamination from the radiative tail.

The log likelihood function can now be extended also to the sideband regions. According to the measured $J/\psi$ mass, each event is weighted in the following way: if the mass value falls in the signal region a weight $W(\text{Mass}) = +1$ is assigned to the event, if the mass value lies in the sidebands the weight $W(\text{Mass})$ is equal to $-1$, otherwise the weight is zero. In this way the likelihood
function becomes:

$$\ln L = N_{\text{tot}} \sum_{i=1}^{N_{\text{tot}}} W(\text{Mass}_i) \times \ln \left[ \frac{P(\cos \theta_i, \phi_i) \times \varepsilon_{\text{tot}}(\cos \theta_i, \phi_i)}{\text{Norm}(\lambda_\theta, \lambda_{\theta\phi}, \lambda_\phi)} \right]$$  \hspace{1cm} (5.7)

where now the sum runs over all the events in data, though only the events in the signal and in the sideband regions give a contribution.

Under the hypothesis that the background shape is linear and that the angular and invariant mass distributions of the background events is the same in the sidebands and signal regions it is possible to assume that the number of background events in the sidebands and in the signal region is the same within statistical fluctuations.

The first hypothesis has been cross-checked by fitting the background with an exponential function. The resulting slope parameter is so small that the linear approximation is fully justified, as shown in Tab. 5.3. The second hypothesis has been cross-checked in the following way. The dimuon mass distribution is studied around 3400 MeV/$c^2$, well far away from the signal region. The signal and sideband regions are redefined around 3400 MeV/$c^2$, by shifting the nominal regions by 3400 MeV/$c^2 - \mu$, where $\mu$ is the Crystal Ball mean value. Since now the signal and sideband regions contain only background events, their angular distributions can be compared. From this check it is found that the shifted sideband and signal distributions are very similar except for a few $\cos \theta$ and $\phi$ points where the signal distribution is in any case between the two sideband distributions. This effect can be correct by making the average of the two nominal sidebands distributions in order to construct the background distribution in the nominal signal region.

With the assumptions discussed above, the likelihood function defined in eq. (5.7) is equivalent to the one in eq. (5.6) within statistical fluctuation.

In the likelihood estimator the total efficiency $\varepsilon_{\text{tot}}$ is determined from MC, but the analytic shape doesn’t have to be known because it only introduces a constant term which is not relevant during the maximization of the likelihood function, and can thus be removed. Finally the estimator can be written as:

$$\ln L = \sum_{i=1}^{N_{\text{tot}}} W(\text{Mass}_i) \times \ln \left[ \frac{P(\cos \theta_i, \phi_i)}{\text{Norm}(\lambda_\theta, \lambda_{\theta\phi}, \lambda_\phi)} \right] + \sum_{i=1}^{N_{\text{tot}}} W(\text{Mass}_i) \times \ln[\varepsilon_{\text{tot}}(\cos \theta_i, \phi_i)]$$  \hspace{1cm} (5.8)

where the constant term doesn’t depend on $\lambda_\theta$, $\lambda_{\theta\phi}$ and $\lambda_\phi$ and will be cancelled out in the derivative.
<table>
<thead>
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<th>$2.5 &lt; y &lt; 3.0$</th>
<th>$3.0 &lt; y &lt; 3.5$</th>
<th>$3.5 &lt; y &lt; 4.0$</th>
<th>$4.0 &lt; y &lt; 4.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean of CB</td>
<td>3096.01±0.07</td>
<td>3094.97±0.04</td>
<td>3094.06±0.04</td>
<td>3093.25±0.06</td>
<td>3091.20±0.12</td>
</tr>
<tr>
<td>σ of CB</td>
<td>11.04±0.07</td>
<td>11.92±0.04</td>
<td>13.60±0.04</td>
<td>15.98±0.05</td>
<td>19.22±0.11</td>
</tr>
<tr>
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<td>0.0006±0.0001</td>
<td>-0.0003±0.0000</td>
<td>-0.0005±0.0000</td>
<td>-0.0004±0.0001</td>
<td>-0.0001±0.0002</td>
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<td>$3 &lt; P_T &lt; 4$</td>
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<td>3090.99±0.15</td>
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<td>11.28±0.07</td>
<td>12.34±0.04</td>
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<td>16.80±0.07</td>
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<td>-0.0002±0.0001</td>
<td>-0.0006±0.0001</td>
<td>-0.0001±0.0001</td>
<td>-0.0002±0.0003</td>
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<td>$4 &lt; P_T &lt; 5$</td>
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<td>$3.5 &lt; y &lt; 4.0$</td>
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<tr>
<td>mean of CB</td>
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<td>3092.88±0.09</td>
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<td>σ of CB</td>
<td>11.83±0.07</td>
<td>12.98±0.05</td>
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<tr>
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<td>3093.20±0.10</td>
<td>3091.53±0.22</td>
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<tr>
<td>σ of CB</td>
<td>12.55±0.06</td>
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<td>18.53±0.09</td>
<td>22.47±0.20</td>
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<td>$2.5 &lt; y &lt; 3.0$</td>
<td>$3.0 &lt; y &lt; 3.5$</td>
<td>$3.5 &lt; y &lt; 4.0$</td>
<td>$4.0 &lt; y &lt; 4.5$</td>
</tr>
<tr>
<td>mean of CB</td>
<td>3095.25±0.10</td>
<td>3094.01±0.09</td>
<td>3093.38±0.12</td>
<td>3093.18±0.18</td>
<td>3092.22±0.43</td>
</tr>
<tr>
<td>σ of CB</td>
<td>13.55±0.09</td>
<td>14.64±0.08</td>
<td>16.82±0.10</td>
<td>20.17±0.16</td>
<td>25.30±0.37</td>
</tr>
<tr>
<td>τ of Exp</td>
<td>0.0005±0.0004</td>
<td>0.0006±0.0004</td>
<td>0.0003±0.0005</td>
<td>0.0002±0.0008</td>
<td>0.0007±0.0020</td>
</tr>
<tr>
<td>$10 &lt; P_T &lt; 15$</td>
<td>$2.0 &lt; y &lt; 2.5$</td>
<td>$2.5 &lt; y &lt; 3.0$</td>
<td>$3.0 &lt; y &lt; 3.5$</td>
<td>$3.5 &lt; y &lt; 4.0$</td>
<td>$4.0 &lt; y &lt; 4.5$</td>
</tr>
<tr>
<td>mean of CB</td>
<td>3095.13±0.20</td>
<td>3093.66±0.20</td>
<td>3093.42±0.27</td>
<td>3092.54±0.45</td>
<td>3093.96±1.14</td>
</tr>
<tr>
<td>σ of CB</td>
<td>15.03±0.17</td>
<td>16.21±0.17</td>
<td>18.77±0.23</td>
<td>23.02±0.40</td>
<td>29.59±1.04</td>
</tr>
<tr>
<td>τ of Exp</td>
<td>-0.0001±0.0008</td>
<td>-0.0000±0.0009</td>
<td>0.0028±0.0016</td>
<td>0.0015±0.0024</td>
<td>0.0000±0.0000</td>
</tr>
</tbody>
</table>

Table 5.3: Main parameters derived from the fit to the invariant mass distribution, with the Crystal Ball and exponential functions for the signal peak and the background respectively.
5.5.2 Determination of normalization factor from Monte Carlo

The normalization factor $\text{Norm}(\lambda_\theta, \lambda_{\theta\phi}, \lambda_\phi)$ is defined as:

\[
\text{Norm}(\lambda_\theta, \lambda_{\theta\phi}, \lambda_\phi) = \int d\Omega P(\cos \theta, \phi) \times \varepsilon_{\text{tot}}(\cos \theta, \phi)
\]

\[
= \int d\Omega (\varepsilon_{\text{tot}}(\Omega) + \lambda_\theta \cos^2 \theta \times \varepsilon_{\text{tot}}(\Omega) + \lambda_{\theta\phi} \sin 2\theta \cos \phi \times \varepsilon_{\text{tot}}(\Omega) + \lambda_\phi \sin^2 \theta \cos 2\phi \times \varepsilon_{\text{tot}}(\Omega))
\]

\[
= \int d\Omega \varepsilon_{\text{tot}}(\Omega) + \lambda_\theta \int d\Omega \cos^2 \theta \varepsilon_{\text{tot}}(\Omega) + \lambda_{\theta\phi} \int d\Omega \sin 2\theta \cos \phi \varepsilon_{\text{tot}}(\Omega) + \lambda_\phi \int d\Omega \sin^2 \theta \cos 2\phi \varepsilon_{\text{tot}}(\Omega)
\]

(5.9)

where $\Omega \equiv (\cos \theta, \phi)$. The four terms in the last member of eq. (5.9) can be calculated from MC as follows:

\[
\int d\Omega \varepsilon_{\text{tot}}(\Omega) = \frac{1}{N_{\text{gen}}} \equiv \varepsilon_{\text{tot}}
\]

\[
\lambda_\theta \int d\Omega \cos^2 \theta \varepsilon_{\text{tot}}(\Omega) = \lambda_\theta \times \frac{\sum_i \cos^2 \theta_i}{N_{\text{gen}}} \equiv \varepsilon_{\text{tot}} \times a\lambda_\theta
\]

\[
\lambda_{\theta\phi} \int d\Omega \sin 2\theta \cos \phi \varepsilon_{\text{tot}}(\Omega) = \lambda_{\theta\phi} \times \frac{\sum_i \sin 2\theta_i \cos \phi_i}{N_{\text{gen}}} \equiv \varepsilon_{\text{tot}} \times b\lambda_{\theta\phi}
\]

\[
\lambda_\phi \int d\Omega \sin^2 \theta \cos 2\phi \varepsilon_{\text{tot}}(\Omega) = \lambda_\phi \times \frac{\sum_i \sin^2 \theta_i \cos 2\phi_i}{N_{\text{gen}}} \equiv \varepsilon_{\text{tot}} \times c\lambda_\phi
\]

(5.10)

The sum is made over all the selected events in the Monte Carlo sample, while $N_{\text{gen}}$ is the number of generated events. If the first term is then normalized to 1 the three coefficients $(a, b, c)$ can be interpreted as the expectation values of $\cos^2 \theta$, $\sin 2\theta \cos \phi$ and $\sin^2 \theta \cos 2\phi$ and can be extracted from the corresponding Monte Carlo distributions. Also in the calculation of the normalization term the analysis is completely unbinned.

5.5.3 Minimization with TMinuit

The function $-\ln L$ can now be minimized with the TMinuit software package which is integrated in ROOT [47]. The set of parameters that minimizes the log likelihood function is the fit result.

5.6 Binned fit methods

Two alternative methods, based on binned fits of the angular distributions in $\cos \theta$ and $\phi$, are described in this section. As already mentioned, the results obtained with this two alternative approaches are used to cross check the DLL method results.
5.6.1 Full angular analysis: alternative method with 1-D fits

A first method consists in the extraction of the three parameters $\lambda_{\theta}$, $\lambda_{\phi}$, and $\lambda_{\theta\phi}$ with a binned fit by considering the projected single angular distribution. The method has been used by HERA-B for the measurements of the $J/\psi$ polarization \[23\].

Integrating the full angular distribution over $\cos \theta$ and $\phi$ in the ranges $[-1; 1]$ and $[-\pi; \pi]$ respectively, the single-variable distributions of eqs. \ref{eq:5.3} and \ref{eq:5.4} are obtained. Since none of them depends on $\lambda_{\theta\phi}$ a third distribution is needed. For this reason a new variable can be introduced, defined as a combination of the previous two variables as following:

$$\phi_{\theta} = \begin{cases} \phi - \frac{3}{4} \pi & \text{for } \cos \theta < 0 \\ \phi - \frac{\pi}{4} & \text{for } \cos \theta > 0 \end{cases} \quad (5.11)$$

Replacing $\phi$ with the new variable $\phi_{\theta}$ in the full angular distribution in eq. \ref{eq:5.1}, and properly integrating over $\cos \theta$ it is possible to get its one-dimensional distribution:

$$\frac{dN}{d\phi_{\theta}} \propto 1 + \sqrt{2} \lambda_{\theta\phi} \, \cos \phi_{\theta}. \quad (5.12)$$

This third distribution together with those of $\cos \theta$ and $\phi$ allows to extract all the three parameters. This must be done performing a simultaneous fit to the three distributions, since they all depend on $\lambda_{\theta}$. 

Selection of the signal events

In order to select the one-dimensional angular distributions of the signal events, the background must be properly normalised and subtracted from the fiducial region in the invariant mass distribution discussed in section \ref{sec:5.5.1}.

The normalization factor is provided by the fit to the invariant mass distribution of the muon pairs. Each of the single-variable distributions in $\cos \theta$, $\phi$ and $\phi_{\theta}$ is built using the events in the fiducial region of the $\mu^+ \mu^-$ invariant mass distribution, defined as the $\pm 3\sigma$ window centered on peak value. The background distribution in the signal region is obtained averaging the distributions of the events coming from the left and right sidebands. The pure signal distributions of $\cos \theta$, $\phi$ and $\phi_{\theta}$ are obtained subtracting the background from the distributions of the events in the fiducial region.

From the fit the number of signal and background events in the signal region is estimated. The number of background events in the sideband regions $N_{\text{sid}}$ is given by the sum of the events in the right and left sidebands (respectively $N_{\text{Rsid}}$ and $N_{\text{Lsid}}$)

$$N_{\text{sid}}^{\text{bk}} = N_{\text{Rsid}} + N_{\text{Lsid}}. \quad (5.13)$$
Thus the background normalization factor is given by:

\[ N = \frac{N_{SFR}^{bkg}}{N_{sid}^{bkg}} \]  \hspace{1cm} (5.14)

where \( N_{SFR}^{bkg} \) is the number of background events in the signal fiducial region.

**Efficiency correction**

The angular distributions obtained after the background subtraction are not yet the theoretical distributions described by the eqs. (5.3), (5.4) and (5.12). In fact the angular distributions can be strongly modified by the detector acceptance and by the efficiency, which are not flat in the two angular variables \( \cos \theta \) and \( \phi \). Taking into account the efficiency effect the observed angular distributions of signal events can be written as products of the theoretical function and the efficiency term:

\[ \frac{d^2 N_{data}}{d \cos \theta d \phi} = P(\cos \theta, \phi; \lambda) \cdot \varepsilon_{tot}(\cos \theta, \phi) \]  \hspace{1cm} (5.15)

where \( \varepsilon_{tot}(\cos \theta, \phi) \) is the total detector efficiency, which depends on the kinematics of the events and \( P(\cos \theta, \phi; \lambda) \) is the theoretical angular distribution defined in eq. (5.1).

To get the one-dimensional distributions in \( \cos \theta \) and \( \phi \), of eqs. (5.3) and (5.4), the formula in eq. (5.15) must be integrated in \( \phi \) and \( \cos \theta \) respectively. To reduce the bias caused by the presence of the efficiency term it is necessary to assume that the two parameters \( \lambda_\phi \) and \( \lambda_{\theta \phi} \) are small. This hypothesis will be verified a posteriori.

Integrating eq. (5.15) over \( \phi \) one obtains:

\[ \frac{dN}{d \cos \theta} \propto (1 + \lambda_\theta \cos^2 \theta) \times \varepsilon_{tot}(\cos \theta) \]  \hspace{1cm} (5.16)

while integrating over \( \theta \):

\[ \frac{dN}{d \phi} \propto \left[ 1 + \frac{2 \lambda_\phi}{3 + \lambda_\theta} \cos(2\phi) \right] \times \varepsilon_{tot}(\phi) \]  \hspace{1cm} (5.17)

To obtain the third angular differential decay rate it is necessary to properly replace \( \phi \) with the new variable \( \theta_\phi \) in the eq. (5.15) and then integrate over \( \cos \theta \).

\[ \frac{dN}{d \phi_\theta} \propto \left[ 1 + \frac{\sqrt{2} \lambda_{\theta \phi}}{3 + \lambda_\theta} \cos \theta_\phi \right] \times \varepsilon_{tot}(\cos \theta_\phi) \]  \hspace{1cm} (5.18)

The efficiency single-variable distribution for \( \cos \theta \), \( \phi \) and \( \phi_\theta \) is evaluated from an unpolarized Monte Carlo sample. In fact since the Monte Carlo sample is unpolarized, \( \lambda_\theta = \lambda_\phi = \lambda_{\theta \phi} = 0 \) and in this case \( P(\cos \theta, \phi; \lambda) = 1 \). Thus the observed angular distributions are equal to the
efficiency:
\[
\frac{1}{N_{MC}} \cdot \frac{dN_{MC}}{d \cos \theta} = \varepsilon_{\text{tot}}(\cos \theta) \tag{5.19}
\]
\[
\frac{1}{N_{MC}} \cdot \frac{dN_{MC}}{d \phi} = \varepsilon_{\text{tot}}(\phi) \tag{5.20}
\]
\[
\frac{1}{N_{MC}} \cdot \frac{dN_{MC}}{d \phi_{\theta}} = \varepsilon_{\text{tot}}(\theta_{\phi}) \tag{5.21}
\]

In this way the unpolarized Monte Carlo angular distributions give exactly the detector efficiency functions.

Dividing the differential decay rate of the data by the differential decay rate of the Monte Carlo simulation, the efficiency term disappears, obtaining the efficiency-corrected angular distribution of signal events:
\[
\frac{dN_{\text{data}}}{d \cos \theta} / \frac{dN_{MC}}{d \cos \theta} \propto 1 + \lambda_{\theta} \cos^{2} \theta \tag{5.22}
\]
\[
\frac{dN_{\text{data}}}{d \cos \phi} / \frac{dN_{MC}}{d \phi} \propto 1 + \frac{2\lambda_{\phi}}{3 + \lambda_{\theta}} \cos(2\phi) \tag{5.23}
\]
\[
\frac{dN_{\text{data}}}{d \cos \phi_{\theta}} / \frac{dN_{MC}}{d \phi_{\theta}} \propto 1 + \frac{\sqrt{2}\lambda_{\theta\phi}}{3 + \lambda_{\theta}} \cos \phi_{\theta} \tag{5.24}
\]

Up to now only continuous distributions have been taken into account but the same arguments can be extended to the discrete distributions, provided that the kinematic of the event is completely known. This is the case of the present analysis since the measurement is done in bins of transverse momentum and rapidity of the \( J/\psi \) candidate, and in the two angular variables \( \cos \theta \) and \( \phi \).

Nevertheless the binning can cause two problems: the bin width can be larger than the \( \cos \theta \) and \( \phi \) range in which a significant variation of the detector efficiency is observed; in addition data and Monte Carlo distributions can be slightly different within each bin. On the other hand the bins should be large enough to be sufficiently populated and make the fit feasible.

The \( \lambda_{\theta}, \lambda_{\theta\phi} \) and \( \lambda_{\phi} \) parameters can be extracted from a simultaneous fit to the corresponding efficiency corrected distributions of signal events.

The 1-dimensional background subtracted \( \cos \theta \) and \( \phi \) distributions are showed in Fig. 5.4 in a particular \( p_{T} \) and \( y \) bin. Both distributions are corrected for the efficiency and fitted by the theoretical function defined in eqs. (5.3) and (5.4).

### 5.6.2 Full angular analysis with binned fit

The two-dimensional binned analysis has also been used, to cross check the results of the polarization measurement. The method consists in building the \( \cos \theta \) and \( \phi \) distributions in the signal fiducial region. To extract the signal distribution the background, properly normalized with the parameter \( N \) defined in eq. (5.14), is subtracted from the observed angular decay rate.

To correct for the efficiency the signal distribution is divided by the bi-dimensional efficiency and
detector acceptance, built from the unpolarized Monte Carlo sample:

\[
\frac{d^2 N_{\text{data}}}{d \cos \theta d \phi} \bigg/ \frac{d^2 N_{\text{MC}}}{d \cos \theta d \phi} = f(\cos \theta, \phi; \lambda)
\] (5.25)

The efficiency-corrected distribution is then fitted by the theoretical function defined in eq. (5.1).

### 5.7 Validation of the polarization measurement

All the three methods used for the polarization measurement have been applied to the Monte Carlo sample. Two generated samples are needed. The first sample is unpolarized and it has been used to determine the detection efficiency. The second one plays the same role of the data in the real analysis: from this the \(\lambda_\theta, \lambda_{\theta \phi}\) and \(\lambda_\phi\) parameters are measured.

The latter has been generated with zero and fully transverse polarization, in such a way that the polarization parameters \((\lambda_\theta; \lambda_{\theta \phi}; \lambda_\phi)\) are respectively \((0, 0, 0)\) and \((+1, 0, 0)\). An additional sample, artificially weighted in order to have a completely longitudinal polarization \((-1, 0, 0)\) has been used. For the zero-polarization case the role of the two Monte Carlo samples (the one used for the efficiency and the second one to get the parameters) can be swapped to check the possible presence of bias in the method.

The frame used by the Monte Carlo generator is the laboratory frame, where the polarization axis is chosen to be the \(z\) axis. Thus also the validation is done in the same laboratory frame, to be able to compare the results with the values used in the simulation. The results obtained with DLL method in the unpolarized Monte Carlo are shown in Figs. 5.5, 5.6, 5.7 along with the results obtained with the swapped Montecarlo samples. The results for the transversely polarized Monte Carlo are presented in Figs. 5.8, 5.9 and 5.10. In the first plot, Fig. 5.8 \(\lambda_\theta - 1\) is shown, which should be nominally 0. There seems to be an apparent dependence of the \(\lambda_\theta\) parameter on the
To study this effect the transverse polarized sample has been split into two subsamples and the \( \lambda_\theta \), \( \lambda_{\theta\phi} \) and \( \lambda_\phi \) parameters are extracted again. No dependence is observed in one of the two samples while only a slight dependence is seen in the other, so it can be concluded that the dependence is only an effect due to a statistical fluctuation. The results obtained with the one dimensional binned method are displayed in Figs. 5.11, 5.12 and 5.13 for the unpolarized Monte Carlo sample. From the results obtained with the polarized and unpolarized Monte Carlo it can be concluded that the methods are able to extract the polarization parameters. Similar conclusion can be drawn also for the two-dimensional binned method and looking at the longitudinally polarized Monte Carlo.

Figure 5.5: Result of \( \lambda_\theta \) in the unpolarized Monte Carlo test in HX frame with the DLL method, in the two plots the roles of the two samples are exchanged. Only statistical errors are shown.

Figure 5.6: Result of \( \lambda_{\theta\phi} \) in the unpolarized Monte Carlo test in HX frame with the DLL method, in the two plots the roles of the two samples are exchanged. Only statistical errors are shown.

### 5.8 Systematic uncertainties

In this section the systematic uncertainties associated to the DLL method, which have been taken into account in the measurement, will be presented.
Figure 5.7: Result of $\lambda_\phi$ in the unpolarized Monte Carlo test in HX frame with the DLL method, in the two plots the roles of the two samples are exchanged. Only statistical errors are shown.

Figure 5.8: Result of $\lambda_\theta$ in the fully transversally polarized Monte Carlo test in laboratory frame with the DLL method. The pull distribution is plotted on the right. Only statistical errors are shown.

Figure 5.9: Result of $\lambda_{\theta\phi}$ in the fully transversally polarized Monte Carlo test in laboratory frame with the DLL method. The pull distribution is plotted on the right. Only statistical errors are shown.
Figure 5.10: Result of $\lambda_\phi$ in the fully transversally polarized Monte Carlo test in laboratory frame with the DLL method. The pull distribution is plotted on the right. Only statistical errors are shown.

Figure 5.11: Result of $\lambda_\theta$ in the unpolarized Monte Carlo test in the HX frame with the 1-D binned method. Results are shown also for the test with the swapped Monte Carlo samples (right). Only statistical errors are shown.

Figure 5.12: Result of $\lambda_{\theta\phi}$ in the unpolarized Monte Carlo test in the HX frame with the 1-D binned method. Results are shown also for the test with the swapped Monte Carlo samples (right). Only statistical errors are shown.
5.8.1 Finite Monte Carlo statistics

The normalization in the DLL method is made with the Monte Carlo sample as described in section 5.5.2. In particular the three parameters $a$, $b$ and $c$ introduced in (5.9) are determined with the Monte Carlo sample. Since the statistics of the Monte Carlo is limited, the associated statistical fluctuation must be taken into account.

To do this the bidimensional Monte Carlo distribution in $\cos \theta$ and $\phi$ is built. Using it 100 new Monte Carlo samples (toy Monte Carlo) with the same statistics of the original one are generated. With these new generated samples the normalization is determined and is used to calculate 100 new values for $\lambda_\theta$, $\lambda_{\theta\phi}$ and $\lambda_\phi$. The results for $\lambda_\theta$, $\lambda_{\theta\phi}$ and $\lambda_\phi$ follow very nicely a gaussian distribution, as shown in Fig. 5.14. The sigma of these distributions are taken as systematic uncertainties on the corresponding parameters due to the finite Monte Carlo statistics. This method is used for all the $p_T$ and rapidity bins.

5.8.2 Background subtraction

The error on $\lambda_\theta$, $\lambda_{\theta\phi}$ and $\lambda_\phi$ parameters are estimated from the maximization of the likelihood function, with the TMinuit method [58, 59]. The errors are computed using the one-confidence level of the log likelihood function:

$$lnL(\lambda_{\text{max}} + \Delta \lambda) = lnL(\lambda_{\text{max}}) - \frac{1}{2}.$$  (5.26)

The previous can be compared with the expansion of the log likelihood function at the second order around its maximum. In the hypothesis of a parabolic shape of the log likelihood function:

$$lnL(\lambda) = \lambda^2 \alpha + \lambda \beta + \gamma$$  (5.27)
the variation around the maximum value is $\Delta \lambda = 1/\sqrt{2\alpha}$.

In the DLL method the log likelihood function is weighed in order to subtract the background, so in principle this method is not correct. In the present case the log likelihood function, defined in eq. (5.7) can be written as:

$$
\ln L = \ln L_{\text{sig}} + \ln L_{\text{bkgSR}} - \ln L_{\text{bkgSID}}
$$

(5.28)

where $\ln L_{\text{sig}}$ is the log likelihood built with the signal events, $\ln L_{\text{bkgSR}}$ and $\ln L_{\text{bkgSID}}$ are built respectively with the background events in the signal and sideband regions ($\text{SR}$, $\text{SID}$). While $\ln L_{\text{sig}}$ is parabolic around the maximum, $\ln L_{\text{bkgSR}}$ and $\ln L_{\text{bkgSID}}$ are both linear and, consequently, do not contribute to the uncertainties on the polarization parameters $\lambda_\theta$, $\lambda_{\theta\phi}$ and $\lambda_\phi$. The good linearity can be seen in Fig. 5.15 for a particular $p_T$ and $y$ bin. Quadratic correlation terms are also studied with 2-dimensional log-likelihood distributions and are found to be negligible.

Therefore, the simple error estimation given by the log-likelihood variation around its maximum does not take properly into account the uncertainty introduced by the background subtraction procedure. To take properly into account the contributions due to the background subtraction a toy Monte Carlo is again used. From the $\cos \theta$-$\phi$ distributions built with the background events taken from the sidebands of the mass distribution, two additional terms are generated and included in

Figure 5.14: The distribution of the three $\lambda$ parameters for the 100 fits with the toy MC samples are shown, for an arbitrary bin of the analysis. Each parameter is subtracted by the mean value of the 100 fits and divided by the likelihood uncertainty.
the likelihood functions:

\[ \ln L' = \ln L + \ln L_{bkg1} - \ln L_{bkg2}. \]  

(5.29)

Both terms contain the same amount of events of the sidebands regions.

From the previous \( \ln L \) new sets of \( \lambda_\theta, \lambda_\theta\phi \) and \( \lambda_\phi \) parameters are calculated. This is done 100 times and the distributions of each parameter has a gaussian shape (as shown in Fig. 5.16) and the sigmas of the three distributions are taken as systematic errors associated to the background subtraction.

The procedure is applied in each \( J/\psi \) rapidity and \( p_T \) bin.

![Figure 5.15: \( \log L_{bkg} \) as a function of \( \lambda_\theta, \lambda_\theta\phi \) and \( \lambda_\phi \) around the best fit values.](image)

![Figure 5.16: Distributions of \( \lambda_\theta, \lambda_\theta\phi \) and \( \lambda_\phi \) parameters (subtracted by the nominal results) obtained from the likelihood fit on the 100 toy Monte Carlo samples. The sigma of the Gaussian distribution is taken as the statistical uncertainty due to background subtraction in the likelihood. The plots are shown for 2 GeV/c < \( p_T \) < 3 GeV/c and 3.5 < \( y \) < 4.](image)

### 5.8.3 Background linearity

To subtract the background contribution in the likelihood it is assumed that the background invariant mass distribution is linear.

In addition the fit results for \( \lambda_\theta, \lambda_\theta\phi \) and \( \lambda_\phi \) could depend on the choice of the sideband position. To evaluate the uncertainty due to these assumptions two solutions have been developed.
• Linearity: an exponential model for the background has been tried. In this case the weights are no more ±1 but the ratio between the number of background events respectively in the signal region and in the sideband regions. The differences between the obtained values and the nominal ones are well below the statistical fluctuations so the effect can be neglected.

• Sideband positions: The sidebands have been moved away from the nominal choice $4\sigma$ to $5\sigma$ keeping the $3\sigma$ width. The difference between the obtained values and the nominal ones is quoted as systematic error associated to the background linearity hypothesis.

5.8.4 $J/\psi p_T$ and rapidity binning effects

The Monte Carlo does not reproduce exactly the kinematic distributions of data, in particular the muon transverse momentum. However since the muon spectrum is correlated with the $J/\psi$ kinematic, the binning in $p_T$ and rapidity reduces the amount of discrepancy between data and Monte Carlo spectrum within the bins and the consequent possible variation in the $\lambda_\theta$, $\lambda_\theta\phi$ and $\lambda_\phi$ results. However, to study further this effect a finer binning is made, dividing each $p_T$ and rapidity bin in two, and recomputing in each one of these sub-bins the $\lambda_\theta$, $\lambda_\theta\phi$ and $\lambda_\phi$ parameters. The difference between the average of the results obtained in the sub-bins and the nominal ones is taken as systematic uncertainty due to the binning.

5.8.5 $t_{zs}$ systematic

The events containing prompt $J/\psi$ are selected in the data putting a cut on the pseudo proper time significance, as explained in section 5.3. But since the muons spectra shows some difference between data and Monte Carlo, also the efficiency associated to the $t_{zs}$ cut is different. For this reason a cut on the $t_{zs}$ will select events with different efficiency and the Monte Carlo normalization needed for the DLL method is not properly calculated. Anyway the contribution of the delayed component is just the 3% of the total amount of events in the right side of the pseudo proper time distribution, if a cut at 4 on the significance is kept, and it is almost 0 on the left side (for negative $t_z$ value). Moreover it has been verified that the data $t_{zs}$ distribution is well reproduced by the Monte Carlo if the last one is rescaled for a proper factor, as shown in Fig. 5.17. Then the scaled Monte Carlo can be used to study the cut efficiency, by looking at the ancestors (short or long lived hadrons, prompt $pp$ collisions) of the $J/\psi$ candidate to determine if it comes from a $b$-hadron decay or if it is promptly produced. For what concerns the data distribution, the cut $|t_{zs}| < 4$ must be kept because it is not possible to distinguish the prompt and from $b$ component without reconstructing the full decay chain (the analysis is based on inclusive events).

To evaluate the systematic associated to this cut the pure prompt Monte Carlo sample is used do calculate the normalization of the DLL method. With the same data sample selected by the cut at $|t_{zs}| < 4$ and the new normalization the polarization parameters are computed again. Any differ-
ence from these results and the nominal ones is due to the removed $b$ events and will be quoted as systematic uncertainty.

The last step is to correct this bias for the different cut efficiency between data and Monte Carlo. Naming $\Delta$ the difference between the new and the nominal results and $r_{\text{data}}$, $r_{\text{MC}}$ the two efficiencies in data and Monte Carlo respectively, this is done in the following way:

$$\delta_{\text{Zs}} = \frac{\Delta}{r_{\text{MC}}}$$

which will give the final estimation of the systematic uncertainty.

![Figure 5.17: $t_{\text{Zs}}$ distribution in data and Monte Carlo sample. The Monte Carlo scaled distribution is also plotted.](image)

**5.8.6 Muon identification**

The efficiency of the muon identification cut can show some differences between Monte Carlo and data and this again will carry a systematic uncertainty. The cut on the muon identification is strongly related to the background subtraction: in fact the effect of this cut is to reduce substantially the background. However the difference between Monte Carlo and data in the muon identification cut efficiency is negligible in the analysis range. So no systematic contribution has been given for this effect.

**5.8.7 Bin migration**

Because of the rapidity and transverse momentum resolution, an event can be assigned to the wrong $y$ and $p_T$ bin. The estimation of the events migrating from a bin to another is made considering the Monte Carlo truth. The migration has been found only between neighbour bins and it is
negligible in the rapidity binning, so just the effect on the $p_T$ binning will be taken into account. The amount of migrating events is $\sim 1\%$ in almost all bins with few cases near $2\%$. In data the $p_T$ resolution cannot be determined but it can be estimated from the mass. In fact the measured mass peak width is $\sim 14$ MeV/c$^2$ and, being the natural decay width only $\sim 100$ KeV/c$^2$, it can be considered as a measurement of the kinematic resolution.
Using the Monte Carlo truth the $J/\psi$ binning and the polar angle $\theta$ are determined and used to calculate the normalization for the DLL method. From the same Monte Carlo sample and using the new normalization terms, the $\lambda_\theta$, $\lambda_\theta\phi$ and $\lambda_\phi$ are determined using the reconstructed $p_T$ and rapidity with the nominal binning. Since the Monte Carlo sample is unpolarized any deviation from $0$ is an estimation of the bias due to the full kinematic resolution. To get the bias correspondent to the different resolution between data and Monte Carlo, the observed deviation $\Delta$ must be multiplied by the difference between the resolution in data and Monte Carlo:

$$\delta_{res} = \frac{\sigma_{data} - \sigma_{MC}}{\sigma_{MC}}$$

and this gives the systematic uncertainty.

The estimated uncertainties are found to be well below the statistical fluctuations and are therefore considered negligible.

### 5.8.8 Contamination due to $J/\psi$ from $b$

The cut on pseudo proper time significance does not remove completely the component of delayed $J/\psi$, and a contamination at the level of a few percent (depending on the $J/\psi$ $p_T$ and rapidity) is present in the prompt $J/\psi$ sample. The Monte Carlo is unpolarized for both the prompt and the delayed component but in the data the delayed component will have a different polarization with respect to the prompt one and this fact will introduce a bias in the measurement. To determine the corresponding uncertainty the fraction of $J/\psi$ from $b$ left after the $t_{zs}$ cut is calculated and then corrected for the difference in the selection efficiency between data and Monte Carlo.

The fraction of $J/\psi$ from $b$ is determined from a fit to the $t_{zs}$ distribution as described in section 4.4. The selection efficiency is estimated from the Monte Carlo sample, by looking at the Monte Carlo truth. If $r_p$ and $r_b$ are the selection efficiencies for prompt and $J/\psi$ from $b$ respectively, the fraction of $J/\psi$ from $b$ will be:

$$F_b = \frac{r_b f_b}{r_b f_b + r_p (1 - f_b)}$$

where $f_b$ is the number of $J/\psi$ from $b$ over the total, as estimated from the fit. The contamination with the cut at $t_{zs} = 4$ is shown in Fig. 5.18 as a function of the $J/\psi$ $p_T$.

The bias introduced is evaluated shifting the $t_{zs}$ cut from 4 to 10, to increase the number of accepted $J/\psi$ from $b$ and recalculating the polarization parameters. The difference between these
new results and the nominal ones are propagated to the contamination of $J/\psi$ from $b$ at $t_{zs} = 4$:

$$
\delta_b = \frac{(F_{b,4} - F_{b,10}) \Delta}{F_{b,4}}
$$

(5.33)

which will give the systematic uncertainty.

Figure 5.18: Fraction of $J/\psi$ candidate from $b$-hadrons decays left by the $t_{zs}$ cut.

### 5.8.9 Tracking systematic uncertainty

The uncertainty on the tracking efficiency is estimated for the different $J/\psi$ pseudorapidity and momentum values and it amounts to 2% per muon track. To take into account the difference in the $J/\psi$ spectra between data and Monte Carlo each event is weighted in the Monte Carlo to recompute the normalization of the DLL method. Several models to describe the dependence of the tracking efficiency on the momentum and pseudorapidity have been considered.

The dependence of the tracking efficiency on the pseudo rapidity has been modelled with linear, parabolic and triangular dependence. The functions used as models are chosen in order to simulate a 10%, 4% and 2% inefficiency at the edges of the pseudorapidity range ($2 < \eta < 5$, being $\eta$ the $J/\psi$ pseudorapidity).

To model the tracking efficiency dependence on the transverse momentum a logarithmic function has been chosen, in such a way that at low momentum a 10% inefficiency is simulated.

With each model a per-event weight is computed and assigned to the Monte Carlo sample. The weighted Monte Carlo sample is used to calculate the normalization of the DLL method and, subsequently, new polarization parameters. The difference between these values and the nominal ones is quoted as systematic uncertainty.
5.8.10 Magnet polarity

The analysed sample includes data taken with both magnet polarities. Reverting the magnet polarity has the same effect as taking the $\mu^-$ to compute the polar and azimuthal angles $\theta$ and $\phi$ instead of $\mu^+$. If the negative muon is chosen the polar and azimuthal angles will become respectively $\theta \rightarrow \pi - \theta$ and $\phi \rightarrow \pi + \phi$. The theoretical distribution in eq. (5.1) is invariant under this transformation but the detection efficiency can be affected by some detector asymmetry. Thus a possible systematic uncertainty must be evaluated. To do this the two magnet polarity samples are treated separately and the difference between the obtained polarization parameters is quoted as uncertainty. It has been found that in almost all the analysis bins the uncertainties due to the magnet polarity are well below the statistical fluctuations.

5.9 Results

The values of the fitted parameters $\lambda_\theta, \lambda_{\theta\phi}, \lambda_\phi$ are shown in Figs. 5.19, 5.20 and 5.21 as a function of the transverse momentum $p_T$. Results for different rapidity bins are shown in the same plots.

![Figure 5.19: $\lambda_\theta$ in different $p_T$ bins for 5 rapidity bins. Statistical and systematic uncertainties are added in quadrature.](image)

In Appendix A all the polarization parameters and the calculated uncertainties are listed.

From the plots it can be seen that $\lambda_\phi$ and $\lambda_{\theta\phi}$ are consistent with zero within a few percent. For this reason also the 1-D binned fit method is reliable because as already mentioned in the section 5.3 the possible distortions of the single variable distributions in eqs. (5.3) and (5.4) are strongly suppressed if the two parameters $\lambda_\phi$ and $\lambda_{\theta\phi}$ are close to zero.

Furthermore, the choice of the helicity frame, made at the beginning of the analysis, was a good choice and this frame is the ”natural” frame for measuring the polarization. In fact since $\lambda_\phi \sim 0$, ...
Figure 5.20: $\lambda_{\theta \phi}$ in different $p_T$ bins for 5 rapidity bins. Statistical and systematic uncertainties are added in quadrature.

Figure 5.21: $\lambda_{\phi}$ in different $p_T$ bins for 5 rapidity bins. Statistical and systematic uncertainties are added in quadrature.
the measured $\lambda_\theta$ parameter is a direct measurement of the $J/\psi$ polarization, because the invariant parameter defined in eq. (5.2) is essentially equal to $\lambda_\theta$ itself. Although with a small value, $\lambda_\theta$ results to be negative ($\sim -0.2$), and the $J/\psi$ is found to have a slight longitudinal polarization. This polarization decreases with increasing $p_T$ and rapidity $y$. The results can be compared with the ones obtained by other experiments. For what concerns the $\lambda_\theta$ parameter also CDF [22], PHENIX [60] and HERA-B [23] obtain a slightly longitudinal polarization but given the different ranges of $J/\psi$ rapidity and transverse momentum and the different collisions energy it’s not possible to draw any conclusion concerning the dependence on $p_T$, especially at high values where the statistics is limited. For what concerns the other parameters only HERA-B measured $\lambda_\phi$ and $\lambda_{\theta\phi}$, obtaining results very close to zero, consistently with LHCb values.

In Figs. 5.22, 5.24, 5.23 the results for the three parameters $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$ are shown integrated over the rapidity range between 2 and 4.5. The error bars represent the statistic uncertainties while the boxes are the statistical and systematics uncertainties added in quadrature.

Figure 5.22: $\lambda_\theta$ in different $p_T$ bins integrated over the rapidity range. Error bars represent the statistical uncertainties while the boxes are the statistical and systematic uncertainties added in quadrature.

In Figs. 5.25, 5.26 and 5.27 the results obtained with the DLL method and those obtained using the two alternative binned fits, are compared in the more significative region ($2.5 < y < 4$) where the statistical sensitivity is higher. Except for few single points, the agreement between the three methods is quite good. Due to the poor statistics in the first and the last rapidity bins the two binned methods give problems in the fit, yielding high $\chi^2$ values. For these reasons the two methods are not taken into account for the comparison in these rapidity regions.
Figure 5.23: $\lambda_{\theta\phi}$ in different $p_T$ bins integrated over the rapidity range. Error bars represent the statistical uncertainties while the boxes are the statistical and systematic uncertainties added in quadrature.

Figure 5.24: $\lambda_\phi$ in different $p_T$ bins integrated over the rapidity range. Error bars represent the statistical uncertainties while the boxes are the statistical and systematic uncertainties added in quadrature.
Figure 5.25: Results of $\lambda_\theta$ obtained with the DLL, 2-D and 1-D binned methods for the three central rapidity bins. Only statistical errors are shown.
Figure 5.26: Results of $\lambda_{\theta\phi}$ obtained with the DLL, 2-D and 1-D binned methods for the three central rapidity bins. Only statistical errors are shown.
Figure 5.27: Results of $\lambda_\phi$ obtained with the DLL, 2-D and 1-D binned methods for the three central rapidity bins. Only statistical errors are shown.
5.10 Comparison with ALICE experiment

The ALICE collaboration studied the $J/\psi$ polarization in the $pp$ collisions at $\sqrt{s} = 7$ TeV, looking at its decay into a muon pair [61]. The polar and azimuthal angles $\theta$ and $\phi$ have been obtained in the helicity frame and their 1-D distributions have been studied to extract the $\lambda_{\theta}$ and $\lambda_{\phi}$ parameters (since only the $\cos \theta$ and $\phi$ single variable distributions have been studied in the ALICE analysis the third parameter $\lambda_{\theta \phi}$ has not been measured). The measurement has been performed in bins of $J/\psi$ transverse momentum and integrating on the rapidity in a kinematic range very similar to the one of LHCb, being $2.5 < y < 4$ and $2$ GeV/$c < p_T < 8$ GeV/$c$. Thus a comparison with the LHCb results is possible. In Fig. 5.28 the ALICE results obtained for $\lambda_{\theta}$ and $\lambda_{\phi}$ are compared the LHCb results in the three central rapidity bins for $2.5 < y < 4$, which is the same rapidity range of the ALICE analysis. The error bars correspond to the statistical uncertainties while the boxes are the sum in quadrature of the statistical and systematic errors. As it can be seen from the plots the results are in good agreement for both the $\lambda_{\theta}$ and $\lambda_{\phi}$ parameters.

Figure 5.28: Results of $\lambda_{\theta}$ and $\lambda_{\phi}$ obtained by the LHCb and ALICE collaborations in the rapidity range $2.5 < y < 4$. The results obtained by LHCb in the three central rapidity bins are shown in different colors. The error bars correspond to the statistical uncertainties while the boxes are the sum in quadrature of the statistical and systematic errors.
Chapter 6

J/ψ cross section measurement

To better understand the quarkonium production mechanism it is of fundamental importance to measure also the J/ψ production cross section.

A measurement of the J/ψ production cross section has been published by LHCb based on data collected in 2010 [63]. In this early analysis the J/ψ was supposed unpolarized and a large systematic uncertainty was assigned to the measured cross section due to the unknown J/ψ polarization. In addition large systematic errors were affecting the result due to the knowledge of the tracking efficiency and to the error on the absolute luminosity.

In this work a new analysis of the 2010 data is described where the J/ψ production cross section is now measured taking into account the results on the polarization discussed in chapter 5 and profiting of a much better understanding of the tracking systematics and of a smaller error on the luminosity that were the largest sources of uncertainty in the previous measurement.

The cross section measurement is made in 5 bins of J/ψ rapidity in the range 2 < y < 4.5 and 14 bins of J/ψ transverse momentum in the range 0 < p_T < 14 GeV/c and is also measured separately for the prompt J/ψ and for the component coming from b decays. The latter is finally used to obtain the total pp → b¯bX production cross section.

6.1 Event selection

This study is based on 5.2 pb⁻¹ of pp collision data taken at a centre-of-mass energy of 7 TeV in September 2010. The data were collected using three trigger configurations, named TCKs (Trigger Configuration Keys), which are three sequences of algorithms defining the trigger lines used to select the events. The three TCKs are identical for what concerns the muon trigger and the lines (and correspondent cuts) are those listed in section 4.1. Each TCK is identified by a 32 bit hexadecimal value as listed below:

- TCK 0x001D0030;
- TCK 0x001E0030;
• TCK 0x001F0029.

In the third TCK the single muon line at HLT1 is prescaled by factor 0.2: this means that only 1/5 of the events passing this line are accepted by the trigger. Because of this prescaling, a different efficiency will be computed for the 0x001D0030, 0x001E0030 and for the 0x001F0029 TCKs. The data used in this analysis were collected with high pile-up beam conditions (see section 2.3). In these conditions very high multiplicity events can occur that cause a high occupancy, especially in the Vertex Locator and in the Outer Tracker. These events generate dead time at HLT level because of the long processing time needed to analyse them. To avoid this, a set of Global Events Cuts are applied at the Level0 trigger on the multiplicity, mainly on the subdetectors involved in the pattern recognition, which are the VELO and the tracking stations. A cut is also applied on the multiplicity of the hits in the Scintillating Pad Detector (SPD) whose information is used by the L0 trigger. The list of Global Event Cuts is shown in Tab. 6.1.

<table>
<thead>
<tr>
<th>trigger level</th>
<th>GEC</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0</td>
<td># SPD hits</td>
<td>&lt; 900</td>
</tr>
<tr>
<td>HLT1</td>
<td># Velo clusters</td>
<td>&lt; 3000</td>
</tr>
<tr>
<td>HLT1</td>
<td># IT clusters</td>
<td>&lt; 3000</td>
</tr>
<tr>
<td>HLT1</td>
<td># OT clusters</td>
<td>&lt; 10000</td>
</tr>
<tr>
<td>HLT2</td>
<td># Velo tracks</td>
<td>&lt; 350</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of Global Event Cuts (GEC).

The events accepted by the trigger are further filtered by the list of cuts described in section 4.2. The trigger configuration and performance for the 2010 data taking are described in [62].

6.2 Cross section measurement

The double differential cross section value in each \( p_T \) and rapidity bin is defined as

\[
\frac{d^2\sigma}{dy dp_T} = \frac{N(J/\psi \rightarrow \mu^+ \mu^-)}{\mathcal{L} \times \varepsilon_{\text{tot}} \times B(J/\psi \rightarrow \mu^+ \mu^-) \times \Delta y \times \Delta p_T}
\]  

(6.1)

where \( N(J/\psi \rightarrow \mu^+ \mu^-) \) is the number of signal events in the examined \( (p_T; y) \) bin, \( \varepsilon_{\text{tot}} \) is the total \( J/\psi \) detection efficiency (including the acceptance, the reconstruction and selection, trigger efficiencies). \( \mathcal{L} \) is the integrated luminosity of the data sample used for the analysis, \( B(J/\psi \rightarrow \mu^+ \mu^-) \) is the branching ratio of the \( J/\psi \rightarrow \mu^+ \mu^- \) decay, which is provided by the Particle data Group [12] \( B(J/\psi \rightarrow \mu^+ \mu^-) = (5.94 \pm 0.06) \times 10^{-2} \). The widths of the \( p_T \) and rapidity bins are respectively \( \Delta y = 0.5 \) and \( \Delta p_T = 1 \text{ GeV}/c \).
6.3 Number of signal events

The number of signal events $N(J/\psi \to \mu^+\mu^-)$ in each analysis bin has been determined with an unbinned fit to the invariant mass distribution of the $\mu^+\mu^-$ pairs, in the range $[2950; 3300]$ GeV/$c^2$. The signal peak and the background are described respectively by a Crystal Ball and an exponential function, as described in section 4.3. The fit is performed in each $p_T$ and $y$ bin of the analysis.

It has been observed a strong dependence of the Crystal Ball parameters $a$, $n$ on the width $\sigma$ of the mass peak. Thus their values have been extracted from the Monte Carlo simulation as follows. The Monte Carlo mass distribution is smeared with a gaussian function with width taking values within $[7; 30]$ MeV/$c^2$. Then the smeared distribution is fitted with the Crystal Ball function fixing the $\sigma$ parameter to the corresponding smearing value. The dependence of $a$ and $n$ with respect to the $\sigma$ value is then obtained and it is shown in Fig. 6.1. The two curves are parametrized with two second order polynomials $a = 2.22 + 0.004\sigma + 0.001\sigma^2$ and $n = 1.04 + 0.041\sigma + 0.002\sigma^2$. These parametrizations are then used in the fit to the data mass distributions. The total signal yield is obtained summing the numbers of signal events of the $14 \times 5$ bins of the analysis and lead to $N_{signal} = 564603 \pm 924$, where the error is statistical. An example of the fit to the mass distribution is shown in Fig. 6.2 for two particular $p_T$ bins, with $1 \text{ GeV}/c < p_T < 2 \text{ GeV}/c$ and $2 \text{ GeV}/c < p_T < 3 \text{ GeV}/c$, and for $3 < y < 3.5$.

6.4 Disentangle the prompt and from $b$ components

The production cross section is measured separately for the prompt and from $b$ $J/\psi$ components. The number of signal events is determined for each component performing a combined fit to the pseudo proper time and to the invariant mass distributions, as described in section 4.4. The fit is made independently for each of the $14 \times 5$ analysis bins. An example is shown in Fig. 6.3.
Figure 6.2: Example of the fit to the mass for two particular bins, with $1 \text{ GeV/c} < p_T < 2 \text{ GeV/c}$ or $2 \text{ GeV/c} < p_T < 3 \text{ GeV/c}$ and $3 < y < 3.5$.

Figure 6.3: Example of the fit to the $t_z$ for a particular bin, with $3 \text{ GeV/c} < p_T < 4 \text{ GeV/c}$ and $2.5 < y < 3$. 
6.5 Luminosity measurement

The integrated luminosity of the analysed sample is measured with the Van der Meer scans \[66\] and with the beam-gas imaging method (BGI) \[64\], in specific periods during the data taking. The Van der Meer method consists in measuring the number of visible pp interactions $\mu_{vis}$ in beam-beam crossing, as a function of the position offsets of the two colliding beams. This is made performing a scan in both the horizontal and vertical directions. From $\mu_{vis}$ the effective cross section $\sigma_{vis}$ is calculated. During the 2010 data taking two scans have been performed during a single fill. The uncertainty, $\sim 10\%$, is mainly due to the beam currents knowledge. The two measurements gave the same value within the error.

In the BGI method the position of the vertices from the interactions between the beam and the residual gas in the machine or between the beam-beam interactions are determined from the reconstructed tracks in LHCb detector. The residual gas consists mainly of light elements such as hydrogen, carbon and oxygen in the beam vacuum pipe. The measurement of the distribution of these vertices allows to determine an image of the transverse bunch profile along the beam direction. The beam angles, profile and relative position are then measured from the bunches images. With the beam-image method, the measurements were taken during six different periods of stable running of LHC using the beam-gas and beam-beam interactions, with the analysis described in \[65\]. The measurements taken with both methods are consistent, within the $10\%$ uncertainty.

The measurements have been subsequently updated to reduce the uncertainty \[67\]. This has been possible thanks to an improved vertex resolution and analysis tools and a better calibration of the bunch current transformers, for the measurements of the bunch intensities. Besides the uncertainty also the integrated luminosity central value has been updated with respect to the previous measurements. The new integrated luminosity for each of the data taking periods with the three different TCKs is given in Tab. 6.5 with the correspondent uncertainty and the total value.

<table>
<thead>
<tr>
<th>TCK</th>
<th>$\mathcal{L}$ (pb$^{-1}$)</th>
<th>err. (pb$^{-1}$)</th>
<th>Rel. err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D0030</td>
<td>0.688</td>
<td>0.024</td>
<td>3.5 %</td>
</tr>
<tr>
<td>1E0030</td>
<td>1.610</td>
<td>0.056</td>
<td>3.5 %</td>
</tr>
<tr>
<td>1F0029</td>
<td>3.193</td>
<td>0.112</td>
<td>3.5 %</td>
</tr>
<tr>
<td>All</td>
<td>5.491</td>
<td>0.192</td>
<td>3.5 %</td>
</tr>
</tbody>
</table>

6.6 Efficiency calculation

The total efficiency is the product of three terms: the detector acceptance $\varepsilon_{acc}$, the detection, reconstruction, selection efficiency $\varepsilon_{rec}$ and the $L0 \times HLT1 \times HLT2^1$ trigger efficiency $\varepsilon_{trig}$:

$$\varepsilon_{tot} = \varepsilon_{acc} \times \varepsilon_{rec} \times \varepsilon_{trig}$$  \hspace{1cm} (6.2)

$^1$The product sign X stands for the logical AND of the various trigger conditions.
where each of the three terms is evaluated using a $J/\psi$ inclusive unpolarized Monte Carlo sample. The first term is the generator level detector acceptance and is defined as:

$$
\varepsilon_{\text{acc}} = \frac{\text{Events in LHCb acceptance in each } p_T \text{ and } y \text{ bin}}{\text{Generated events in each } p_T \text{ and } y \text{ bin}}. \quad (6.3)
$$

It is the ratio between the number of $J/\psi$ candidates with both muons contained in the LHCb acceptance over the total number of generated events, in each $(p_T; y)$ bin. The request for a muon to be in the LHCb acceptance means that it must have the momentum direction in $[10, 400]$ mrad before the magnet. To avoid possible event losses due to the magnetic field, the acceptance is enlarged a bit with respect to the nominal one, $[15, 350]$ mrad. The Monte Carlo sample used for the computation of the acceptance is generated with the $J/\psi$ momentum contained in the forward region $z > 0$.

The second term includes the detection, reconstruction and selection efficiencies:

$$
\varepsilon_{\text{rec}} = \frac{\text{Reconstructed events in each } p_T \text{ and } y \text{ bin}}{\text{Events in LHCb acceptance in each } p_T \text{ and } y \text{ bin}}. \quad (6.4)
$$

It is defined as the number of events detected, reconstructed by the reconstruction algorithms and passing the selection cuts over the number of events in acceptance, in each $(p_T; y)$ bin. This term contains also the efficiency of matching the reconstructed $J/\psi$ candidate with the generated $J/\psi \rightarrow \mu^+\mu^-$ decay, asking for the Monte Carlo truth (which is $\sim 99.9\%$).

The last term is the $L0 \times HLT1 \times HLT2$ efficiency:

$$
\varepsilon_{\text{trig}} = \frac{\text{Triggered events in each } p_T \text{ and } y \text{ bin}}{\text{Reconstructed events in each } p_T \text{ and } y \text{ bin}}. \quad (6.5)
$$

given by the ratio of the number of events accepted by the three trigger steps over the number of reconstructed events, in each $(p_T; y)$ bin.

For each efficiency term it has been checked that it is the same for the prompt and from $b$ component, within the statistical errors.

### 6.6.1 Polarization correction

The polarization leads to a distortion in the angular distribution of the two muons, which is reflected in a distortion of the efficiency distribution as a function of the polar angles. Since the three efficiency terms $\varepsilon_{\text{acc}}, \varepsilon_{\text{rec}}$ and $\varepsilon_{\text{trig}}$ are evaluated using an unpolarized Monte Carlo sample, they have to be corrected for the polarization.

To do this each event is weighted with the angular distribution of the muons produced in the $J/\psi$ decay normalized to 1, in the following way: for each event the reconstructed polar and azimuthal angles $\theta$ and $\phi$ in the helicity frame are computed and the normalised angular distribution
of the two muons is evaluated for these $\theta$ and $\phi$ values:

$$f_P = \frac{d^2 N}{d \cos \theta d\phi} = \left(\frac{3}{4\pi(3 + \lambda_\theta)}\right) \left(1 + \lambda_\theta \cos^2 \theta + \lambda_{\theta\phi} \sin 2\theta \cos \phi + \lambda_{\phi} \sin^2 \theta \cos 2\phi\right)$$  \hspace{1cm} (6.6)

The efficiencies can be written as:

$$\varepsilon_{\text{acc}} = \frac{\sum_{i=1}^{N_{\text{ACC}}} f_P(\theta, \phi)}{\sum_{j=1}^{N_{\text{GEN}}} f_P(\theta, \phi)}$$  \hspace{1cm} (6.7)

$$\varepsilon_{\text{rec}} = \frac{\sum_{i=1}^{N_{\text{REC}}} f_P(\theta, \phi)}{\sum_{j=1}^{N_{\text{ACC}}} f_P(\theta, \phi)}$$  \hspace{1cm} (6.8)

$$\varepsilon_{\text{trig}} = \frac{\sum_{i=1}^{N_{\text{TRIG}}} f_P(\theta, \phi)}{\sum_{j=1}^{N_{\text{REC}}} f_P(\theta, \phi)}$$  \hspace{1cm} (6.9)

where $N_{\text{GEN}}$, $N_{\text{ACC}}$, $N_{\text{REC}}$ and $N_{\text{TRIG}}$ are respectively the number of generated events, events in LHCb acceptance, reconstructed and triggered events.

Since the binning of the polarization measurement is different from the one used for the cross section analysis the correction is applied in the following way:

- no correction is applied for $p_T < 2$ GeV/c;
- the cross section calculated in each $p_T$ bin between 2 and 5 GeV/c is corrected using the $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$ evaluated in the correspondent $p_T$ bin.
- the cross section calculated in both the $p_T$ bins between 5 and 7 GeV/c is corrected using the $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$ evaluated between 5 and 7 GeV/c.
- the cross section in each $p_T$ bin between 7 and 10 GeV/c is corrected using the $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$ evaluated between 7 and 10 GeV/c.
- the cross section in each $p_T$ bin between 10 and 15 GeV/c is corrected using the $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$ evaluated between 10 and 15 GeV/c.

### 6.6.2 Polarization systematic error

To propagate the uncertainties on the $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$ parameters on the efficiency determination (which eventually reflects in an error on the cross section measurement), a toy Monte Carlo is employed. For each bin of $J/\psi$ rapidity and transverse momentum 100 new sets of $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$ parameters are generated according to a gaussian distribution centered on the measured values. This has to be done separately for the correlated and uncorrelated uncertainties on the three polarization parameters, listed in Tabs. A.1[A.2] A.3[A.4] and A.5. The uncertainties on the $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$ coming from the maximization of the likelihood function are correlated while the statistical
uncertainties related to the Monte Carlo and the systematics can be safely considered uncorrelated. While for uncorrelated uncertainties, single disjoint gaussian distributions can be used to generate the three parameters, for the correlated errors a full three-dimensional gaussian distribution must be used:

\[
P = \frac{1}{(2\pi)^{3/2} \sqrt{|V|}} \cdot e^{\frac{1}{2}(\lambda - \bar{\lambda})^T V^{-1}(\lambda - \bar{\lambda})}
\]

(6.10)

where \( \bar{\lambda} = (\lambda_\theta, \lambda_\phi, \lambda_{\theta\phi}) \). The covariance matrix \( V \) associated to the correlated uncertainties is given by the likelihood fit, while for the uncorrelated uncertainties it is a diagonal matrix containing the sum in quadrature of all the contributions:

\[
V = \begin{pmatrix}
cov(\lambda_\theta, \lambda_\theta) & cov(\lambda_\theta, \lambda_{\theta\phi}) & cov(\lambda_\theta, \lambda_\phi) \\
cov(\lambda_{\theta\phi}, \lambda_\theta) & cov(\lambda_{\theta\phi}, \lambda_{\theta\phi}) & cov(\lambda_{\theta\phi}, \lambda_\phi) \\
cov(\lambda_\phi, \lambda_\theta) & cov(\lambda_\phi, \lambda_{\theta\phi}) & cov(\lambda_\phi, \lambda_\phi)
\end{pmatrix}
\]

(6.11)

With each set of generated parameters the efficiencies are calculated and a distribution is built whose RMS is taken as systematic uncertainty due to the three polarization parameters. The binomial uncertainty, due to the finite number of Monte Carlo events must also be calculated for the three efficiencies, in the following way:

\[
\sigma_{\text{acc}}^{\text{stat}} = \sqrt{\frac{\varepsilon_{\text{acc}}(1 - \varepsilon_{\text{acc}})}{N_{\text{GEN}}}}
\]

\[
\sigma_{\text{rec}}^{\text{stat}} = \sqrt{\frac{\varepsilon_{\text{rec}}(1 - \varepsilon_{\text{rec}})}{N_{\text{ACC}}}}
\]

\[
\sigma_{\text{trig}}^{\text{stat}} = \sqrt{\frac{\varepsilon_{\text{trig}}(1 - \varepsilon_{\text{trig}})}{N_{\text{REC}}}}
\]

(6.12)

(6.13)

Thus the total uncertainty on each efficiency term is given by the sum in quadrature of the three terms:

\[
\sigma_{\varepsilon_{\text{acc}}} = \sqrt{(\sigma_{\varepsilon_{\text{acc}}}^{\text{corr}})^2 + (\sigma_{\varepsilon_{\text{acc}}}^{\text{uncorr}})^2 + (\sigma_{\varepsilon_{\text{acc}}}^{\text{stat}})^2}
\]

(6.14)

\[
\sigma_{\varepsilon_{\text{rec}}} = \sqrt{(\sigma_{\varepsilon_{\text{rec}}}^{\text{corr}})^2 + (\sigma_{\varepsilon_{\text{rec}}}^{\text{uncorr}})^2 + (\sigma_{\varepsilon_{\text{rec}}}^{\text{stat}})^2}
\]

(6.15)

\[
\sigma_{\varepsilon_{\text{trig}}} = \sqrt{(\sigma_{\varepsilon_{\text{trig}}}^{\text{corr}})^2 + (\sigma_{\varepsilon_{\text{trig}}}^{\text{uncorr}})^2 + (\sigma_{\varepsilon_{\text{trig}}}^{\text{stat}})^2}
\]

(6.16)

where \( \sigma_{\varepsilon_{\text{corr}}}^i \) and \( \sigma_{\varepsilon_{\text{uncorr}}}^i \) \((i = \text{acc, rec, trig})\) are respectively the propagated correlated and uncorrelated uncertainties on the polarization measurement, determined with the toy Monte Carlo.

**6.6.3 Efficiency results**

In Figs. 6.4, 6.5, 6.6 and 6.7 the results for acceptance, reconstruction and trigger efficiency are shown, corrected for the polarization distortion. The TCK 0x001F0029 is prescaled at the HLT1 single muon line, so the correspondent trigger efficiency \( \varepsilon_{\text{pre}} \) saturates at 50%, unlike the trigger
efficiencies $\varepsilon_{\text{unpresc}}$ for the TCK $\times$001D0030 and $0\times$001E0030. Almost no difference is observed in the reconstruction and trigger efficiencies, after applying the polarization correction, while the acceptance efficiency is slightly higher than in the zero-polarization hypothesis. No correction is applied to the first two $p_T$ bins. The drop in the reconstruction efficiency at 3 GeV/c is caused by the selection cut on the muon transverse momentum, shown in section 4.2.

Finally the total efficiencies $\varepsilon_{\text{unpresc}}$ and $\varepsilon_{\text{presc}}$ for the two TCKs are shown in Fig. 6.8. The total efficiency $\varepsilon_{\text{tot}}$ used to calculate the cross section is obtained averaging the two efficiencies $\varepsilon_{\text{unpresc}}$ and $\varepsilon_{\text{presc}}$, weighted with the integrated luminosity corresponding to the prescaled and unprescaled TCKs:

$$\varepsilon_{\text{tot}} = \frac{L_{\text{unpresc}}}{L_{\text{TOT}}} \times \varepsilon_{\text{unpresc}} + \frac{L_{\text{presc}}}{L_{\text{TOT}}} \times \varepsilon_{\text{presc}}$$  \hspace{1cm} (6.17)

where $L_{\text{unpresc}}$ and $L_{\text{presc}}$ are respectively the integrated luminosity of the two samples taken with TCKs $\times$001D0030 plus $0\times$001E0030 and TCK $\times$001F0029. The result is shown in Fig. 6.9.

### 6.7 Systematic uncertainties

Several possible sources of systematic uncertainties have been studied, to evaluate the effects on the final result. In the following paragraphs they will be described in details. For each contribution
Figure 6.5: Monte Carlo-based reconstruction efficiency $\varepsilon_{\text{rec}}$ for prompt $J/\psi$, corrected for the polarization effect, in bins of $y$.

Figure 6.6: Monte Carlo-based trigger efficiency $\varepsilon_{\text{trig}}$ for TCK $0 \times 001D0030$ and for prompt $J/\psi$, corrected for the polarization effect, in bins of $y$. A similar efficiency distribution can be obtained for TCK $0 \times 001E0030$. 
Figure 6.7: Monte Carlo-based trigger efficiency $\varepsilon_{\text{trig}}$ for TCK 0×001F0029 and for prompt $J/\psi$, corrected for the polarization effect, in bins of $y$.

Figure 6.8: Total efficiency for not-prescaled (left) and prescaled (right) TCK.
it is specified if the uncertainty is correlated between the analysis $p_T$ and rapidity bins or not.

6.7.1 Systematic on tracking efficiency

The tracking efficiency contains three different contributions. The first one comes from the discrepancy between the data and Monte Carlo efficiency distributions. The ratio between the data and Monte Carlo efficiencies (relative efficiency) has been evaluated in different muon momentum $p$ and pseudorapidity $\eta$ bins. The second and third sources come from the correlated systematic error associated to each track and the uncertainty in the material interaction inside the detector: the two contributions are estimated to be respectively 0.7% and 2%.

In the first measurement of $J/\psi$ cross section [63] a 4% systematic contribution was assigned to each muon track, due to the tracking uncertainty. This led to a resulting 8% contribution for the two muons tracks. With the reduction of the luminosity and the polarization uncertainty (thanks to the polarization correction) the tracking error became the dominant source of systematic. Improved estimates of the relative tracking efficiency between data and Monte Carlo allowed to recompute the uncertainty. This has been done in the following way. Starting from the relative efficiency values between data and Monte Carlo, for each $p$ and $\eta$, a smeared efficiency table has been generated with a toy Monte Carlo: each generated efficiency table correspond to a different simulated experiment (1000 in total). For each simulated experiment all the events in the Monte Carlo sample are weighted with the relative efficiency according to the muon track $p$ and $\eta$. Then the overall efficiency of the experiment is determined averaging on all the events and a distribution is made. The RMS of this distribution gives the uncertainty due to the discrepancy between data

![Graph showing total efficiency $\varepsilon_{\text{tot}}$.](graph.png)

Figure 6.9: Total efficiency $\varepsilon_{\text{tot}}$. 

...
The total systematic associated to the tracking is found to be 1.5% for each track, which means 3% for the two muons tracks. The errors are correlated between each analysis bin.

6.7.2 Trigger efficiency

Since the trigger efficiency is determined using an unpolarized Monte Carlo sample, the systematic uncertainty is given by the comparison between the data and the simulation. The trigger efficiency in the data is computed using a particular trigger unbiased event sample. The sample contains events which would be triggered also if the $J/\psi$ candidate were removed (Trigger Independent of Signal, or TIS). The TIS events used to calculate the efficiency are triggered with respect at least to one trigger line. The numerator of the efficiency is the number of events Triggered On the Signal (TOS), i.e. asking explicitly for the $J/\psi$ candidate. Thus the efficiency in each $(p_T; y)$ bin is given by:

$$\varepsilon_{\text{trig data}} = \frac{\text{TIS&TOS events in each } p_T \text{ and } y \text{ bin}}{\text{TIS events in each } p_T \text{ and } y \text{ bin}} \quad (6.18)$$

To determine the number of events in each $p_T$ and $y$ bin a fit to the mass distribution is performed, as described in section 4.3.

The comparison between the trigger efficiency evaluated in data and Monte Carlo is shown in Fig.[6.10] for the unprescaled (right) and prescaled (left) TCKs as a function of the $J/\psi$ transverse momentum and in each rapidity bin. The systematic uncertainty is then given by the difference between the efficiency calculated in the data and in the Monte Carlo distribution.

6.7.3 Global Event Cuts systematic

The effect of the Global Event Cuts can be evaluated in the data sample with respect to a particular independent trigger, which accepts all the events having just a track segment reconstructed in the Vertex Locator. The only cut which significantly affect the efficiency is the requirement on the
Figure 6.11: Muon tracks normalized $\chi^2$ distribution in data (black squares) and Monte Carlo (red circles) sample (left). On the right the efficiency of the tracks normalized $\chi^2$ cut is shown: the selection requirement is at 4.

VELO clusters, listed in Tab. 6.1, while the effect of the other cuts, on the SPD multiplicity and VELO tracks is found to be negligible. Comparing the Monte Carlo and data efficiency a 2\% difference has been found and is quoted as systematic uncertainty. This uncertainty is correlated between the analysis bins.

### 6.7.4 Muons tracks $\chi^2$ fit

The signal selection applies a cut on the $\chi^2$ of the fit of the muon tracks, normalized to the number of degrees of freedom. Thus the difference in the data and Monte Carlo should be taken into account as systematic uncertainty. In Fig. 6.11 (left plot) the distribution in data and Monte Carlo sample of the $\chi^2$ is shown, normalized to the same area for comparison and superimposed. For the data the signal distribution is extracted using the sPlot technique, described in Appendix C. In Fig. 6.11 (right plot) the efficiency of this cut is shown, for data and Monte Carlo sample. The relative difference of 0.5\% can be quoted as systematic error, correlated between the bins of the analysis.

### 6.7.5 Vertex $\chi^2$ systematic uncertainty

The selection requires also a cut on the $\chi^2$ associated to the $J/\psi$ decay vertex fit, normalized to the number of degrees of freedom. The distribution in data and Monte Carlo sample are both shown in Fig. 6.12 normalized to the same area for comparison. The signal distribution in data is extracted with the sPlot technique. The data and Monte Carlo efficiencies of the cut on this variable are also shown on the right histogram, as a function of the logarithm of the $\chi^2$ probability. At the cut value used in the selection, the difference between the two efficiencies is 1.6\%. Thus the Monte Carlo efficiency is decreased of 1.6\% and half of the correction 0.8\% is quoted as systematic uncertainty. Also in this case the uncertainty is correlated between the analysis bins.
Figure 6.12: On the left the $J/\psi$ decay vertex $\chi^2$ distribution normalized to the number of degrees of freedom, is shown in data (black squares) and Monte Carlo (red circles) sample. On the right the efficiency of the vertex normalized $\chi^2$ cut is shown: the selection requirement is on the $\chi^2$ probability $P(\chi^2) > 0.5\%$, corresponding to $\log_{10}[P(\chi^2)] > 2.3\%$.

### 6.7.6 Mass fit uncertainty

A systematic uncertainty could come from the fit to the invariant mass distribution fit. There are two possible sources of systematic related to this item.

- The number of signal events entering in the cross section formula could depend on the choice of the function used to model the signal peak. A sum of two Crystal Ball functions is also used to perform the fit: the total number of signal events got with this choice is $558338 \pm 1422$. This number is compared to the nominal number $564603 \pm 924$ and the 1\% relative difference is quoted as systematic error.

- The number of signal events can be lower due to the loss of the events in the radiative tail. With Monte Carlo studies the number of events in the tail outside the mass distribution range (below 2.7 GeV/$c^2$), not counted by the fit, is estimated to be 2\% of the signal. Thus the number is corrected by 2\% and a 1\% uncertainty is assigned as systematic.

Both contributions coming from the mass distribution fit are correlated between the analysis bins.

### 6.7.7 Unknown $J/\psi$ spectrum

The total efficiency is averaged within each $J/\psi$ $p_T$ and $y$, thus a contribution to the systematic could come from the unknown $J/\psi$ spectrum. To evaluate the uncertainty each $p_T$ and $y$ bin is divided in two. In this way the $p_T$ and $y$ ranges are divided in bins of 500 MeV/$c$ and 0.25 respectively. The efficiency is computed in each sub-bin and the average is compared with the nominal value in the correspondent bin. The relative efficiency is taken as systematic error.
6.7.8 Pseudo proper time fit

A possible source of systematic uncertainty comes from the fit to the pseudo proper time distribution. This can be studied looking at the most populated analysis bin, in $2.5 < y < 3$ and $3 \text{ GeV}/c < p_T < 4 \text{ GeV}/c$.

The central value $\mu$ of the prompt distribution estimated by the fit is different from zero, probably because of a wrong description of the background close to this value. The fit is repeated fixing $\mu$ to two extreme values $\mu = \pm 3 \text{ fs}$ and calculating the number of events from $b$. The variation with respect to the nominal number is $3.6\%$ and is quoted as systematic uncertainty.

Another possible contribution could come from the description of the long tail, due to the assignment of the wrong primary vertex to the $J/\psi$ candidate. The next-event method, used to model the tail, assumes that the reconstruction efficiency $\varepsilon_{PV}$ of the primary vertex does not depend on its position. The efficiency has been studied with the Monte Carlo and in the data sample. A small dependence on the $z$ coordinate of the primary vertex position has been found in the first one and this can cause an asymmetry in the tail. The variation of the number of $J/\psi$ from $b$ with this asymmetry is measured by redoing the fit using the tail simulated with Monte Carlo sample. The variation is found to be small and thus this effect can be neglected in the systematic estimation.

6.7.9 Muon identification

The muon identification efficiency calculated in the Monte Carlo sample can be cross-checked with the data, to estimate a possible source of uncertainty. The efficiency in the data can be evaluated using the tag and probe procedure: the $J/\psi$ candidate is reconstructed from a first muon track identified by the muon system (tag) and a second track which has left a deposit of minimum ionizing particle in the calorimeter system (probe). To avoid possible bias the second track is independent from the muon trigger lines. Thus the efficiency is computed as function of the transverse momentum of the probe muon. Comparing the data and Monte Carlo results, a $97\%$ correction factor has been found and applied to the efficiency. Then a $1.1\%$ per track systematic uncertainty is assigned to the measurement. The uncertainty is correlated between bins of the analysis. This value is a combination of various contribution, coming from the statistical error due to the finite statistics of the Monte Carlo sample, the background estimation, the different efficiencies of the calorimeter selection criteria. Minor bias can be introduced also by the trigger and the dependence of the correction applied to the efficiency on the muon transverse momentum.

6.7.10 Bin migration

As for the polarization measurement, because of the $J/\psi \ p_T$ and rapidity resolution an event could be assigned to the wrong bin. The resolution is calculated using the Monte Carlo sample: it has been estimated $12.7 \pm 0.2 \text{ MeV}/c$ for the transverse momentum and $(1.4 \pm 0.1) \times 10^{-3}$ for the rapidity. Compared to the bin width, the $y$ resolution is negligible, so no uncertainty is assigned
related to this item. For what concerns the transverse momentum binning, the efficiency is recomputed smearing the $p_T$ resolution with a gaussian of 20 MeV/$c$, larger than the measured value to take into account a possible worse resolution in data. The new efficiency tables are compared with the nominal ones and a 0.5% deviation is found and assigned as systematic uncertainty. The error is correlated between the analysis bins.

6.7.11 $b\bar{b}$ cross section extrapolation

The extrapolation to the LHCb acceptance and to the full acceptance uses the average branching fraction if inclusive $b$-hadrons decays to $J/\psi$ measured at LEP. The underlying assumption is that the $b$ hadronization fractions at $pp$ collisions at $\sqrt{s} = 7$ TeV are identical to those seen by the LEP experiments. However CDF recently showed that the $b$-baryons hadronization fractions could be larger at higher energy [72].

In order to estimate the systematic error due to the potential different $b$ hadronization fractions for $pp$ collisions at $\sqrt{s} = 7$ TeV, the $b$-hadron to $J/\psi$ branching fraction is calculated assuming that the central values of the fragmentation fraction are those measured at Tevatron as calculated by the Heavy Flavour Averaging Group (HFAG [71]). The relative difference between the estimates of the branching fraction at LEP energies and Tevatron energies, 2%, is taken as systematic error, which affects the extrapolation to the $b\bar{b}$ cross section to the LHCb and to the full acceptance.

6.8 Prompt cross section results

The results for the double differential cross section for the prompt $J/\psi$ are shown in Fig. 6.13 corrected for the polarization effect. The cross section values, together with the statistical, systematic and polarization uncertainties, are listed in Tabs. [B.1] [B.3] [B.5] [B.7] and [B.9] The integrated cross section for $2 < y < 4.5$ and $p_T < 14$ GeV/$c$ is

$$\sigma_{\text{prompt}}(2 < y < 4.5, p_T < 14 \text{ GeV}/c) = \left[9.49 \pm 0.04 \pm 0.53^{+0.14}_{-0.13}\right] \mu b \quad (6.19)$$

No correction for the polarization is applied in the first two $p_T$ bins. The uncertainty due to the polarization applied below 2 GeV/$c$ is computed as the difference between the fully transverse and the fully longitudinal scenario, as in [63]. The integrated cross section in the polarization analysis range, with $2 < y < 4.5$ and 2 GeV/$c < p_T < 14$ GeV/$c$ is

$$\sigma_{\text{prompt}}(2 < y < 4.5, 2 \text{ GeV}/c < p_T < 14 \text{ GeV}/c) = \left[4.90 \pm 0.01 \pm 0.27 \pm 0.14\right] \mu b \quad (6.20)$$

In both measurement the first error is statistical, the second is systematic while the third one is propagated from the polarization measurement.
Figure 6.13: Differential cross section of prompt $J/\psi$ component as function of $p_T$ and in bins of $y$. The displayed uncertainties are the statistical errors.

### 6.9 $J/\psi$ from $b$ cross section

The double differential cross section of $J/\psi$ from $b$ as function of $J/\psi$ transverse momentum in each rapidity bin is shown in Fig. 6.14. The displayed errors are only statistical. No correction is applied for the polarization on the delayed $J/\psi$, assuming it is not polarized in the laboratory frame. The values, together with the errors, are listed in Tabs. B.2, B.4, B.6, B.8 and B.10.

The integrated cross section is also evaluated in the analysis range, with $2 < y < 4.5$ and $p_T < 14$ GeV/c:

$$\sigma\text{from } b(2 < y < 4.5, p_T < 14 \text{ GeV/c}) = [1.14 \pm 0.01 \pm 0.08] \mu b$$ (6.21)

where the first and the second errors are respectively statistical and systematic.

### 6.10 $b\bar{b}$ cross section

From the cross section measurement of the $J/\psi$ from $b$, the $b\bar{b}$ cross section is extrapolated to the full solid angle in the following way:

$$\sigma(pp \to b\bar{b}X) = \alpha_{4\pi} \frac{\sigma(J/\psi\text{ from } b, p_T < 14 \text{ GeV}, 2 < y < 4.5)}{2B(b \to J/\psi X)}$$ (6.22)

where $\alpha_{4\pi} = 5.88$ is obtained from the Monte Carlo simulation, based on PYTHIA and EvtGen, and is basically the ratio between the number of $J/\psi$ from $b$ candidates in the $4\pi$ solid angle and
the number of $J/\psi$ from $b$ candidates in the analysis range ($2 < y < 4.5$ and $p_T < 14$ GeV/c).

$\mathcal{B}(b \rightarrow J/\psi X) = (1.16 \pm 0.10)\%$ is the branching fraction of a $b$-hadron decay into a $J/\psi$ plus any other particle [68, 69, 70]. The result is:

$$\sigma(pp \rightarrow bbX) = [295 \pm 4 \pm 33] \mu b$$

where the uncertainties are respectively statistical and systematic.

This measurement is in good agreement with the one obtained by the LHCb collaboration studying the semileptonic $b$ decays in $D^0\mu\nu X$ in the first 14 nb$^{-1}$ sample acquired by the experiment [73]:

$$\sigma(pp \rightarrow b\overline{b}X) = [284 \pm 20 \pm 49] \mu b$$

being the first and the second uncertainty the statistical and systematic respectively.

### 6.11 Comparison with theoretical models

A full discussion of the theoretical interpretation of the results presented in this chapter is beyond the scope of this thesis. In addition no theoretical calculation are available yet based on the present results. However, since the present results are fully consistent with the previous LHCb results published in [63], for illustration purposes the comparison of those results with the theoretical prediction is shown in this section.
6.11.1 Prompt cross section

The measurements presented in [63] have been compared with several theoretical models. On the left plot in Fig. 6.15 the cross section measurement of the unpolarized $J/\psi$ is compared with the Non Relativistic QCD prediction for the direct $J/\psi$ production (without including any contribution from heavier charmonium states) at the leading order (LO) and next to leading order (NLO) in $\alpha_s$ (respectively the orange and green uncertainty band). The predictions are obtained summing the contributions coming from the COM and the CSM in the LHCb acceptance region. On the right plot in Fig. 6.15 the same measurement is compared with the Non Relativistic QCD prediction for the prompt $J/\psi$ component, at the next to leading order, taking into account also the feed down from the $\chi_c$ state. Also in this case the NRQCD prediction, computed in the LHCb acceptance region, includes the contribution from both the COM and CSM.

The experimental results are in good agreement with the theoretical predictions.

6.11.2 $J/\psi$ from $b$ cross section

The differential cross section of the $J/\psi$ from $b$ is shown in Fig. 6.16 compared with the calculation made with the Fixed Order Next to Leading Logarithm (FONLL) [74, 75] model. Within the FONLL formalism the $b$ quark cross section is predicted including its fragmentation to the $b$-hadrons and their decays to the $J/\psi$. The calculation of the cross section is done in powers of $\alpha_s$, taking advantage of the possibility to resum terms proportional to the logarithm of the heavy quark $p_T$, at high transverse momentum. In this case terms up to the second order in $\alpha_s$ are included. The agreement between the theory and the experimental results is good.
6.12 Comparison with CMS results

The LHCb results have been compared also with the ones obtained by the CMS experiment. The comparison is shown in Fig. 6.17 for the prompt component, assuming no polarization, and for the delayed component, respectively on the left and right plot. The measurements have been compared in a common rapidity range, i.e. $2 < y < 2.5$ for LHCb and $1.6 < |y| < 2.4$ for CMS, and they show a good agreement.
Figure 6.17: Comparison of the LHCb and CMS prompt (left) and from $b$ (right) $J/\psi$ production cross-sections, as a function of $p_T$ in a common rapidity interval. The measurements are obtained assuming no $J/\psi$ polarization.
In this work a preliminary study of the charmonia production in hadronic collisions, with the LHCb detector, is presented. The study is mainly focused on the $J/\psi$ polarization and production cross section measurement with the data acquired by the experiment during the 2010 operation, with proton-proton collisions at $\sqrt{s} = 7$ TeV at the Large Hadron Collider. To have a more exhaustive view of the charmonia production the polarization and the cross section are both needed for a better comparison among the different theoretical prediction. In particular in this study the $J/\psi$ mesons have been studied, through their decays into muon pairs. The polarization has been measured using a dataset of about $\sim 25$ pb$^{-1}$ of integrated luminosity, corresponding to roughly the 60% of the full 2010 LHCb sample. It is possible to derive the polarization of a $J = 1^{--}$ state such as the $J/\psi$, from the angular distribution of the two leptons produces in its decay (in the present case the muon pairs). From the shape of this distribution the polarization parameters are extracted, using an unbinned maximum likelihood fit to the angular distribution. Two binned methods are also used to cross check the results. The measurement has been done for $2 \text{ GeV}/c < p_T < 15 \text{ GeV}/c$ and for $2 < y < 4.5$ and it is the first preliminary estimation of the polarization performed at LHC in 6 bins of $J/\psi$ transverse momentum and 5 bins of rapidity. The results obtained for the $\lambda_\theta$ parameter show a slightly longitudinal polarization, increasing with the increasing $p_T$. The $\lambda_{\theta\phi}$ and $\lambda_\phi$ parameters are both consistent with 0. The production cross section has been measured separately for the prompt component (directly produced in the $pp$ collisions or feed down by the higher charmonia states decay, such as the $\psi(2S)$ and the $\chi_c$) and the component produced in the decays of various $b$-hadrons: it is possible to refer to the latter as the delayed component, since the $J/\psi$ is produced far away from the interaction vertex. The analysis has been done over a sample of $\sim 5$ pb$^{-1}$ integrated luminosity, out of the full 2010 sample, selecting the subset taken with higher data acquisition efficiency and in the lower pile-up conditions: the inclusive cross section and differential cross section, in $J/\psi$ transverse momentum and rapidity, have been measured for $p_T < 14 \text{ GeV}/c$ and $2 < y < 4.5$. The measurement has been performed selecting the $J/\psi$ decaying into a muon pair and takes advantage of the large sample of $J/\psi$ candidates reconstructed by the LHCb experiments (more
than 560000 in the analysed $\sim 5 \text{ pb}^{-1}$ sample and almost $\sim 10^5$ reconstructed candidates per $\text{pb}^{-1}$.

The $J/\psi$ polarization introduces a distortion in the efficiency distribution, modifying the detector acceptance, on which the cross section depends. Thus the efficiency has been corrected weighting each event with the measured polarization function. The results for the prompt cross section is:

$$\sigma_{\text{prompt}}(2 < y < 4.5, 2 \text{ GeV}/c < p_T < 14 \text{ GeV}/c) = [4.90 \pm 0.01 \pm 0.27 \pm 0.14] \mu\text{b}$$

where the errors are respectively statistical, systematic and due the polarization. The latter is computed propagating the uncertainty on the polarization measurement on the cross section. This measurement supersedes the one published in [63], performed in the hypothesis of unpolarized $J/\psi$. To take the polarization effect into account a $\sim 20\%$ systematic uncertainty was added to the published cross section measurement of the prompt $J/\psi$ component. This was by far the dominant uncertainty, over the other statistical and systematic errors.

From the inclusive production cross section of the $J/\psi$ from $b$ component the inclusive $b\bar{b}$ cross section is extrapolated from the analysis range (in rapidity it roughly corresponds to the detector angular coverage) to the $4\pi$ solid angle. The results obtained for the inclusive cross section for the $J/\psi$ from $b$ and for the $b\bar{b}$ production are respectively:

$$\sigma_{\text{from } b}(2 < y < 4.5, p_T < 14 \text{ GeV}/c) = [1.14 \pm 0.01 \pm 0.16] \mu\text{b}$$

$$\sigma(pp \rightarrow b\bar{b}X) = [295 \pm 4 \pm 48] \mu\text{b}$$

where the errors are respectively statistical and systematic. The cross section results are found to be consistent with other measurements from CMS experiment and with the most accurate theoretical models.
Appendix A

Polarization results
Table A.1: Measured polarization parameters, statistical errors and systematic uncertainties. LL stat are the statistical errors returned by likelihood estimators, MC stat are the statistical errors coming from MC fluctuations, SB stat are the statistical errors coming from background subtraction, SB/Specsys are the background subtraction/unknown $J/\psi$ spectrum related systematic uncertainties, tzs sys are systematic uncertainties coming from tzs cuts. TrkSys are systematic uncertainties related to the difference of tracking efficiency between MC and data. BContamSys are systematic uncertainties coming from the remaining polarization of $J/\psi$ from b. stat sum is the quadratically added statistical errors while sys sum is the quadratically added systematic uncertainty. Resolution systematic uncertainty is so small that they are neglected. $2.0 < y < 2.5$.

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Table A.2: Measured polarization parameters, statistical errors and systematic uncertainties. LL stat are the statistical errors returned by likelihood estimators, MC stat are the statistical errors coming from MC fluctuations, SB stat are the statistical errors coming from background subtraction, SB/Specsys are the background subtraction/unknown J/ψ spectrum related systematic uncertainties, tzs sys are systematic uncertainties coming from tzs cuts. TrkSys are systematic uncertainties related to the difference of tracking efficiency between MC and data. BContamSys are systematic uncertainties coming from the remaining polarization of J/ψ from b. stat sum is the quadratically added statistical errors while sys sum is the quadratically added systematic uncertainty. Resolution systematic uncertainty is so small that they are neglected. 2.5 < y < 3.0.

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Table A.3: Measured polarization parameters, statistical errors and systematic uncertainties. LL stat are the statistical errors returned by likelihood estimators, MC stat are the statistical errors coming from MC fluctuations, SB stat are the statistical errors coming from background subtraction, SB/Specsys are the background subtraction/unknown $J/\psi$ spectrum related systematic uncertainties, tzs sys are systematic uncertainties coming from tzs cuts. TrkSys are systematic uncertainties related to the difference of tracking efficiency between MC and data. BContamSys are systematic uncertainties coming from the remaining polarization of $J/\psi$ from b. stat sum is the quadratically added statistical errors while sys sum is the quadratically added systematic uncertainty. Resolution systematic uncertainty is so small that they are neglected. $3.0 < y < 3.5$.

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Table A.5: Measured polarization parameters, statistical errors and systematic uncertainties. LL stat are the statistical errors returned by likelihood estimators, MC stat are the statistical errors coming from MC fluctuations, SB stat are the statistical errors coming from background subtraction, SB/Specsys are the background subtraction/unknown $J/\psi$ spectrum related systematic uncertainties, tzs sys are systematic uncertainties coming from tzs cuts. TrkSys are systematic uncertainties related to the difference of tracking efficiency between MC and data. BContamSys are systematic uncertainties coming from the remaining polarization of $J/\psi$ from b. stat sum is the quadratically added statistical errors while sys sum is the quadratically added systematic uncertainty. Resolution systematic uncertainty is so small that they are neglected. $4.0 < y < 4.5$.

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<th>SB sys</th>
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Appendix B

Cross section results

B.1 2.0 < y < 2.5
Table B.1: Cross section results for prompt $J/\psi$, with the statistical, systematic and polarization uncertainties, for $2 < y < 2.5$.

<table>
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<th>$p_T$ (GeV/c)</th>
<th>2 - 3</th>
<th>3 - 4</th>
<th>4 - 5</th>
<th>5 - 6</th>
<th>6 - 7</th>
<th>7 - 8</th>
<th>8 - 9</th>
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<tr>
<td>$\frac{d\sigma}{dydp_T}$ (nb/(GeV/c))</td>
<td>927.374</td>
<td>656.824</td>
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<td>15.3944</td>
<td>9.12986</td>
<td>4.97816</td>
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<td>Pol. errors (nb/(GeV/c))</td>
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<td>64.4692</td>
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Table B.2: Cross section results for $J/\psi$ from $b$, with the statistical and systematic uncertainties, for $2 < y < 2.5$.

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<th>12 - 13</th>
<th>13 - 14</th>
<th>14 - 15</th>
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<td>$\frac{d^2\sigma}{dp_T^2dy}$ (nb/(GeV/c))</td>
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<th>$p_T$ (GeV/c)</th>
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<th>9 - 10</th>
<th>10 - 11</th>
<th>11 - 12</th>
<th>12 - 13</th>
<th>13 - 14</th>
<th>14 - 15</th>
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<td>$\frac{d^2\sigma}{dp_T^2dy}$ (nb/(GeV/c))</td>
<td>0.3</td>
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<td>0.1</td>
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Table B.3: Cross section results for prompt $J/\psi$, with the statistical, systematic and polarization uncertainties, for $2.5 < y < 3$.

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<th>5 – 6</th>
<th>6 – 7</th>
<th>7 – 8</th>
<th>8 – 9</th>
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</thead>
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<td>$d\sigma/dydp_T$ (nb/(GeV/c))</td>
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<td>188.069</td>
<td>94.4364</td>
<td>51.0606</td>
<td>26.5302</td>
</tr>
<tr>
<td>Stat. errors (nb/(GeV/c))</td>
<td>7.8949</td>
<td>5.0437</td>
<td>2.9914</td>
<td>1.99353</td>
<td>1.31267</td>
<td>0.929303</td>
<td>0.623459</td>
</tr>
<tr>
<td>Pol. errors (nb/(GeV/c))</td>
<td>88.4222</td>
<td>46.0365</td>
<td>23.8504</td>
<td>11.4596</td>
<td>5.31234</td>
<td>3.18367</td>
<td>1.53608</td>
</tr>
<tr>
<td>Sys. errors (nb/(GeV/c))</td>
<td>99.3016</td>
<td>60.8835</td>
<td>34.0109</td>
<td>18.6511</td>
<td>9.4691</td>
<td>5.31104</td>
<td>2.59549</td>
</tr>
<tr>
<td>$p_T$ (GeV/c)</td>
<td>9 – 10</td>
<td>10 – 11</td>
<td>11 – 12</td>
<td>12 – 13</td>
<td>13 – 14</td>
<td>14 – 15</td>
<td></td>
</tr>
<tr>
<td>$d\sigma/dydp_T$ (nb/(GeV/c))</td>
<td>15.4619</td>
<td>10.5301</td>
<td>5.70276</td>
<td>3.49345</td>
<td>2.00366</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Stat. errors (nb/(GeV/c))</td>
<td>0.623115</td>
<td>0.542301</td>
<td>0.380374</td>
<td>0.262008</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Pol. errors (nb/(GeV/c))</td>
<td>0.833551</td>
<td>0.865986</td>
<td>0.452608</td>
<td>0.261797</td>
<td>0.138922</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Sys. errors (nb/(GeV/c))</td>
<td>1.51266</td>
<td>1.03018</td>
<td>0.55791</td>
<td>0.341769</td>
<td>0.196021</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**B.2 2.5 < y < 3**

Table B.4: Cross section results for $J/\psi$ from $b$, with the statistical and systematic uncertainties, for $2.5 < y < 3$.

<table>
<thead>
<tr>
<th>$p_T$ (GeV/c)</th>
<th>0 – 1</th>
<th>1 – 2</th>
<th>2 – 3</th>
<th>3 – 4</th>
<th>4 – 5</th>
<th>5 – 6</th>
<th>6 – 7</th>
<th>7 – 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d^2\sigma/d\eta dp_T$ (nb/(GeV/c))</td>
<td>75.3</td>
<td>147.4</td>
<td>139.9</td>
<td>98.3</td>
<td>57.0</td>
<td>35.3</td>
<td>22.1</td>
<td>12.1</td>
</tr>
<tr>
<td>stat. error (nb/(GeV/c))</td>
<td>4.0</td>
<td>3.8</td>
<td>3.0</td>
<td>2.1</td>
<td>1.3</td>
<td>1.0</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>cor. syst. error (nb/(GeV/c))</td>
<td>10.4</td>
<td>20.4</td>
<td>19.3</td>
<td>13.6</td>
<td>7.9</td>
<td>4.9</td>
<td>3.0</td>
<td>1.7</td>
</tr>
<tr>
<td>uncor. syst. error (nb/(GeV/c))</td>
<td>11.8</td>
<td>3.9</td>
<td>2.8</td>
<td>2.5</td>
<td>1.7</td>
<td>1.3</td>
<td>0.9</td>
<td>0.6</td>
</tr>
<tr>
<td>total error (nb/(GeV/c))</td>
<td>16.3</td>
<td>21.1</td>
<td>19.8</td>
<td>14.0</td>
<td>8.2</td>
<td>5.1</td>
<td>3.2</td>
<td>1.8</td>
</tr>
<tr>
<td>$p_T$ (GeV/c)</td>
<td>8 – 9</td>
<td>9 – 10</td>
<td>10 – 11</td>
<td>11 – 12</td>
<td>12 – 13</td>
<td>13 – 14</td>
<td>14 – 15</td>
<td></td>
</tr>
<tr>
<td>$d^2\sigma/d\eta dp_T$ (nb/(GeV/c))</td>
<td>8.2</td>
<td>5.2</td>
<td>3.2</td>
<td>2.2</td>
<td>1.6</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>stat. error (nb/(GeV/c))</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cor. syst. error (nb/(GeV/c))</td>
<td>1.1</td>
<td>0.7</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>uncor. syst. error (nb/(GeV/c))</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>total error (nb)</td>
<td>1.2</td>
<td>0.8</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B.5: Cross section results for prompt $J/\psi$, with the statistical, systematic and polarization uncertainties, for $3 < y < 3.5$.

<table>
<thead>
<tr>
<th>$3.0 &lt; y &lt; 3.5$</th>
<th>$p_T$ (GeV/c)</th>
<th>2 – 3</th>
<th>3 – 4</th>
<th>4 – 5</th>
<th>5 – 6</th>
<th>6 – 7</th>
<th>7 – 8</th>
<th>8 – 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\sigma}{dydp_T}$ (nb/(GeV/c))</td>
<td>949.696</td>
<td>561.157</td>
<td>296.244</td>
<td>156.198</td>
<td>76.130</td>
<td>40.9116</td>
<td>21.7749</td>
<td></td>
</tr>
<tr>
<td>Stat. errors (nb/(GeV/c))</td>
<td>5.98308</td>
<td>4.15256</td>
<td>2.51808</td>
<td>1.78066</td>
<td>1.13434</td>
<td>0.805959</td>
<td>0.574857</td>
<td></td>
</tr>
<tr>
<td>Pol. errors (nb/(GeV/c))</td>
<td>57.0182</td>
<td>31.7508</td>
<td>15.8407</td>
<td>8.39495</td>
<td>3.87416</td>
<td>2.35049</td>
<td>1.19325</td>
<td></td>
</tr>
<tr>
<td>Sys. errors (nb/(GeV/c))</td>
<td>89.5891</td>
<td>53.3866</td>
<td>28.6286</td>
<td>15.2811</td>
<td>7.60502</td>
<td>4.0156</td>
<td>2.13727</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_T$ (GeV/c)</th>
<th>9 – 10</th>
<th>10 – 11</th>
<th>11 – 12</th>
<th>12 – 13</th>
<th>13 – 14</th>
<th>14 – 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\sigma}{dydp_T}$ (nb/(GeV/c))</td>
<td>11.2638</td>
<td>6.88602</td>
<td>4.05288</td>
<td>2.63716</td>
<td>1.2832</td>
<td>0</td>
</tr>
<tr>
<td>Stat. errors (nb/(GeV/c))</td>
<td>0.504617</td>
<td>0.394569</td>
<td>0.28208</td>
<td>0.295626</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pol. errors (nb/(GeV/c))</td>
<td>0.57973</td>
<td>0.520262</td>
<td>0.260476</td>
<td>0.196074</td>
<td>0.0951032</td>
<td>0</td>
</tr>
<tr>
<td>Sys. errors (nb/(GeV/c))</td>
<td>1.10557</td>
<td>0.675883</td>
<td>0.397802</td>
<td>0.258845</td>
<td>0.12595</td>
<td>0</td>
</tr>
</tbody>
</table>

B.3 $3 < y < 3.5$

Table B.6: Cross section results for $J/\psi$ from $b$, with the statistical and systematic uncertainties, for $3 < y < 3.5$.

<table>
<thead>
<tr>
<th>$3.0 &lt; y &lt; 3.5$</th>
<th>$p_T$ (GeV/c)</th>
<th>0 – 1</th>
<th>1 – 2</th>
<th>2 – 3</th>
<th>3 – 4</th>
<th>4 – 5</th>
<th>5 – 6</th>
<th>6 – 7</th>
<th>7 – 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d^2\sigma}{dp_Tdy}$ (nb/(GeV/c))</td>
<td>60.4</td>
<td>122.5</td>
<td>112.7</td>
<td>75.3</td>
<td>43.7</td>
<td>26.1</td>
<td>14.9</td>
<td>9.6</td>
<td></td>
</tr>
<tr>
<td>stat. error (nb/(GeV/c))</td>
<td>2.3</td>
<td>2.6</td>
<td>2.2</td>
<td>1.6</td>
<td>1.1</td>
<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>cor. syst. error (nb/(GeV/c))</td>
<td>8.4</td>
<td>16.9</td>
<td>15.6</td>
<td>10.4</td>
<td>6.0</td>
<td>3.6</td>
<td>2.1</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>uncor. syst. error (nb/(GeV/c))</td>
<td>3.7</td>
<td>2.4</td>
<td>2.0</td>
<td>1.7</td>
<td>1.2</td>
<td>0.8</td>
<td>0.6</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>total error (nb)</td>
<td>9.4</td>
<td>17.3</td>
<td>15.9</td>
<td>10.7</td>
<td>6.3</td>
<td>3.8</td>
<td>2.2</td>
<td>1.4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_T$ (GeV/c)</th>
<th>8 – 9</th>
<th>9 – 10</th>
<th>10 – 11</th>
<th>11 – 12</th>
<th>12 – 13</th>
<th>13 – 14</th>
<th>14 – 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d^2\sigma}{dp_Tdy}$ (nb/(GeV/c))</td>
<td>5.3</td>
<td>3.4</td>
<td>2.0</td>
<td>1.5</td>
<td>0.9</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>stat. error (nb/(GeV/c))</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>cor. syst. error (nb/(GeV/c))</td>
<td>0.7</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>uncor. syst. error (nb/(GeV/c))</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>total error (nb/(GeV/c))</td>
<td>0.8</td>
<td>0.5</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>
Table B.7: Cross section results for prompt $J/\psi$, with the statistical, systematic and polarization uncertainties, for $3.5 < y < 4$.

<table>
<thead>
<tr>
<th>$3.5 &lt; y &lt; 4.0$</th>
<th>$p_T$ (GeV/c)</th>
<th>2 – 3</th>
<th>3 – 4</th>
<th>4 – 5</th>
<th>5 – 6</th>
<th>6 – 7</th>
<th>7 – 8</th>
<th>8 – 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\sigma}{dydp_T}$ (nb/(GeV/c))</td>
<td>781.492</td>
<td>453.059</td>
<td>228.024</td>
<td>111.066</td>
<td>56.298</td>
<td>26.7951</td>
<td>14.2502</td>
<td></td>
</tr>
<tr>
<td>Stat. errors (nb/(GeV/c))</td>
<td>5.47045</td>
<td>3.66978</td>
<td>2.39425</td>
<td>1.49939</td>
<td>1.02462</td>
<td>0.661839</td>
<td>0.464557</td>
<td></td>
</tr>
<tr>
<td>Pol. errors (nb/(GeV/c))</td>
<td>54.3711</td>
<td>28.0369</td>
<td>15.1677</td>
<td>6.6183</td>
<td>3.14903</td>
<td>1.79771</td>
<td>0.883083</td>
<td></td>
</tr>
<tr>
<td>Sys. errors (nb/(GeV/c))</td>
<td>74.525</td>
<td>43.783</td>
<td>22.2367</td>
<td>11.3124</td>
<td>5.52581</td>
<td>2.63002</td>
<td>1.3987</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_T$ (GeV/c)</th>
<th>9 – 10</th>
<th>10 – 11</th>
<th>11 – 12</th>
<th>12 – 13</th>
<th>13 – 14</th>
<th>14 – 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\sigma}{dydp_T}$ (nb/(GeV/c))</td>
<td>8.17068</td>
<td>4.67649</td>
<td>2.79174</td>
<td>1.14429</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Stat. errors (nb/(GeV/c))</td>
<td>0.474716</td>
<td>0.357752</td>
<td>0.317142</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pol. errors (nb/(GeV/c))</td>
<td>0.490303</td>
<td>0.349213</td>
<td>0.227571</td>
<td>0.0804964</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sys. errors (nb/(GeV/c))</td>
<td>0.801976</td>
<td>0.459012</td>
<td>0.274018</td>
<td>0.112315</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table B.8: Cross section results for $J/\psi$ from $b$, with the statistical and systematic uncertainties, for $3.5 < y < 4$.

<table>
<thead>
<tr>
<th>$3.5 &lt; y &lt; 4.0$</th>
<th>$p_T$ (GeV/c)</th>
<th>0 – 1</th>
<th>1 – 2</th>
<th>2 – 3</th>
<th>3 – 4</th>
<th>4 – 5</th>
<th>5 – 6</th>
<th>6 – 7</th>
<th>7 – 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d^2\sigma}{dydp_T}$ (nb/(GeV/c))</td>
<td>41.0</td>
<td>81.5</td>
<td>71.4</td>
<td>48.4</td>
<td>27.6</td>
<td>15.6</td>
<td>8.6</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>stat. error (nb/(GeV/c))</td>
<td>1.9</td>
<td>2.2</td>
<td>1.9</td>
<td>1.4</td>
<td>1.0</td>
<td>0.7</td>
<td>0.4</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>cor. syst. error (nb/(GeV/c))</td>
<td>5.7</td>
<td>11.3</td>
<td>9.9</td>
<td>6.7</td>
<td>3.8</td>
<td>2.3</td>
<td>1.2</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>uncor. syst. error (nb/(GeV/c))</td>
<td>1.5</td>
<td>1.7</td>
<td>1.6</td>
<td>1.3</td>
<td>0.8</td>
<td>0.7</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>total error (nb/(GeV/c))</td>
<td>6.2</td>
<td>11.6</td>
<td>10.2</td>
<td>7.0</td>
<td>4.0</td>
<td>2.3</td>
<td>1.3</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_T$ (GeV/c)</th>
<th>8 – 9</th>
<th>9 – 10</th>
<th>10 – 11</th>
<th>11 – 12</th>
<th>12 – 13</th>
<th>13 – 14</th>
<th>14 – 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d^2\sigma}{dydp_T}$ (nb/(GeV/c))</td>
<td>3.2</td>
<td>1.8</td>
<td>1.2</td>
<td>0.6</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>stat. error (nb/(GeV/c))</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cor. syst. error (nb/(GeV/c))</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>uncor. syst. error (nb/(GeV/c))</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>total error (nb/(GeV/c))</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B.9: Cross section results for prompt $J/\psi$, with the statistical, systematic and polarization uncertainties, for $4 < y < 4.5$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
$p_T$ (GeV/c) & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
$d\sigma/dydp_T$ (nb/(GeV/c)) & 587.631 & 303.025 & 159.711 & 73.0094 & 30.7834 & 15.1902 & 8.13827 & \\
Stat. errors (nb/(GeV/c)) & 5.81755 & 4.03024 & 2.66717 & 1.62811 & 0.963519 & 0.624318 & 0.445977 & \\
Pol. errors (nb/(GeV/c)) & 66.185 & 38.8477 & 20.1859 & 7.40642 & 2.76549 & 1.6475 & 0.851499 & \\
Sys. errors (nb/(GeV/c)) & 57.872 & 30.6191 & 15.5749 & 7.11982 & 3.00197 & 1.48134 & 0.793638 & \\
\hline
\end{tabular}
\end{table}

B.5 $4 < y < 4.5$

Table B.10: Cross section results for $J/\psi$ from $b$, with the statistical and systematic uncertainties, for $4 < y < 4.5$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
$p_T$ (GeV/c) & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\hline
$d\sigma/dydp_T$ (nb/(GeV/c)) & 3.52674 & 1.73797 & 0 & 0 & 0 & 0 & \\
Stat. errors (nb/(GeV/c)) & 0.355848 & 0 & 0 & 0 & 0 & 0 & \\
Pol. errors (nb/(GeV/c)) & 0.36551 & 0.230705 & 0 & 0 & 0 & 0 & \\
Sys. errors (nb/(GeV/c)) & 0.343925 & 0.169486 & 0 & 0 & 0 & 0 & \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$p_T$ (GeV/c) & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
$d^2\sigma/dp_Tdy$ (nb/(GeV/c)) & 22.4 & 52.4 & 42.3 & 27.8 & 15.0 & 9.0 & 5.2 & 2.8 & \\
Stat. error (nb/(GeV/c)) & 1.7 & 2.2 & 1.9 & 1.4 & 1.0 & 0.7 & 0.5 & 0.3 & \\
Cor. syst. error (nb/(GeV/c)) & 3.1 & 7.2 & 5.8 & 3.8 & 2.1 & 1.2 & 0.7 & 0.4 & \\
Uncor. syst. error (nb/(GeV/c)) & 1.4 & 1.8 & 1.6 & 1.2 & 0.6 & 0.3 & 0.2 & 0.1 & \\
Total error (nb/(GeV/c)) & 3.8 & 7.8 & 6.4 & 4.3 & 2.4 & 1.5 & 0.9 & 0.5 & \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
$p_T$ (GeV/c) & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\hline
$d^2\sigma/dp_Tdy$ (nb/(GeV/c)) & 1.5 & 0.8 & 0.5 & \\
Stat. error (nb/(GeV/c)) & 0.2 & 0.2 & 0.1 & \\
Cor. syst. error (nb/(GeV/c)) & 0.2 & 0.1 & 0.1 & \\
Uncor. syst. error (nb/(GeV/c)) & 0.1 & 0.0 & 0.0 & \\
Total error (nb/(GeV/c)) & 0.3 & 0.2 & 0.1 & \\
\hline
\end{tabular}
\end{table}
Appendix C

The sPlot technique

C.1 The sPlot formalism

The sPlot \cite{76} is a statistical technique derived directly from the Maximum likelihood method and extended to the case in which different sources give a contribution to the data sample to be analysed.

The events will be characterized by a certain number of variables, that can be divided in two categories: discriminant variables and control variables. The first category contains the variables whose distributions are known for all the sources that contribute to the data sample. Variables belonging to the second categories have instead unknown distributions.

The sPlot technique allows to build the distributions of the control variable independently for each sources, just starting from the knowledge of the discriminant variables distributions. The technique rely on the fact that the two sets of variables are completely uncorrelated.

The procedure is applied as follows. The maximum likelihood function is built for a dataset populated by various sources:

\[
L = \sum_{e=1}^{N} \ln \left\{ \sum_{i=1}^{N_s} N_i f_i(y_e) \right\} - \sum_{i=1}^{N_s} N_i
\]  

(C.1)

where \(N\) is the total number of events, \(N_s\) is the number of sources giving contribution to the data sample, \(N_i\) is the expected number of events for the \(i\)-th source, \(f_i(y_e)\) is the probability density function for the discriminant variable \(y\), for the \(e\) event and the \(i\)-th source. \(L\) is a function of \(N_s\) yield \(N_i\). The function \(L\) is maximized performing a fit to the distributions of the discriminant variables \(y\) with respect to \(N_i\), which is the expected number of events in each source.

A weight (sWeight) is computed in the following way:

\[
sP_{\omega}(y_e) = \frac{\sum_{j=1}^{N_s} V_{nj} f_j(y_e)}{\sum_{k=1}^{N_s} N_k f_k(y_e)}
\]  

(C.2)
where $V_{n,j}$ is the covariance matrix calculated by maximizing the $\mathcal{L}$ function. The weight $sP_{n}(y_e)$ is assigned to each event in the dataset and for each source. From the definition it is possible to derive the following normalization rules for the $s$Weights:

$$\sum_{e=1}^{N} sP_{n}(y_e) = N_n \sum_{l=1}^{N_n} sP_{n}(y_e) = 1$$ (C.3)

where $N_n$ is the expected number of events for the $n$-th source.

The distribution of the control variable $y$ is built for a particular source $n$ weighting each event with its own weight $sP_{n}(y_e)$. In this way the distributions of each control variable is built for each source, knowing just the discriminant variables.

### C.2 Signal selection with sPlot

The sPlot can be used in the particular case in which the dataset is populated by two sources, as in the present analysis: the signal and the background. Here the discriminant value can be chosen to be the invariant mass of the muons pairs, relying on the fact that its distribution is well known for both the signal and the background sources.

The $M(\mu^+\mu^-)$ distribution is fitted a first time as described in 4.3, using a Crystal Ball function for the signal peak and a first order polynomial for the background. This first fit provides the parameters of the functions used to model the signal and the background.

A second fit is performed fixing all the parameters but the number of signal and background events. This second fit provides the covariance matrix, used to calculate the weights for each event, as defined in (C.2).

Finally the distributions of interesting variables for the signal or background events are calculated for the signal or the background source, weighting each event with its own weight.

A validation of this technique can be obtained by comparing the background distribution obtained with the sPlot and the one built from the sideband events of the mass distribution. In Fig. C.1 the background distribution of the pseudo proper time $t_z$ obtained with the sPlot technique is shown, compared with one built with the sideband events of the invariant mass distribution. As it can be seen from the figure the sPlot background is well reproduced by the sideband distribution.
Figure C.1: Background distribution for the pseudo proper time, built using the sPlot technique (black circles) and the events taken from the sidebands in the invariant mass distribution (blue triangles).
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